
TI InterActive!™ in the Classroom

Sample activities for teaching and learning mathematics and science

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Preface

By the end of 2002, the T3 Europe project supported the foundation of so-called T3 Interest Groups. T3 Europe (www.T3ww.org/wweurope.htm) is the European project 'Teachers Teaching with Technology'. This project aims to help teachers integrate technology into their teaching. T3 Europe is funded by Texas Instruments.

A 'T3 Europe Interest Group' is a small group of interested educators with a focus on a specific kind of technology. Our group considered TI InterActive! to be highly relevant and interesting software with fascinating potential and so we had the idea to devote one of the Interest Groups to the development of TI InterActive! worksheets for use in the classroom. 'We' means a group of six educators – whose biographical data are presented in appendix B – from different European countries. During the year 2003, the group met twice for two-day sessions to discuss the focus of our work and the materials under construction. The result of the work is now in your hands. Of course, the group was supported by others. We thank Texas Instruments and particularly Hubert Colombat and Koen Stulens for their help, as well as Lynda Ball for the language improvements she contributed.

It goes without saying that efficient working in such an international group is far from easy. Each of us spoke a different language, had a different background, came from a different educational culture and had specific pedagogical preferences. Looking back we can say that participation in such a group was an interesting and enriching experience, which led to an increased mutual understanding and to a convergence of ideas. As a result, we hope that the materials presented here form a coherent collection, which may inspire both teachers and students to discover the potential of TI InterActive! software for the learning of mathematics and science!

Paul Drijvers
Chair

Rationale of the materials

Why TI InterActive! ?

Why develop teaching materials in the TI InterActive! environment? As we see it, TI InterActive! has several characteristics that are powerful for educational purposes. TI InterActive! integrates different kinds of functionality, for which one needed different software packages in the past. We mention the combination of text editing software (cf. Word), graphing software (cf. graphing calculators), computer algebra systems (cf. Derive, Maple), spreadsheet software (cf. Excel), and an Internet browser (cf. Internet Explorer). On the one hand TI InterActive! is the flexible integration of these features within one single environment, which is powerful. On the other hand, each of these software specific applications provides the full range of options which one would expect. For example, the text editing features of TI InterActive! include importing pictures and exporting files to Word or html formats. The computer algebra module covers the whole range of algebraic options that are usually found in computer algebra systems. Spreadsheet files can be read with Excel. Furthermore, TI InterActive! offers a programming language and the ability to connect to data loggers such as the CBL 2 and CBR, as well as Texas Instruments graphing calculators. An additional advantage is that the TI InterActive! interface is compatible to standards set by Windows and Office software. For mathematical operations the commands are similar to those for graphing and symbolic handheld devices such as the TI-83 Plus and Voyage 200.

The integration of general features such as text editing and a browser, with mathematical features for graphing and algebra allows TI InterActive! to be used as a learning environment for students. Tasks can be delivered in TI InterActive!; meanwhile, students can work on such tasks within the same environment, using it as their notebook, which they may hand in to be assessed by their teacher.

How to use TI InterActive! ?

How can we capitalize on the potentially powerful features of the TI InterActive! software environment? This, in fact, is the main challenge of our work. We have already indicated that TI InterActive! may be used as both an environment to deliver tasks, say, as the students' textbook, and also as the students' notebook. Because many teachers don't want students to write in their textbooks, and because we wanted to maintain a clear distinction between the text presented to the students and their elaboration of it, we decided to have separated notebook files, in which the students work. These notebook files are the only files that need to be handed in to be evaluated by the teacher. In these files, students are required to present their mathematical calculations (often in the form of formulas, graphs and tables) and their explanations, reflections and arguments, usually in natural language.

As far as 'textbook files' are concerned, we used the TI InterActive! hyperlink option to structure the materials so that students can follow their personal learning trajectory through the materials. This contrasts with the linear, sequential format of hardcopy materials. We hoped that this hyperlink organization of the texts would help students to keep track of the main issues, to get more information or help if needed and to advance more quickly through material if possible.

For mathematical purposes, TI InterActive! acts in two ways. First, it can be used as a 'mathematical assistant' to carry out operations that students could do by themselves, but which would be time consuming and the source of calculation errors. Secondly, TI InterActive! is an exploration environment in which 'mathematical experiments' can be carried out. The results of such experiments may enhance students' thinking and foster mathematical insight.

As far as physics, chemistry and biology are concerned, TI InterActive! is used to support data collection with data loggers. After the experimental phase TI InterActive!'s mathematical and statistical features can be used for data analysis.

Although TI InterActive! 1.2 was recently released, we decided to use version 1.1 in order to make materials available to those who did not upgrade to version 1.2. However all materials can be used in TI InterActive! 1.2 as well.

How to use this collection?

This collection of materials consists of two parts. Part I contains student units, which are ready for use in the classroom. Each of these units is comprised of a number of TI InterActive! files which are connected by hyperlinks so that students can find their way through. A teacher guide is provided to assist the teacher with lesson preparation. The units cover a range of subjects and levels. The table of contents provides an overview and the teacher guides contain more detailed information. To use Part I materials please read the installation instructions in the introduction to Part I.

Part II contains teaching ideas which are less elaborated than the Part I student units. Most of these ideas can be used as an inspiration source for teaching, but may need some customizing for your educational situation. Again, the table of contents provides an overview. Please read the introduction to Part II before using one of the teaching ideas.

Part 1: Student units

Part 1 contains student units, which are ready for use in the classroom. Each of these units consists of a number of TI InterActive! files, which are connected by hyperlinks, so that students can find their way through. For the hyperlinks to work, however, it is necessary that all the files of one unit are in the same folder on disk C of the computer. The name of the folder can be found in the table of contents on the CD, as well as in the Teacher guides files.

There are two ways to install a unit into the right folder. The first way is to use the executable zip files, which can be recognized by the extension 'exe'. If you double-click on such a file, all the files of the corresponding unit will be installed in the folder with the correct name. This is the easiest way to install a unit. However, some systems don't allow for downloading executable files. In that case, the second way of installing is to download the 'ordinary' zip files, which can be recognized by the extension 'zip'. Each of the zip files then needs to be extracted into the folder. The user has to define the folder name according to the name provided in the table of contents.

After installation, we recommend that the teacher reads the 'Teacher Guide' in the specified folder. This is a Word document, which aims to explain the main ideas of the unit, and to make the teacher's preparation as efficient as possible. For the student, the starting point is the TI InterActive! notebook file in the folder. This notebook file contains hyperlinks to the task files and other materials. It is the student's notebook, which is the file to hand in. The other files of the unit ideally are not changed by the student. Of course, students can copy expressions, graphs or text from 'textbook files' into their personal notebook files. Units 3 and 4 are exceptions in the sense that these have an overarching starting file, which links to different subunits, with a subfolder and a notebook file for each of the subunits.

There are different kinds of textbook files, such as 'Main Task', 'Help', 'Hint', 'Control Task', 'Additional Task' and 'Solution' files. The teacher should decide if a hyperlink to the 'Solution' file is appropriate. Eventually, it may be deleted. Unit 3 also contains video animation files which can be recognized by the extension smc.

1. Introduction TI InterActive!

Goal of the task

The goal of this unit is to help students become familiar with the TI InterActive! environment. They should learn to find their way in this software package and be able to use the built-in help features to use the program independent of the teacher.

The unit is split up into five main tasks:

- A TI InterActive! as a word processor
- B TI InterActive! as a calculator
- C TI InterActive! as a graphing environment
- D TI InterActive! as an algebra environment
- E TI InterActive! as a function investigator

A solution file is provided. If the teacher wants to restrict students' access to the solution file then the hyperlinks need to be removed.

Target group and required time

There is no clearly defined target group for this unit, however the main tasks A and B, parts of C and the first part of D are recommended for ninth grade and the second part of D and E for grade 10. Each of the tasks requires about one lesson in the computer lab.

Preliminary TI InterActive! skills

Students do not need any skills in using TII (TI InterActive!), but basic skills in using Windows applications are useful.

Preliminary mathematical skills

The required preliminary mathematical knowledge is minimal in tasks A and B. For task C, the students should be familiar with quadratic formulas and parabolas. In task D, the general solution of a quadratic equation appears, as well as some more advanced algebraic procedures. For task E differentiation is used although this part of the unit can be skipped easily.

File organization

The task consists of the following linked TII files:

- IntroNotebook.tii: This file is the starting point, containing hyperlinks to other files and also the file in which students record solutions.
- IntroMainTaskA.tii: the file containing main task A.
- IntroMainTaskB.tii: the file containing main task B.
- IntroMainTaskC.tii: the file containing main task C.
- IntroMainTaskD.tii: the file containing main task D.
- IntroMainTaskE.tii: the file containing main task E.
- IntroSolution.tii: contains the solutions.

The hyperlinks only work if files are installed in the map c:\TII\Intro.

Classroom organisation

We suggest a short introduction to TII of about 10 minutes where students are shown some important and impressive features of the software. The general structure of the menu bars may also be explained.

Students can then work through the tasks. We recommend some plenary summaries, demonstrations and student presentations to guarantee a kind of convergence of machine techniques and to discuss interesting findings.

Technical hints

Students start by opening the notebook file in TII. They then open the main task in a separate window, using the hyperlink. In fact, two versions of TII are now activated. By means of a right mouse click in the Windows menu bar, the option 'Cascade Windows' offers opportunities for switching easily between the files. However, if the same hyperlink is used twice, two versions of the same file will be opened. This may be a source of confusion and therefore requires some attention by the teacher.

Because TII only supports absolute hyperlinks and not relative links, the links need to be adjusted after installation of the files, for example on the school network.

Didactical suggestions

While the students are working, alone or in pairs, we recommend that teachers ask them to carefully observe the result obtained. For example, the teacher could ask students if they understand the TII output, if the output could have been found in another way, or if the answer makes sense.

The students may try to skip the questions that address explanations of the TII output, so it is up to the teacher to stress the importance of the interpretative skills.

