How many different ways can you find to solve
"A number plus its square = …?"

This activity worked particularly well for me as an introductory/induction activity for sixth form students in a variety of situations because it was accessible to everyone and encouraged sharing ideas and collaborative work as well as further research and investigation. In the past students were encouraged to use a variety of different forms of ICT such as spreadsheets, short programs, graphical calculators etc. This activity could now be tackled using just a variety of TI-Nspire pages and the sharing aspect would work well with Navigator. **Linda Tetlow June 2010**

Introduction

How many different ways can you find to solve "A number plus its square = …?" (for example 16)
Change part of the problem; how does this affect your choice of methods?

Classroom potential

- This activity is easily accessible and can be used with a wide range of students of varying backgrounds and aptitudes (both in mathematics and ICT).
- It lends itself to cooperative group work, sharing ideas and presenting different methods to the class.
- It has worked particularly well as a starting activity for 6^{th} form/induction classes as it provides an opportunity to refresh memories on previous work after the long summer break and encourages sharing ideas and strategies between students with different backgrounds and experience. It can also introduce future work possibilities such as iterative methods, complex numbers etc.

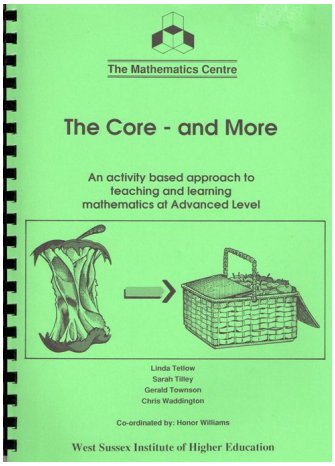
Rationale

- Encourages research from textbooks.
- Encourages cooperative group work.
- Encourages looking for alternative methods of solution and comparing and evaluating them.
- Possibilities for presentations on different methods by students.
- Graphical methods are possible encouraging the use of graphing packages/ graphical calculators etc.
- Spreadsheet and short program use are also encouraged.
- Provides good opportunities for covering a range of Core A level mathematical content and extending into discrete procedures, limiting processes, proof and complex numbers.
- Trial and improvement and iterative methods will be necessary and can be developed further if students are encouraged to extend the activity to equations in higher powers.

Possible content

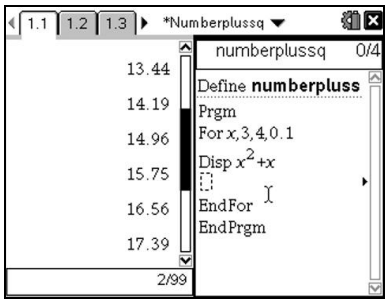
Solution by factorising (for certain numbers), completing the square, quadratic equation formula, various graphical methods, iterative methods including Newton-Raphson, convergence and divergence, cobweb and staircase diagram.
Proof of quadratic equation formula and other results
Complex numbers
Factors and factor theorem
Properties of roots and of quadratic and higher order equations

Background information

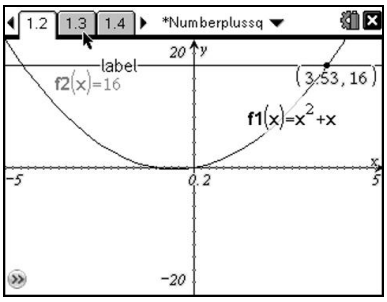


Between 1988 and 1995 a group of schools operated a revolutionary A level course linked to the Raising Achievement in Mathematics Project (RAMP) and the University of Chichester (then the West Sussex Institute of Higher Education).
The course was assessed by a portfolio of student's work which had to meet a range of stringent assessment criteria that included providing evidence of covering the content of the Interboard Common Core. Demonstrating coverage of the Core was the student's responsibility.
A group of teachers from the schools put together a booklet of activities that had worked for them entitled "The Core - and More". These activities have been found very useful by teachers operating standard A level courses and the booklet is still available from the Maths Centre at Chichester University.
This activity was used extensively during the RAMP A level, but was also successful later for a traditional A level course in an FE college where classes contained students from a wide range of previous institutions. The activity can also be used with GCSE students.

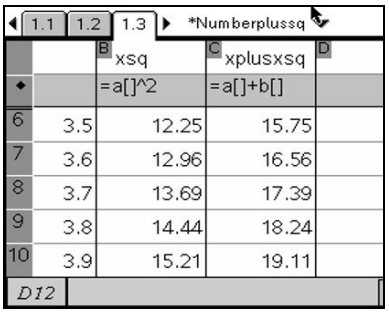
A few possibilities using TI-Nspire



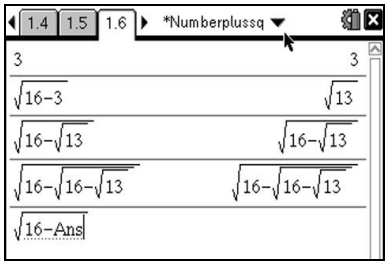
Programming using a 'For Next loop' to do a decimal search, homing in on 16. (Formerly done on a TI 85 or BBC B.)
How can the program be changed to get closer to the solution?
Programs could also be used for iterative methods.



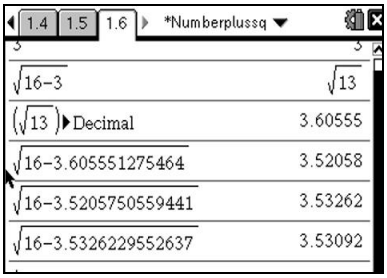
There are lots of possible graphing solutions using different equations and different ways in which TI-Nspire can find the x coordinates of the intersection points.
This example uses Trace with two equations.



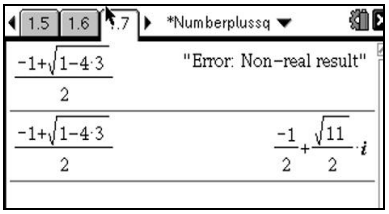
A spreadsheet can also be used. In this case for a decimal search but it could also be used for a variety of iterative solutions.
Iterations could be plotted to produce cobweb or staircase diagrams.



The two screens above and right show variations on an iterative routine with 3 as a first approximation.
A Calculator page has been used with repeated use of $\sqrt{(16\text{-ans})}$
(This may work better using the handheld or emulator)



How many different iterative methods are possible? Which converge to a solution? Does this work for *both* solutions of the equation?



If the number in the equation is changed to a negative one different results can be obtained depending on the number settings (real/ rectangular/ polar). This could lead to interesting discussions.