

Volumes of Revolution

Teacher Notes

Introduction

The aim of this activity is to enable students to explore the volume generated by rotating a curve around the x-axis. The task will enable students to explore the problem visually and will lead them through the process of deriving the standard formula for finding the volume of the generated shape.

Students explore how the volume can be approximated by considering a series of slices, and approximating each slice as a cylinder. Students will briefly look at how the approximate volume can be made more accurate by increasing the number of slices and how, taking the limit as we increase the number of slice to infinity, we can replace the sum of these slices with an integral. Finally students will examine under what circumstance this approximation is an overestimate of the true volume and under which it is an underestimate.

Resources

This activity is made up of a tns file and these associated Teacher Notes. Student notes are provided within the tns file.

Skills required

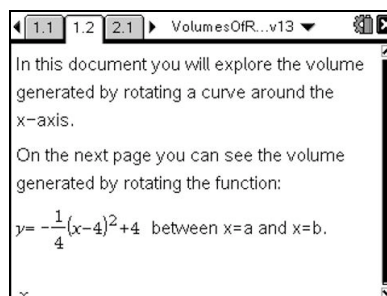
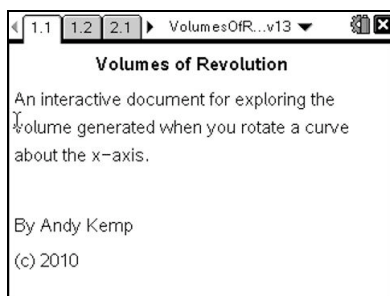
Students should have an appreciation of basic integration and the volume of a cylinder.

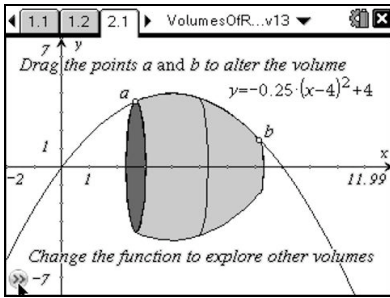
Students only need basic skills related to TI-Nspire document manipulation. They need to be able to move between pages of the document (⌘▶ and ⌘◀) and to check their answers to questions (⌘2).

The activity

The activity can be used either with students working individually on TI-Nspire handhelds or as a teacher-led discussion with the TI-Nspire software projected onto a screen.

Below the various sections of the tns file are explained and indications given of how best to use the document.



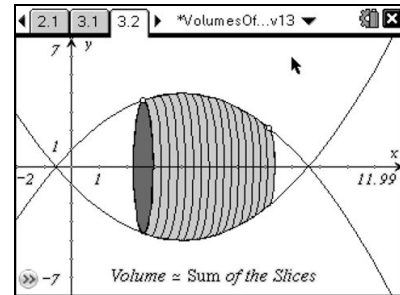


On page 2.1 students are encouraged to explore the volume generated by adjusting the bounds a and b by dragging the open circles. They can also explore different functions by adjusting the function $f_1(x)$ either by editing the displayed label or by pressing $\text{ctrl} + \text{G}$ and adjusting the function.

On the following pages students are encouraged to consider the effect of slicing up the volume and considering each slice as a cylinder.

How can we find the volume of this generated shape? Imagine dividing the shape into a series of slices and approximating the volume of each slice by a cylinder whose radius is given by the y -coordinate at the left end of the interval.

Explore the diagram on the next page then look at the question that follows.



On page 3.3 students are encouraged to reflect upon their exploration by finding the volume for one of the slices. They can check their response by pressing $\text{menu} + 2$.

If there are 16 slices between a and b and the radius is of the first slice is given by y_1 , what is the volume of the first slice?

Student types answer here

If there are 16 slices between a and b and the radius is of the first slice is given by y_1 , what is the volume of the first slice?

$\pi * y_1^2 * (a-b)/16$

Suggested Response:

$Vol = \pi * y_1^2 * \frac{b-a}{16}$

The next section leads the students through the process of moving from the approximate volume to the exact volume by considering the limit of the sum of the slices.