1.2 Main task B: TI InterActive! as a calculator

intro

TI InterActive! is a perfect mathematical assistant. One of the first mathematical features that you might want to use is the regular calculation feature. Keep in mind that TI InterActive! calculates exact as well as approximate results! Furthermore, big numbers are no longer a problem.

task

If you want to use TI InterActive! as a calculator, you need to open the Math Palette.

B1. Open the Math Palette by pressing . Move the Palette so that you can read this text.

Two things happened simultaneously. First, you see a rectangle around your cursor. This is the Math Box, where you can enter your calculation. It will also contain the result. Second, you see the Math Palette, which looks somewhat like an ordinary calculator. You use it for entering numbers, and later formulas and commands. The Math Palette can be moved to any convenient position on your screen.



The Input and Output fields on the lower side of the palette allow you to control the input and output styles. The More field gives access to advanced settings.

You can close both the Math Box and the Math Palette with Esc.

B2. Just for practise: close the Math Box and Palette, and then open it again.

B3. Enter in your notebook file: Task B3.

Then open a Math Box and enter: 11^2 . Finish with Enter.

You see that the square sign is represented nicely, and that the answer appears immediately in red.

A new Math Box is opened automatically, and the Math Palette is still there.


The Math Palette contains a button with ANS on it. This stands for the answer of the last calculation.

B4. Enter in the open Math Box: $\sqrt{\text{ans}}$.
Verify the calculation that was carried out by TI InterActive!.

B5. Calculate 111^2 , 1111^2 , 11111^2 . Nice pattern, isn't it?

If you want to get rid of a previously made Math Box, just click on it and press the Del button or the Backspace on your keyboard.

Fractions can be done as well. A fraction such as $1/3$ can be entered like exactly this way in a Math Box.


Another option is to start with the fraction bar by pressing  in the Math Palette.

B6. Open a Math Box in your notebook file and enter: $1/3 + 1/5$.
Verify the answer by hand!

The answer is a fraction again. If you want a decimal answer, you can use the command **approx**, which is short for approximate.

B7. Enter in a Math Box: **approx(ANS)**.

If an approximation with more decimals is required, one can change the settings.

B8. Press the Mode Settings icon  in the mathematics toolbar. Set the number of digits to Fixed 12 in the Display Digits field and click OK. Open a new Math Box and enter: **approx(1/3+1/5)**. What is the result of changing the Mode Settings?

B9. Calculate $1/3+0.5$.
Explain why you get a decimal answer without using **approx**.

In a calculation, the position of parentheses may determine the results. Be wary of this!

B10. Open a Math Box. You can also use **Ctrl-M** for that.
Calculate $1+2/3*4$.
How did TI InterActive! find the result?

B11. Double-click on $1+2/3*4$ in the Math Box and change it into $1+2/(3*4)$.
Explain the difference in the outcome.

B12. Insert parentheses in $1+2/3*4$ so that the result is 4.
Change the position of the parentheses in order to get two different results.

You may know about factorials. For example, $10!$ ('10 factorial') is defined as $1*2*3*4*5*6*7*8*9*10$.


B13. Calculate both $10!$ and $1*2*3*4*5*6*7*8*9*10$. You should get the same result.

B14. Factorize the answer with **factor(ANS)**. You can either type in the word **factor**, or find it in the **Math Algebra** menu of the Math Palette.
Explain the two factors 5.

B15. Calculate $25!$ How many times does the factor 5 appear?

B16. Calculate a bigger factorial number, factorize it and explain the numbers of times the factor 5 is present.

Summary

For calculations you have to open a Math Box, which can be done with Ctrl-M or . The Math Box can be moved, deleted or edited. Commands can be typed in directly or picked from the Math Palette Menus.

We finish task B with two additional assignments.

B17. Carry out the following calculation by double-clicking on it and changing the 'Hide output' setting in the Math Palette:

$$6 - \frac{-1 - 3^2}{\frac{-5}{4}} + \sqrt{\frac{8}{9-7}}$$

B18. Try to find out a calculation yourself which contains each of the digits 1, 2, 3, ..., 9 once and not more than once. Have one of your colleagues carry out the calculation. The answer should be an integer value.


1.3 Main task C: TI InterActive! as a graphing environment


intro

TI InterActive! can be used as a graphing environment. It draws all graphs for you, in a fast and accurate way and in the colours you choose. Setting an appropriate viewing window is important for getting a good view of the graph.

task

Before we can graph a function, it first needs to be defined. You can do this with the command Define.


C1. In your notebook file open a Math Box with  and enter define $f(x) = x^3 - x$.

The icon  gives access to the Graphing window.


C2. Press .

Two windows pop up, a Functions window which contains the functions to be graphed, and a Graph window, where the graphs will appear.

C3. Enter $f(x)$ for $y1(x)$ in the Functions window. Immediately, the graph appears.

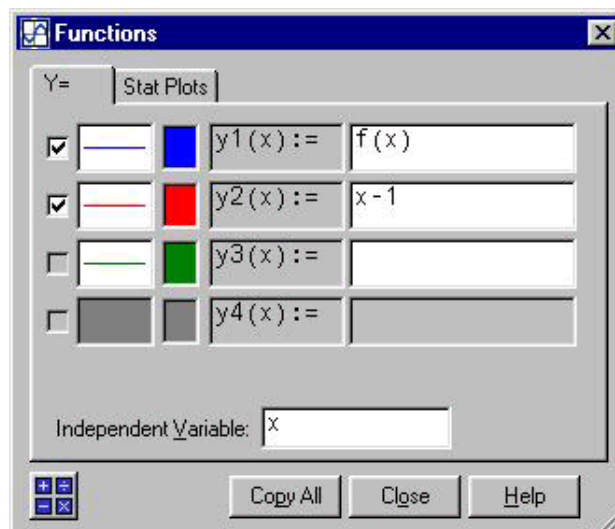
The viewing window is indicated by the numbers in the margin of the graph. You can either change these dimensions directly, or in the  window.

C4. Change the viewing window to $[-2, 2]$ by $[-3, 3]$.

C5. Press  in the Graph window. As a result, the Functions window is closed and the graph is copied into your notebook document. If you click on the graph window in the notebook file, you can change the dimensions of its appearance.

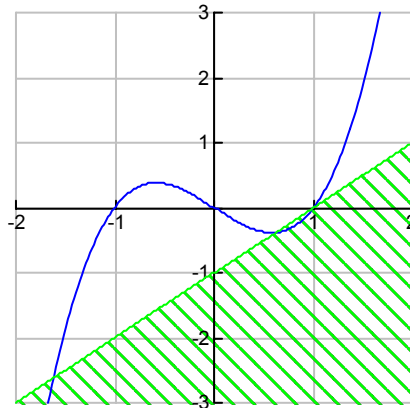
C6. Re-open the graph window by means of a double click.
Enter a second function $x - 1$ in $y2$. This graph appears immediately as well.

As you see in the picture below, you can define a function in the Home screen and refer to it in the Functions window, as was done for the function f . A second option is to enter the formula directly into the Functions window, as you did for function $y2$. You can choose the colour and the thickness of the graph. By means of ticking the box, you can activate or de-activate the function.



C7. Click on the colour to the left of y_2 and choose a colour that you like. Click on the line to the left of y_2 to change the style of drawing. Copy the graph into your notebook file.

Your graph might look like this, for example:




In the Graph window you see several buttons and options:

The first line offers options such as **Zoom** for changing the view of the graph and **Calculate** for calculating intersection points, zeros and extreme values.


The second menu bar offers icons, such as the  icon for animated drawing of graphs and the  symbol palette for entering expressions.

The third menu bar offers icons for calculations in the graph and window settings.

The fourth line contains the main buttons , ,  and .

The  button allows for tracing the curve to see the coordinates of the cursor position.



C8. Re-open the graph and use **Trace** to approximate the coordinates of the intersection points. You can use the arrows in the Trace Value window to move the cursor. Use **Close** to end tracing.

C9. Click on . This opens the Calculate Intersection window. Move the black cross towards one of the intersection points and choose **Calculate** in the Calculate Intersection window. Write down the coordinates of the first intersection point. Do the same for the second and the third points.

C10. Verify that $(1, 0)$ is the coordinate of an exact intersection point by calculating $f(1)$ and $y_2(1)$ in the notebook file.


C11. Enter $y_1(x) - y_2(x)$ as a third function in the Functions window. Explain why the graph of y_3 intersects the horizontal axis for the same values of x for which y_1 and y_2 intersect.

Summary

For graphing you have to open the Functions window and the Graph window by clicking on . You can scale the viewing window by setting the dimensions, either by entering the endpoints for x and y directly or by using .

We finish task C with three additional assignments.

C12. Define a function f : define $f(x) = -2x^2 - 2x + 12$ Draw the graph of f and find the coordinates of the zeros and the maximum.

C13. Enter $y1(x) := \sqrt{x^2 + 1}$ in the Functions window. The Symbol Palette  can be used to enter the square root sign. Graph $y2(x) := x$ and $y3(x) := -x$. Why do the lines seem to touch the curved graph?

C14. Enter $x^4 + b \cdot x^2 + 1$ | $b = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$ as function $y1$ in the Functions window: Using this instruction, the values $-4, 0, 3, \dots, 4$ are substituted into b . This leads to a 'sheaf' of graphs. Find an appropriate viewing window for this sheaf.


1.4 Main task D: TI InterActive! as an algebra environment

intro

In contrast to many graphing software packages, TI InterActive! also has algebraic features. It can rewrite algebraic expressions, solve equations and remove the calculational drudgery of algebra. It remains important, though, that you are fully aware of your problem solving strategies.

task

In Main Task A you used Math Boxes to carry out numerical calculations. In the same Math Boxes you can enter algebraic calculations. The Math Palette menus provide the commands.

D1. In your notebook file open a Math Box with  and enter $a + a$.
TI InterActive! automatically simplifies this as $2a$.

D2. Enter $a*b*c*a*b/c$.
Is the answer correct for all values of a , b and c ?

D3. Enter $\sqrt{x^2}$.
Also enter $\sqrt{x^2} | x > 0$ and $\sqrt{x^2} | x < 0$.
The vertical bar $|$ is the 'wherein-bar' or the 'with-bar'. It is a means to indicate a condition.

D4. The wherein-bar can be used for substitution. Enter for example:

$$x^3 | x = 2$$

$$x^3 | x = y$$

$$x^3 | x = \text{monkey}$$

$$x^3 | x = a + 1$$

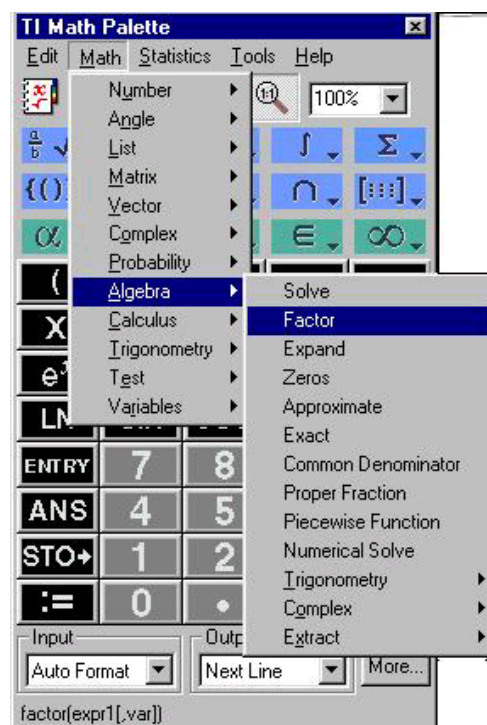
D5. You probably know that $a^2 + 2 \cdot a \cdot b + b^2$ can be written as $(a + b)^2$.
Enter $a^2 + 2 \cdot a \cdot b + b^2$. Nothing happens, as TI InterActive! does not know which of the two forms is the most simple for you.

If you want to get the second form in assignment D5, you need to use the factor command. There are two ways of entering this command; either just type it in, or select it from the Math Algebra-menu in the Math Palette. In this case, you first click on the Math button in the Math Palette to enter the menu for mathematical calculations. Then choose Algebra, and click on the Factor command in the pop-up menu. This method has the advantage of offering help concerning the syntax. In the last line of the Math Palette you see `Factor(expr [,Var])`, indicating that Factor should be followed by an expression between parentheses, with an optional comma and name of a variable.

D6. Enter `factor(a^2 + 2*a*b + b^2)`. Mind the multiplication sign $*$ between a and b .

If you do not enter the multiplication sign $*$ between a and b , TI InterActive! will interpret ab as one variable name.

D7. Verify that `factor(a^2 + ab + b^2)` does not give the same result as above.



D8. In some cases, you should add the variable to the factor command. For example, try:

```
factor(x2 - 2)
factor(x2 - 2, x)
factor(x2 - 2.0, x)
factor(x2 + y2 - 2·y + 1, x)
factor(x2 + y2 - 2·y + 1, y)
```

To do the opposite and expand the brackets, use the expand command. Like the factor command, it can be entered as such, or picked from the Math Algebra menu in the Math Palette.

D9. Enter in a Math Box: `expand((a + b)2)`.
Mind the parentheses!

D10. Expand the parentheses in $(a + b)^{10}$.

D11. Factor the answer of the previous assignment.

D12. Find a value of a so that $(x - a)^3$ equals $x^3 - 12x^2 + 48x - 64$.

TI InterActive! can solve equations. The command, **Solve**, can be found in the Math Algebra menu of the Math Palette. You can also type in the word **Solve** using your keyboard. The syntax for solving an equation is `solve(equation, unknown)`.

D13. Enter in a Math Box: `solve(x2 = 5, x)`.
Approximate the solutions.
What happens if you enter `solve(x2 = 5.0, x)`?

D14. Solve the generic quadratic equation $a \cdot x^2 + b \cdot x + c = 0$.
Do you recognize the solution?
What happens if you solve the equation with respect to a instead of x ?

Summary

Important algebraic features of the TI InterActive! algebra module are:

- the automatic simplification that occurs in the Math Box,
- the commands factor and expand, which can be typed in or selected from the Math Algebra menu in the Math Palette,
- substitution, which is done with the 'wherein-bar' | ,
- the solve command for solving equations. The syntax is `solve(equation, unknown)`.

Two additional assignments end this task.

D15. Enter $\frac{\frac{2}{3}a + 2b}{a + 3b}$ and explain the simplified form.

D16. In task C you defined the functions $f(x)$ and $y2(x)$:

```
define f(x) = x3 - x
define y2(x) = x - 1
```

You found the intersection of the two graphs.

Verify the results you found with a solve command: `solve(f(x) = y2(x), x)`.

Approximate the solutions.

1.5 Main task E: TI InterActive! as a function investigator

intro

In Main Tasks A - D, you learned several TI InterActive! techniques. In this final part of the introductory unit, you will combine these techniques and apply them to investigate a function.

task

In this task you will investigate the function f with define $f(x) = \frac{4x^2 + 7x}{8x - 2}$.

- E1. Enter the function in your notebook file.*
- E2. Select an appropriate viewing window and copy the graph into your notebook file.*
- E3. Calculate the exact coordinates of the zeros of f , as well as their approximate values. Verify your answers in the graph.*
- E4. Factor the formula of $f(x)$. In factorized form you can recognize the zeroes as well. Explain how.*
- E5. The factorized form of $f(x)$ also reveals the vertical asymptote. Give the equation of this asymptote. How did you find it?*
- E6. Expand the formula of $f(x)$. In the expanded form, you can recognize the linear function, which represents the second asymptote of the graph of f . Explain how you can recognise the asymptote and add this asymptote to the graph.*
- E7. If we want to calculate the extreme values of f , we should find the derivative of f . Select the derivative command from the **Math** **Calculus** menu of the Math Palette. Complete the command as `derivative(f(x),x)`.*
- E8. Find the values of x for which the derivative is equal to zero, and calculate the exact and approximate coordinates of the extrema.*

2. Sweet and sour with a hat

Goal of the task

This task has three goals:

- To familiarize students with some features of TI InterActive! that have not been encountered during the introduction session. In particular, students will learn to draw on a Graph, use Stat Plots, learn the syntax for piecewise functions and use sliders in order to obtain parameter variations.
- To test the mathematical skills mentioned before and evaluate the extent to which students have integrated the notion of function and can compose elementary functions in order to obtain, for example, a positive and increasing function for some given interval of the variable.
- To encourage students' creativity.



Target group and required time

The target group consists of students in 11th grade after first semester, or 10th grade at the end of the academic year. Estimated time to perform:

Main Task Part A: 2 "hours" ("hour" means 1 time period of 50 minutes)

Main Task Part B: 3 "hours"

Additional Task: 2 "hours"

Control Task: 1 "hour"



Preliminary TI InterActive! skills

The students need some basic skills in using TII, in particular the use of Function Windows and Graph Windows. For some parts of the work, it will be useful for students to be able to easily manipulate a Math Box and particularly the functionalities of the Algebra Menu.

Preliminary mathematical skills

Preliminary knowledge in calculus/algebra includes: elementary functions (polynomial function of degree 2, square root function, homographic function), piecewise functions, transformations of the plane applying the graph of f on the graph of $a \cdot f(mx + p) + b$, determination of the parameter(s) appearing in the analytic expression of a function in order that the graph is tangent to a given line (in one of its points).

Eventually, the notion of derivative.

Preliminary knowledge in analytic geometry: perpendicular lines, equation of a circle.



File organization

The unit consists of the following linked TII files:

- Start file: File containing links to all the other files of the group Sweet & Sour except for the different Help and Hint files. Links to these files are situated in Main Task A and B files or Additional Task file or Control Task file.

- Notebook file: Starting file for the student. The student is supposed to insert his or her answers to the various parts of the exercise in this file.
- Main Task file, part A: Introductory file, describing the first part of the task.
- Main Task file, part B: Contains the second and principal part of the task.
- Help 1 and Help 2 files: Contains some technical help to perform some of the required tasks for Main task A.
- Hint 1 file: Contains suggestions that could be useful to the student who does not know how to solve some of the questions from Main task A.
- Hint 2 to Hint 4 files: Contains suggestions that could be useful to the student who does not know how to solve some of the questions from Main task B.
- Additional Task file: Contains an additional task for the brighter or faster student.
- Hint 5 file: Mathematical help to perform the additional task.
- Control Task file: Contains a control question to evaluate the understanding of some of the suggestions made in Hint 4 file.
- Solution file: Contains a possible solution for each of the exercises proposed in Main Tasks A and B, in the Additional Task and in the Control Task file. Ideally, the teacher should delete the link that exists from the Start file to the Solution file before the students start to work!

The hyperlinks only work if the files are installed in the map c:\TII\SweetandSour.

Technical hints

Students start by opening the notebook file in TII. It is recommended that the main task is opened in a separate window so that two versions of TII are activated. By means of a right mouse click in the Windows menu bar the option 'Windows Cascade' offers opportunities for switching easily between the files. However, if the same hyperlink is used twice two versions of the same file will be opened! This may be a source of confusion and therefore requires some attention by the teacher.

As TII only supports absolute hyperlinks and no relative links, the links need to be adjusted after installation of the files, for example on the school network.

Classroom organisation

The students work either individually or in pairs on computers during normal mathematics classes or at any other time. The above files are organized and detailed in such a way that students should be able to work without the presence of their mathematics teacher.

Didactical suggestions

We tried out this unit with students in 11th grade. For technical reasons related to the poor state of the computers in the school, the students worked alone or in groups of two or three at home. Though all of the students were beginners with TI InterActive!, none of them encountered technical difficulties related to the use of the software itself.