Adding together the volume of the 16 slices would obviously give us an approximation of the actual volume, but how could this be used to find the exact volume?

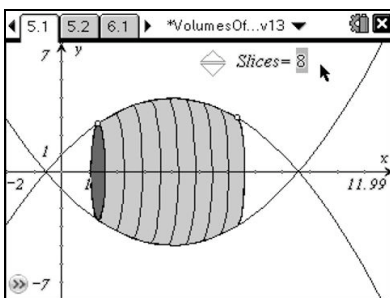
Volume:

We have seen that we can approximate the area by a series of slices. Assuming that we divide the shape into n slices the volume would be given by:

$$Volume = \pi \cdot \frac{b-a}{n} \sum_{i=1}^n (y_i^2)$$

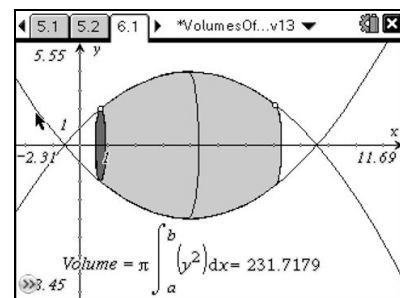
Now consider the limit as the number of slices is increased. The sum gets closer to the actual volume.

You can see this by reducing the interval $[a,b]$ on page 5.1 and seeing how the cylinders would more accurately represent the volume or by adjusting the number of slices.



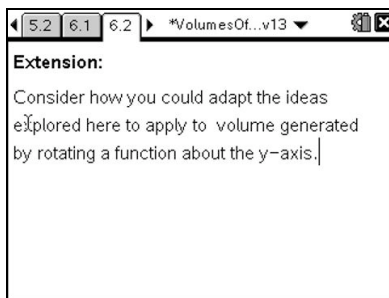
So by taking the limit as $n \rightarrow \infty$ we can replace the summation with the integral:

$$Volume = \lim_{n \rightarrow \infty} \left[\pi \cdot \frac{b-a}{n} \sum_{i=1}^n (y_i^2) \right]$$

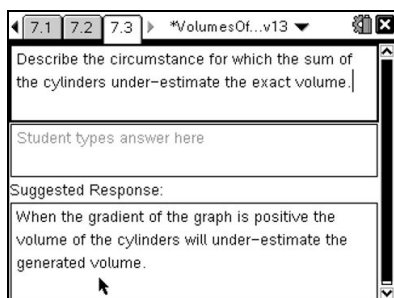
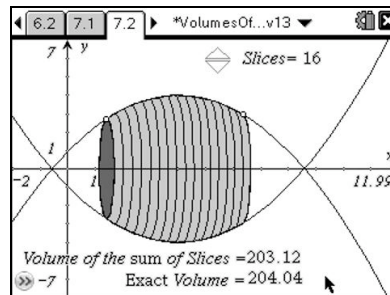
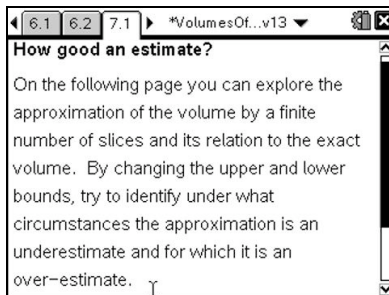
$$= \pi \int_a^b (y^2) dx$$


Students are then encouraged to consider how this approach could be

adapted to consider the volume when rotated around the y-axis.



In this final section students are asked to consider under which circumstances the approximate volume generated by the cylinder would be an overestimate or underestimate.



Students are then encouraged to explain the findings and can check their interpretation by pressing (menu) (2).

Additional Information:

All of the diagrams are fully dynamic and you could use them to explore the volume generated for various different functions by changing the function f_1 and by changing the limits a and b . If you wish to change these limits to a exact values you can display their coordinates and then edit the x-coordinates. This can be done pressing (menu) (1) (7) and moving over the point. Double click and press (esc). Finally double click on the x-value and type in your desired value.