On the contrary, none of the students was able to generate the simultaneous movements of the mouth and chin of Sweet and Sour with only one slider bar. Most of them succeeded with two slider bars, but this was not the goal of this part of the task. This probably means that the students did not entirely grasp the concept of function.

A few students even have had problems in choosing an adequate function to represent the mouth and in limiting the function representing the mouth to a certain interval.

The additional task was partially successful in the sense that everybody obtained the linear part of the hat, but the portion of hyperbola did not look as it should have.

2.1 Main task A

intro

The main task consists of two parts, part A and part B, in which you will use, among other features, the:

- graphing tool of TI InterActive!
- stat plot tool
- slider bars
- definition of piecewise functions.

If you do not feel familiar with one of the above options or if you do not know the syntax to be used for piecewise functions, feel free to consult the *help topics* provided in this document or use the TI InterActive!

Help  button.

The mathematical skills you need concern:

- elementary functions and their graphs
- translating and stretching curves
- tangent lines to curves

Some *mathematical hints* are also provided in this document.

Try to be imaginative, and success !

task

a. The neutral face

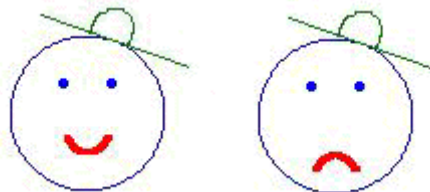
Sketch a small face similar to the one in the following figure using the drawing tool for the circle, mouth and hat, and the stat plot tool for the eyes.



Help 1

b. Sweet changes into Sour

Sketch a face similar to the one in the preceding figure whose mouth shape varies by means of a slider bar, from "smile" to "sad" as illustrated below.



Help 2

Hint 1

2.2 Main task B

task

a. Once more the neutral face

Sketch a small face, similar to the one of the figure below, using:

- *two functions* to draw the circle,
- the drawing tool for the hat and the mouth
- the Stat Plot tool for the eyes.

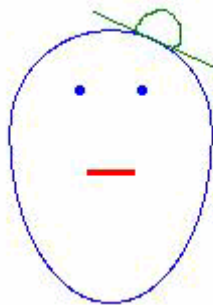
Hint 2



b. Sweet changes into Sour

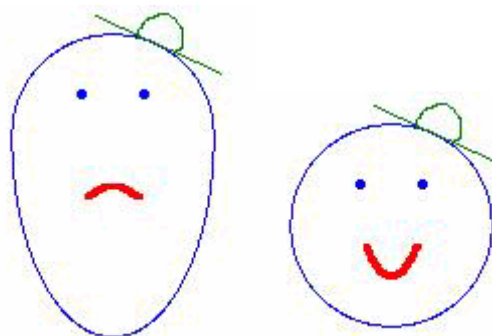
Sketch a face similar to the one in the preceding figure, but this time with a chin becoming longer and longer by means of a slider bar.

Hint 3



c. Using *only one slider*, sketch Sweet and Sour in such a way that the movements of the chin of Sour correspond simultaneously to a variation of the mouth, as indicated on the following figures.

Hint 4



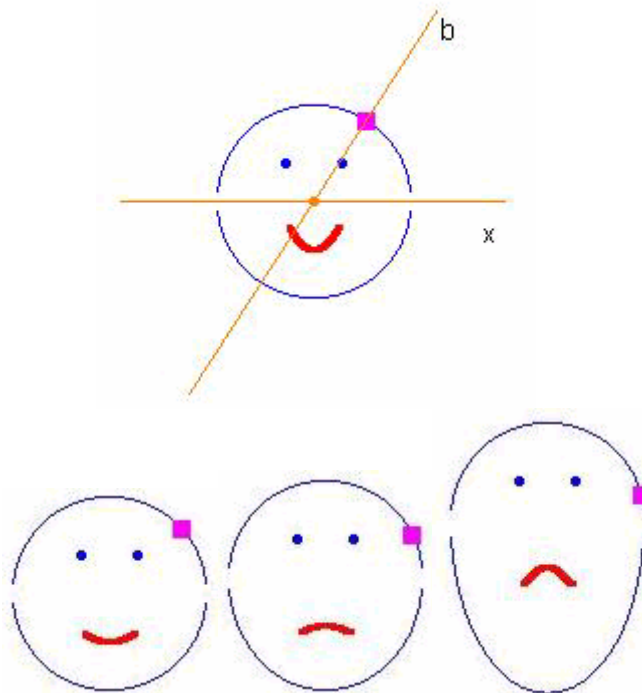
2.3 Control task

task

Elvis Pompilio has created a nice tiny hat for our friend ...
Here it is:



- Sketch the same face as in Task B, delete the green hat (you double-click on the segment then delete it and do the same for the green circle).
Using Stat Plots, create the tiny hat knowing that it is situated on the upper semi-circle of the face and that the angle (x,b) is equal to 1 radian.
- Put a parameter in the hat's coordinates in such a way that the hat moves from its initial angular position of 1 radian to an angular position of a quarter of a radian when you slide the button into the slider bar *controlling the mouth and chin* of Sweet&Sour.



2.4 Additional task

task

Elvis Pompilio has created a nice hat for our friend ...

Here it is.

Sketch the same face as in Task B but delete the green hat (double click on the segment then delete it and do the same for the green circle).
Using well chosen functions, try to create the same type of hat for our fellow!



Hint 5

2.5 Hints

hint 1

Let us consider three functions defined the following way:

$$\text{define } f(x) = \sqrt{x-1}$$

$$\text{define } g(x) = k \cdot \sqrt{x-1}$$

$$\text{define } h(x) = \sqrt{k \cdot x - 1}$$

In the figure below, the graph of f is the black curve (half a parabola).

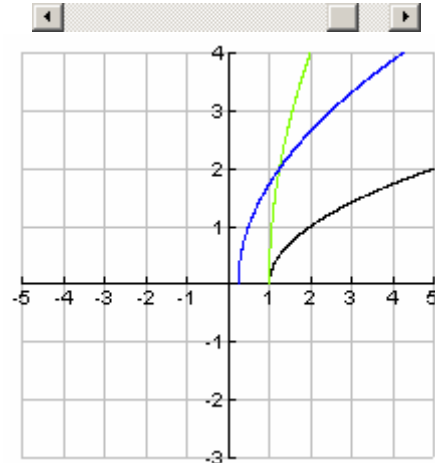
Using a slider to change the values of k , you can observe the effect of these changes on the green graph of g and the blue graph of h .

$$\text{We have } g(x) = k \cdot f(x).$$

From this result we know that the green curve can be obtained by stretching the black one in the direction Oy, axes Ox, factor k . The effect of this transformation is just a change in the opening of the half parabola.

$$\text{We have } h(x) = f(k \cdot x).$$

Hence the blue curve can be obtained by stretching the black one according to direction Ox, axes Oy, factor $\frac{1}{k}$. The effect of this transformation is not only a change in the opening but the intersection point with Ox also changes.



Can you explain what happens when k is equal to zero or equal to one ?

Another way of transforming the graph of f would be to add k to $f(x)$ or replace x by $x+k$ inside the algebraic expression for $f(x)$...

Do you think that this is the type of transformation we need here to change the shape of the mouth of Sweet and Sour ?

hint 2

Drawing a circle with given center and radius

The equation of the circle with center $(0, 0)$ and radius r is $x^2 + y^2 = r^2$.

This circle is the union of the graphs of two functions whose analytic expressions are obtained by solving the preceding equation with respect to y .

$$\text{solve}(x^2 + y^2 = r^2, y)$$

$$y = \sqrt{r^2 - x^2} \text{ and } r^2 - x^2 \geq 0 \text{ or } y = -\sqrt{r^2 - x^2} \text{ and } r^2 - x^2 \geq 0$$

The equation of the circle with center (x_0, y_0) and radius r is $(x - x_0)^2 + (y - y_0)^2 = r^2$.

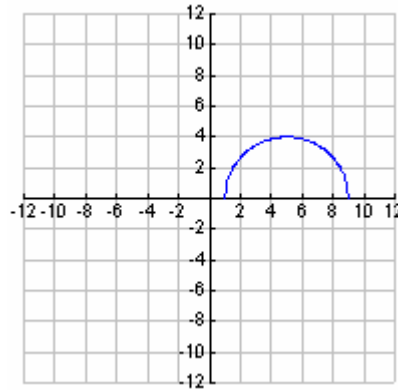
This circle is the union of the graphs of two functions whose analytic expressions are given by

$$y1(x) := y_0 + \sqrt{r^2 - (x - x_0)^2} \text{ and } y2(x) := y_0 - \sqrt{r^2 - (x - x_0)^2}.$$

Hint 3.1

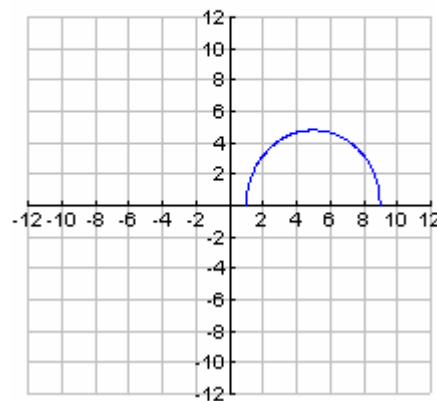
define $f(x) = \sqrt{16 - (5 - x)^2}$

The graph of this function is half a circle with center $(5, 0)$ and radius.



Hint 3.2

Let us introduce a parameter k varying from 0.2 to 5 and consider the function $g(x) := k \cdot \sqrt{16 - (x - 5)^2}$ and its graph. Use the slider bar to see the effect on the initial circle when k is varied.



The half circle is stretched in the direction of Oy, the axes of the stretching is Ox and the factor is k . The circle is transformed into an ellipse whose intersection points with Ox are fixed.

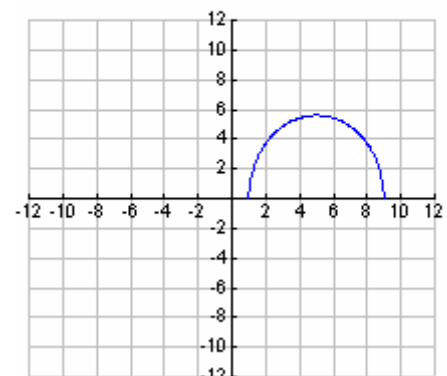
Remark: The maximum value of k is too big with respect to the chosen window giving the impression that the ellipse is getting "open".



Hint 3.3

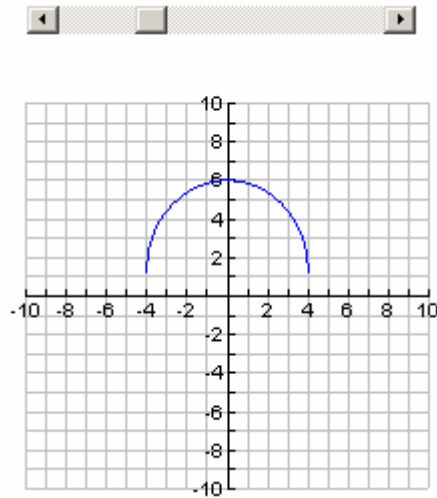
Now observe what is happening if you move the slider bar.

The graph appearing in the figure above is the graph of $k \cdot f(x)$ but the parameter k takes negative values (right click on the slider bar, choose open/activate to see the new domain of k).



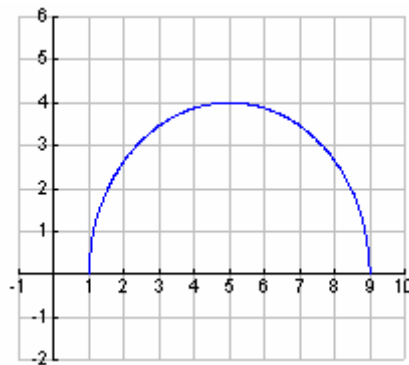
Hint 3.4

What happens if the semi-circle does not cut the Ox axes?
Explain your answer using the following figure and the corresponding slider bar.



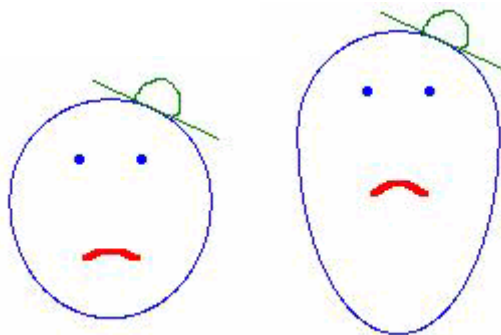
Hint 3.5

And now, explain what you observe in the figure below.
Is the equation of this curve the equation of an ellipse or a circle?



hint 4

Using only one slider would require that the parameter generating the movements of the chin would depend on (be a function of) the parameter generating the variations of the mouth.



To make the preceding figures, the parameter m that was chosen for the mouth is strictly negative and decreasing when Sour is sad and getting sadder and sadder.

The parameter ch generating the movements of the chin had to be greater than 1 and increasing to assure that the chin became longer and longer and to keep the right concavity.

So, in the preceding figures, the problem was to find a function ch of m that would be positive, greater than 1, increasing when m was negative and decreasing and that would not increase too rapidly in order to avoid the chin "opening" itself.

In the following figures, you see some of the misadventures that would have happened to Sour if one had not chosen the right function $ch(m)$:

$ch(m)$ is not greater than 1 .



Here $ch(m)$ increases too quickly.



Here $ch(m)$ is not positive !



How can you find the function that fits this case ?

First of all, do not look for a sophisticated function. You know a lot of rather elementary functions among which you will find what you are looking for: polynomial functions of degrees one to four, the homographic function, the absolute value and the square root.

Use TI InterActive! to graph the function you have chosen for $ch(m)$ (if these are the names of your parameters) and check that this function corresponds to the one you are looking for.

hint 5

- a. The hat contains a segment which is tangent to the face at the point T, abscissa 1 and positive ordinate.

It is easy to determine the equation $y = c \cdot x + d$ of a line which is tangent to the upper semi-circle at T.

To obtain a segment instead of a line, it suffices to limit the graph of $c \cdot x + d$ making use of piecewise functions.

- b. The hat contains a portion of curve (equation $y = \frac{n}{x} + q$) that is tangent to the preceding segment at T.



You know different techniques to find n and q in such a way that the graphs of $f(x) = c \cdot x + d$

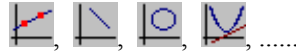
and $g(x) = \frac{n}{x} + q$ are tangent at one given point:

- Express that the two curves intersect each other into the *double* point T or
- Express that the graph of $g(x)$ goes through point T and that the slope of the tangent at T is the right one (using derivatives).

2.6 TI InterActive! help

a. Drawing on a graph in a Graph Window

To draw a line using two points, a segment, a circle or a tangent line without needing equations and without affecting any already graphed function use the Draw menu or the corresponding toolbar buttons

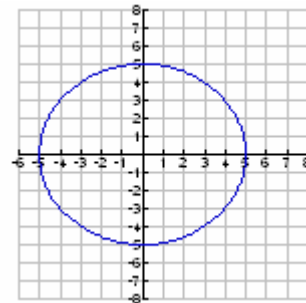
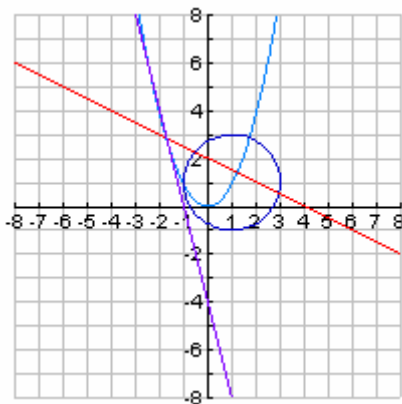


Example:

In one Graph Window produce graphs of the function $f(x) = x^2$, the circle with center (1, 1) radius 2, the line through points (2, 1) and (-2, 3) and the tangent line to the graph of f in its point of abscissa -2.

To see how this was done, double-click on the figure below, then on the circle, line or tangent line

Remark: This was also supposed to be a circle! Do you understand why it is not ?

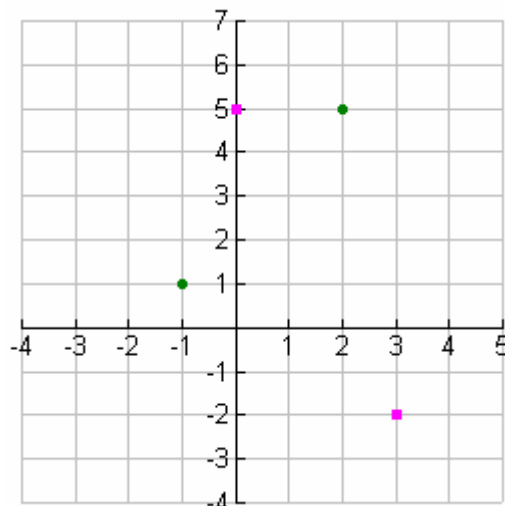


b. Drawing several points in a Graph Window.

Assume that you want to draw the points with coordinates (-1, 1), (2,5), (3,-2),(0,5):

- In a Functions Window click on the Stat Plots tab.
- On the first line insert the abscissas using braces and commas : {-1,2,3,0}.
- On the second line, insert the ordinates in the same order : {1,5,-2,5}.

Again, to see how this was done, double-click on the figure below and click on the Stat Plots tab in the Functions Window.



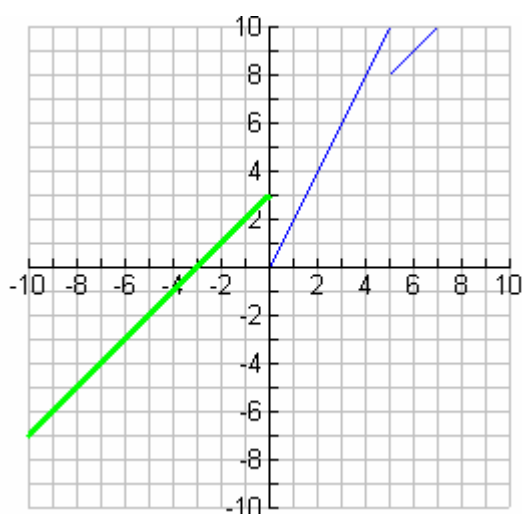
a. Piecewise functions

The function $f(x) := \begin{cases} x+3 & x < 0 \text{ or } x > 5 \\ 2x & 0 \leq x \text{ and } x \leq 5 \end{cases}$ will be defined using the syntax

when($x < 0$ or $x > 5, x+3, 2x$).

The function $f(x) := \begin{cases} x+3 & x > 0 \end{cases}$ without any other alternative, will be defined using the syntax when($x < 0, x+3$).

Click on the figure below to see how this was done.



b. Using a slider

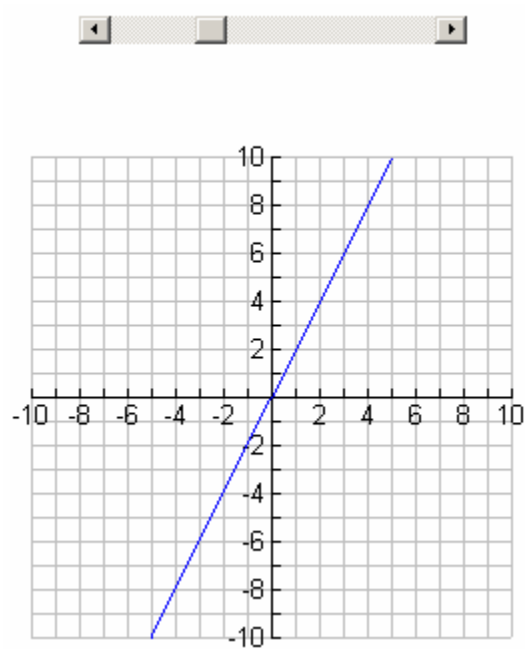
Consider the function $f(x) := 2 \cdot x + m$ where m is a parameter taking values between -4 and 10 with step $\Delta m = 2$.

With TI InterActive! it is possible to illustrate the corresponding variations of the graph of f :

First click on the button  of the TII toolbar. Fill in the following fields:

- Variable name: Enter the name of the parameter, in this case, m ,
- Initial value: Default value given to m ,
- Choose the minimum and maximum values allowed for m ,
- Step amount: Step $\pm \Delta m$ will be used if one clicks on the arrows of the slider,
- Page amount: Step width that will be used to increment or decrement m if you click anywhere in a blank area between an arrow and the slider bar.

Below you see the graph of f . Use the slider to investigate the effect of varying the parameter m .



3. Growth

Goal of the task

The Growth unit is divided into four sub-units: Linear Growth, Exponential Growth, Logistic Growth and Comparison.

Before starting with this unit on Growth, the students have been made familiar with exponential functions and logarithmic functions for about 8 -10 lessons. Mathematics in this unit, developed for a business college, is closely related to business and our aim is to teach our pupils applications of the mathematics, rather than the theory. One of the main topics in the students' second year of Mathematics is growth.

- The students should become familiar with the basic types of growth (linear, exponential, logistic).
- TII is used in notebook-classes only and has to be used in each lesson for calculating and exploring “the jungle”.
- The worksheets are available on the internet for users of TII only and are intended for classroom usage. These worksheets consist of two kinds; one which features various situations and incorporates sliders and one which contains concrete tasks. The tasks are aimed at students of different levels.

The worksheets presented here do not replace the traditional course on growth, but must be seen as an extension or an addition. They do not represent a separate course on that topic.

Target group and required time

The target group of this unit is 10th grade students with an interest in business applications of mathematical models. The Growth unit requires approximately 20 to 25 lessons, including practice time.

Preliminary TI InterActive! skills

The students need the basic skills in using TII which are addressed in the introductory unit of this booklet. Some basic skills in using Windows applications are also useful.

Preliminary mathematical skills

The required preliminary mathematical knowledge consists of a thorough knowledge of exponential and linear functions. Knowledge of differential calculus is not required.

File organization

The task consists of linked TII files stored in the following folders:

- C:\TII\Growth contains the starting file and the teacher guide. Students start work by opening Start_Growth in TI InterActive!. This file contains hyperlinks to the tasks, which can be found in the folders C:\TII\Growth\linear, C:\TII\Growth\exponential, C:\TII\Growth\logistic and C:\TII\comparison.
- C:\TII\Growth\video_animation_help contains hyperlinks to video animations showing how specific tasks can be carried out in TI InterActive!. The videos are in the same folder and can be recognized by the extension scm, which stands for Screencam, the Lotus software that is used for the videos.

- C:\TII\Growth\linear contains the files concerning linear growth. The starting file for linear growth is linear_growth_Notebook, in which the students write their solutions. The folder contains hint files for mathematical assistance.
- C:\TII\Growth\exponential contains the files concerning exponential growth. The starting file for exponential growth is expo_growth_notebook, in which the students write their solutions. The folder contains hint files for mathematical assistance.
- C:\TII\Growth\logistic contains the files concerning logistic growth. The starting file is logi_growth_Notebook, in which the students write their solutions. The folder contains hint files for mathematical assistance.
- C:\TII\Growth\comparison contains the files which address the comparison of the three different growth models. The starting file here is comp_Notebook, in which the students write their solutions. The folder contains hint files for mathematical assistance.

The hyperlinks only work if the files are installed in the appropriate folders.

Suggestions for the teacher

The topic should be started with a general introduction and the presentation of introductory examples by the teacher (about 20 minutes). The teacher should also repeat the most relevant TII commands. If desired, the students can work in pairs. Students can explore the problem graphically with the help of sliders or by substituting values for the existing parameters. It is recommended that the teacher draws up an inventory of the students' findings and leads a classroom discussion on the issues that arise. The teacher may also present a correct solution or invite students to do so.

Technical hints

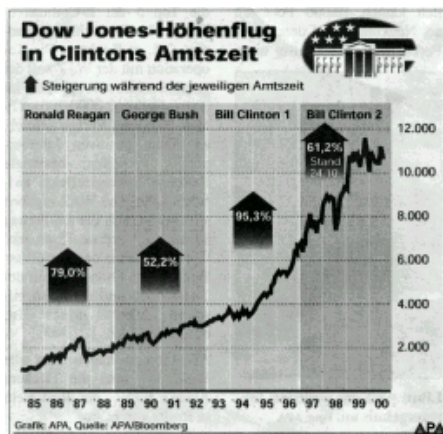
Students start by opening the file Start_Growth in TII. It is recommended that the different TII-files are opened in separate windows so that several versions of TII are activated. By means of a right mouse click in the Windows menu bar the option 'Windows Cascade' offers opportunities for switching easily between the files. However, if the same hyperlink is used twice two versions of the same file will be opened! This may be a source of confusion and therefore requires some attention by the teacher.

As TII only supports absolute hyperlinks and no relative links, the links need to be adjusted after installation of the files, for example on the school network.

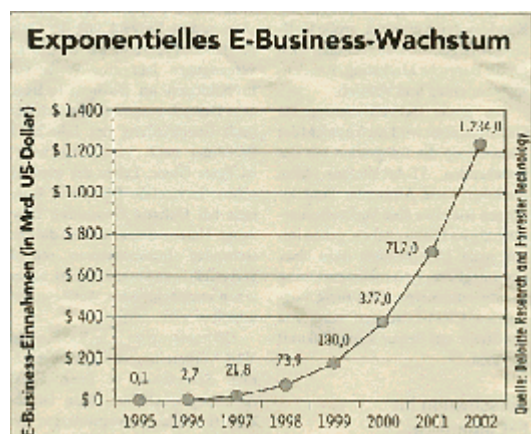
Examples

The following pictures show some examples of various kinds of growth. What kind of growth would one expect in these cases? In this unit you will develop models to describe the most important kinds of growth.

The Dow Jones Index

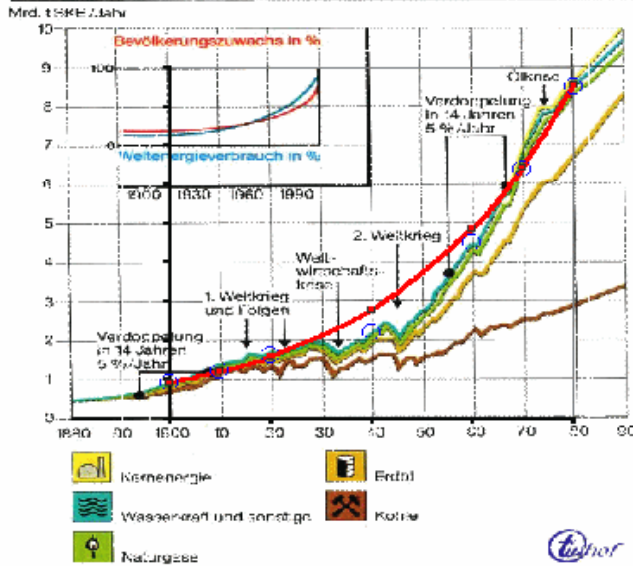


The Growth of e-business

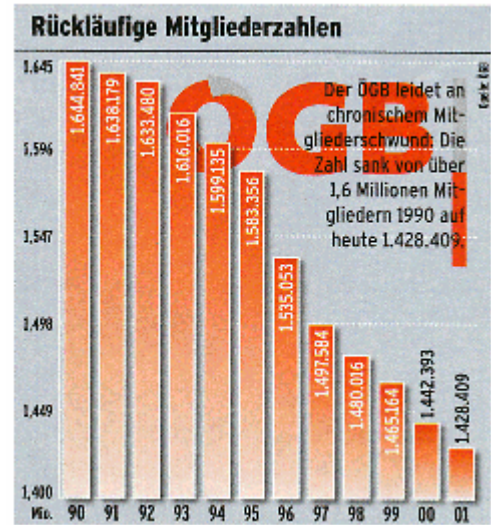


The World Energy Consumption

Weltenergieverbrauch nach Primärenergieträgern 1880 – 1990



The Number of Tradeunionists



Three types of mathematical growth models

$y(t)$	population at time t
$y_0 = y(0)$	population at the beginning of exploration
$\Delta y = y(t+1) - y(t)$	absolute increment
$\frac{\Delta y}{y} = \frac{y(t+1) - y(t)}{y(t)}$	relative increment; rate of growth

_____ formula _____

Linear growth (red)

$$y(t) = y_0 + t \cdot a$$

_____ formula _____

Exponential growth (blue)

$$y(t) = y_0 \cdot (1+i)^t \text{ or } y(t) = y_0 \cdot e^{(k \cdot t)}$$

_____ formula _____

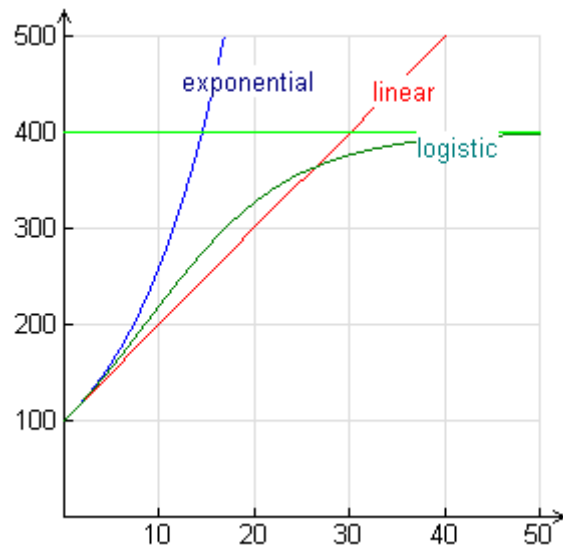
Logistic growth (green)

M maximum; saturation:

$$y(t) = \frac{M}{1 + b \cdot e^{-t}} \text{ with } b = \frac{M - y_0}{y_0}$$

or

$$y(t) = \frac{M}{1 + b \cdot e^{(-k \cdot t)}} \text{ with } b = \frac{M - y_0}{y_0}$$



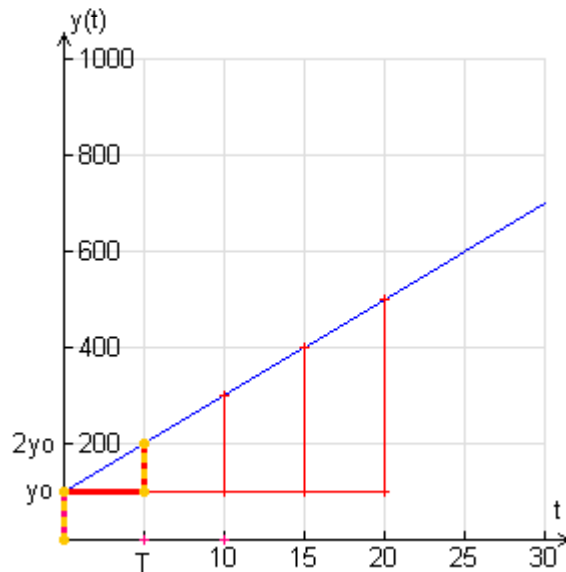
3.1 Linear growth

intro

We remember that the graphs of linear functions are defined by: (*linear growth*)

$$y(t) = y_0 + a \cdot t$$

On the right is the graph of the function for $y_0=100$ and $a = 20$.



task

Main task

Answer the following questions.

Hint 1

- Define $a = 20$.
What is the effect of the value of y_0 on the graph of the function?
Illustrate the answer using a Graph window.
- Define $y_0 = 100$.
What is the effect of the value of a on the graph of the function?
Illustrate the answer using a Graph window.
- Define $y_0 = 50$, $a = 20$.
In what period T does the function value y_0 duplicate?
- What happens to T if the value of y_0 is changed?
Calculate the values of T for $y_0 = 50; 100; 150; 200; 300$ and $a = 20$.
What happens to T if the value of a is changed?
Calculate the values of T for $a = 10; 20; 30; 50; 100$ and $y_0 = 50$.

task

Control task

- Calculate a general equation for the double period T for linear growth. $y(T) = 2 \cdot y_0$.
- Define $y_0 = 90$ and $a = 15$.
Calculate the period in which the function value y_0 is triple the initial value.
- Dora has 80.00 in a box.
At the end of each of the following weeks, she adds 20.00.
How much money is there after 5, 6, 7, t weeks in Dora's box?
- After how many months is the amount of Dora's money in the box 200.00?

Hint 2

3.2 Exponential growth

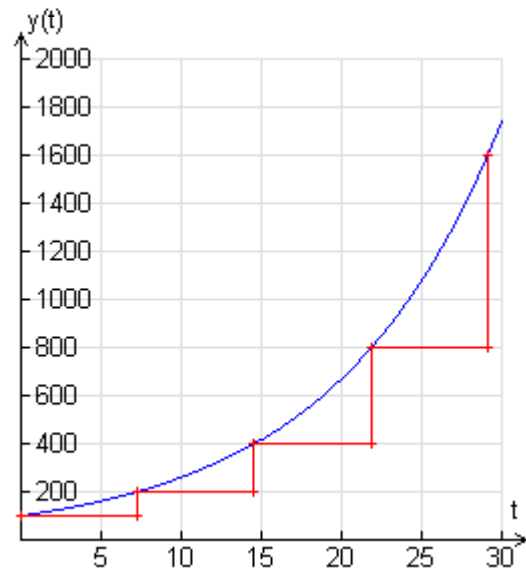
intro

We remember that the graph of exponential functions are defined by:

$$y(t) = y_0 \cdot (1+i)^t \quad \text{exponential growth}$$

On the right is the graph of the function for

$$y_0=100 \quad \text{and} \quad i = 10\% ; i = 0.1.$$



task

Main task A

Hint 3

- Define $i = 0.1$.
What is the influence of the value of y_0 on the graph of the function?
In what period T is the duplication of the function value $y(t)$?
- Define $y_0 = 200$.
What is the influence of the value of i on the graph of the function?
In what period T is the duplication of the function value $y(t)$?
- Define $y_0 = 100$, $i = 0.1$.
In what period T is the duplication of the function value $y(t)$?
What happens with T , if the value of y_0 is changed?

task

Control task A

Hint 4

- Calculate a general equation for the double period T .
Is T constant?
Does T depend on y_0 ?
Does T depend on i ?
- Define $i = 20\%$ and $y_0 = 300$.
Calculate the period for the function value to be trebled.
- Define $i = 20\%$ and $y_0 = 300$.
In what period does the function value becomes $n \cdot y_0$.

task

Main task B

Hint 5

- $y(3) = 25$
 $y(4) = 31.5$
Calculate the absolute increment.
Calculate the relative increment.

- b.** $y(2) = 25$
 $y(7) = 31.5$
 Calculate the absolute increment.
 Calculate the relative increment.
- c.** Define $y_0 = 300$, $i = 0.1$.
 Plot the graph of a function $\text{deltay}(t)$ [$\text{deltay}(t) := y(t) - y(t-1)$] for the absolute increment of the exponential growth.
 Plot the graph of a function $\text{rely}(t)$ [$\text{rely}(t) := \frac{y(t) - y(t-1)}{y(t-1)}$] for the relative increment of the exponential growth.
 Describe both graphs and describe the difference between the two increment functions.
- d.** Change the value of y_0 .
 What is the influence of this value on the two increment functions?
- e.** Change the value of i .
 What is the influence of this value on the two increment functions?

task

Control task B

Hint 6

- a.** $y_0 = 0.9$
 $y(80) = 8.5$
 Calculate the absolute increment and the relative rate of growth for 80 years.
- b.** $y_0 = 0.9$
 $y(80) = 8.5$
 Calculate the relative rate of growth i for *one* period.
 Write the equation of exponential growth and plot the function.

task

Additional task

Hint 7

C-14 is a radioactive isotope with a half-life period of $T = 5730$ (+/- 1%) years.

- a.** How old is a skeleton with 10% of the native C-14 content?
- b.** Calculate an interval for the age of the skeleton.

Comparison between exponential and linear growth

Suppose in the first year both kinds of growth have the same increment.

$$y_0 = 24.1$$

$$a = 15$$

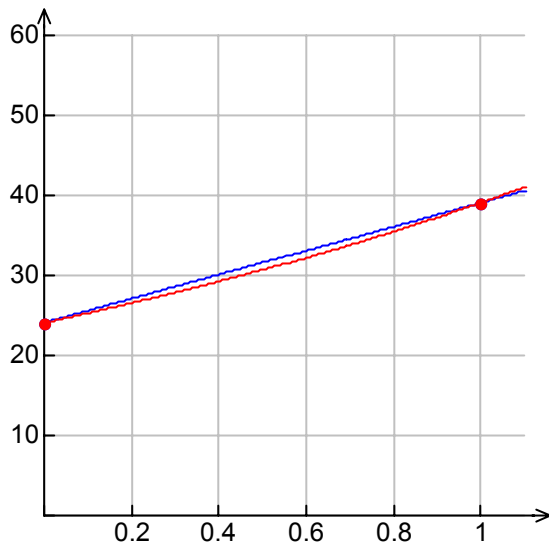
$$i := \frac{a}{y_0} = .622406639$$

$$\text{seq}(a, a, 0, 20) \rightarrow L1$$

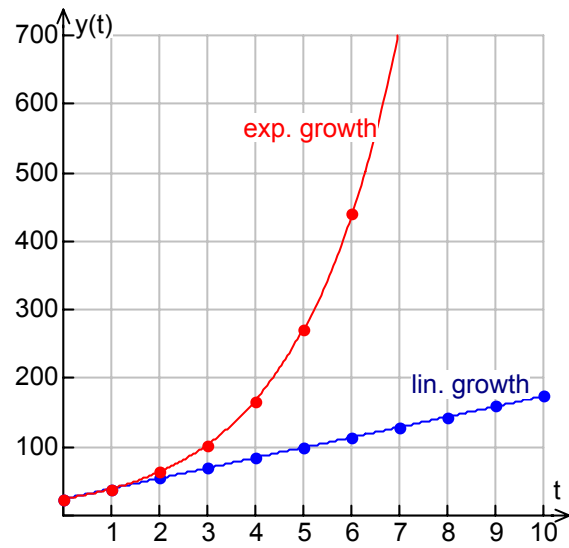
$$y_0 + 11 \cdot a \rightarrow L2$$

$$y_0 + (1+i)^{11} \rightarrow L3$$

Interval [0 ; 1]



Interval [0 ; 10]



3.3 Logistic growth

intro

We recall that logistic functions are defined by:

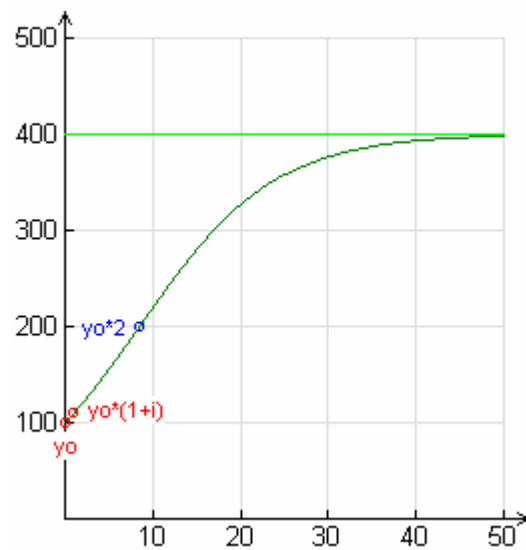
$$y(t) := \frac{M}{1 + b \cdot c^t} \quad \text{or} \quad y(t) = \frac{M}{1 + b \cdot e^{(-k \cdot t)}},$$

$$\text{with } b := \frac{M - y_0}{y_0}.$$

On the right you see the graph of the function for

$y_0=100$ and $i = 10\% = 0.1$.

Rate of growth in the first period: $M = 400$.



task

Main task A

Hint 8

- Define $y_0 = 100$, $i = 0.1$ and $M = 400$ with sliders. Calculate b and c and graph the function. What is the influence of the value of y_0 on the graph of the function?
- Define $y_0 = 100$, $i = 0.1$ and M with a slider. What is the influence of the value of M on the graph of the function?
- Define $y_0 = 100$, i and $M = 400$ with a slider. What is the influence of the value of i on the graph of the function?
- Define y_0 , $i = 0.1$ and $M = 400$. At what time T does the duplication of the function value y_0 occur? What happens to T if the value of y_0 is changed?

task

Control task A

Hint 9

- Define $i = 15\%$, $y_0 = 90$ and $M = 450$ in the logistic growth model. Find the time for the function value to become double the initial value y_0 .
- Define $i = 15\%$, $y_0 = 50$ and $M = 450$. Find the time when the value of the function is triple the initial value of y_0 .

task

Main task B

Hint 10

- Define $y_0 = 100$, $y(1) = 130$ and $M = 400$. Plot a function $\text{deltay}(t)$ [$\text{deltay}(t) := y(t) - y(t-1)$] for the *absolute increment* of the logistic growth. Plot a function $\text{rely}(t)$ [$\text{rely}(t) := \frac{y(t) - y(t-1)}{y(t-1)}$] for the *relative increment* of the logistic growth. Describe the function of the absolute increment.

- b. Change the value of $y(1)$.
What is the influence of this value on the function $\text{deltay}(t)$?

task

Control task B

Hint 11

Suppose $y_0 = 15$, $y(4) = 23$ and $M = 50$.

- Calculate the formula for logistic growth function.
- Plot the graph of the logistic function and the function with absolute increment.
- Calculate the maximum of the absolute increment.

task

Additional task

Hint 12

In 1980 4.5 billion people lived on the Earth.
In 1999 6 billion lived on our planet.

- One model of growth predicts an exponential growth for the number of people on Earth.
Calculate the exponential growth function which fits the data.
- Another growth model assumes logistic growth with a maximum of 15 billion people on Earth.
Calculate the logistic growth function with $y_0 = 4.5$, $M = 15$ and $y(19) = 6$.
- Plot the graphs of both functions on one graph. Which model is more realistic, do you think?

3.4 Comparison

intro

We remember that the graphs of functions for the most important types of growth are defined by:

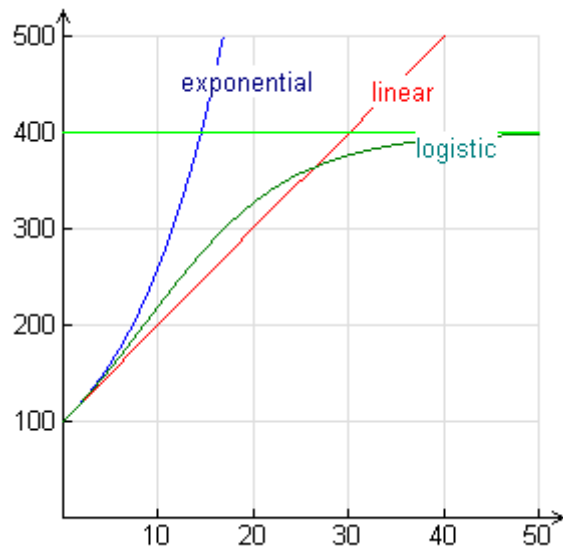
$$\text{lin}(t) := y_0 + a \cdot t \quad \text{linear growth}$$

$$\text{ex}(t) := y_0 \cdot (1 + i)^t \quad \text{exponential growth}$$

$$\text{logi}(t) := \frac{M}{1 + b \cdot c^t} \quad \text{logistic growth}$$

$$b := \frac{M - y_0}{y_0}$$

On the right you see the graphs of the three functions for: $y_0=100$, $i = 10\%$, $a = 10$ and $M = 400$.



task

Main task

- Define $i = 0.1$, $a = i \cdot y_0$, $M = 400$.
What is the influence of the value of y_0 on the three functions?
- Define $y_0 = 100$, $a = i \cdot y_0$, $M = 400$.
What is the influence of the value of i on the three functions?
- Define $y_0 = 100$, $i = 0.1$, $a = i \cdot y_0$.
What is the influence of the value of M on the three functions?

Hint 13

Hint 14

Hint 15

task

Control task

Hint 16

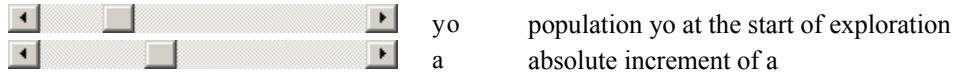
- Define $M = 100$, $i = -10\%$ and $y_0 = 500$.
Calculate the terms of the linear, exponential and the logistic function.
- Explain what happens with the growth functions (exponential and logistic) when the value of i is negative and $y_0 > M$.

3.5 Hints

hint 1

a/b Define the formula of linear growth in a new MathBox: $y(t) := y_0 + a \cdot t$.

Use slider bars to choose values for y_0 and a .



Show the graph of the function.

To calculate the double period (for the first period) use: $\text{solve}(y(t) = 2 \cdot y_0, t)$.

$\text{solve}(y(t) = 2 \cdot y_0, t)$ double period $T = Th$; $y(T) = 2 \cdot y_0$

c. Delete the values of y_0 and a for the following calculations: $\text{delvar}(y_0) :: \text{delvar}(a)$.

Use $\text{solve}(y(t) = 2 \cdot y_0, t)$ to calculate the double period T . Store the result as Th .

Use $Th \mid y_0 = 100$ and $a = 20$ to define $y_0 = 100$, $a = 20$ and to calculate Th .

$Th \mid y_0 = 100$ and $a = 20$.

d. Use $Th \mid y_0 = 50$ and $a = 20$ to define $y_0 = 100$, $a = 20$ and to calculate Th .

For the second assignment in task D, use $Th \mid y_0 = 50$ and $a = 10$ to define $y_0 = 100$, $a = 20$ and to calculate Th .

hint 2

a. Define the formula of linear growth $y(t) := y_0 + a \cdot t$.

Use $\text{solve}(y(t) = 2 \cdot y_0, t)$ to calculate t .

b. Use $\text{solve}(y(t) = n \cdot y_0, t)$ and the "|" to calculate the periods.

c. Define $y(t) := y_0 + a \cdot t$ with y_0 and a .

Use a table to calculate the values of $y(t)$.

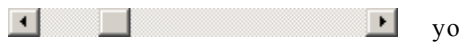
d. Use $\text{solve}(y(t) = 20, t)$ to find the time needed to get a value of 20.

hint 3

a. Define the formula for exponential growth: $y(t) := y_0 \cdot (1 + i)^t$.

Insert slider bars to define the values of y_0 and i :

population at the beginning of exploration



y_0

relative growth rate



i

Plot the graph of the function $y(t)$ to answer the questions.

Calculation of doubling period T.

use: $\text{solve}(y(t) = 2 \cdot y_0, t)$

store the right side of the answer as Th:

$\text{right}(\text{ans}) \rightarrow Th$

Calculation of doubling period T: $y(T) = 2 \cdot y_0$.

Delete variable y_0 and a for further calculations:

$\text{delvar}(y_0) :: \text{delvar}(i)$.

doubling period T:

$\text{solve}(y(t) = 2 \cdot y_0, t)$

use "| $y_0=100$ and $i=0.1$ "

$\text{solve}(y(t) = 2 \cdot y_0, t) | y_0 = 100 \text{ and } i = 0.1$

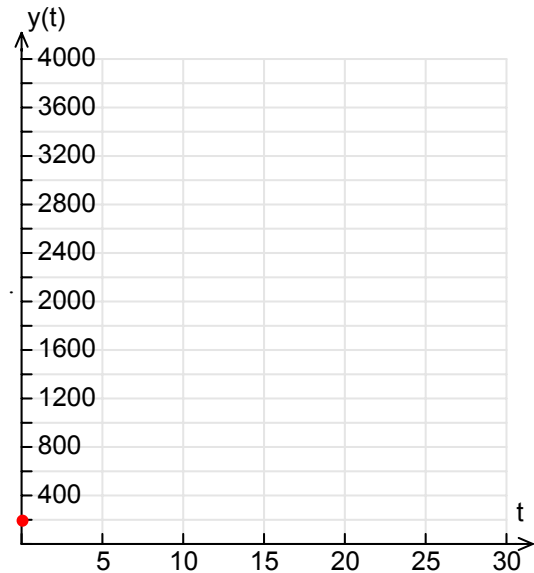
trebling period:

$\text{solve}(y(t) = 3 \cdot y_0, t)$

factor n period:

$\text{solve}(y(t) = n \cdot y_0, t)$

$\text{solve}(y(t) = n \cdot y_0, t) | y_0 = 100 \text{ and } i = 0.1$



- b. Give an answer like $T_1(i) = \dots$ (You must *not* use the function name $T(i)$).
Use: $\text{solve}(y(t) = 2 \cdot y_0, t)$.
- c. Use $\text{solve}(y(t) = 2 \cdot y_0, t)$ for the doubling period T
and $\text{solve}(y(t) = 2 \cdot y_0, t) | y_0 = 100 \text{ and } i = 0.1$.

hint 4

- a. Define the formula for exponential growth: $y(t) := y_0 \cdot (1 + i)^t$.

Insert slider bars to define the values of y_0 and i :

population at the start of exploration

y_0

relative growth rate

i

Print the graph of the function $y(t)$ to answer the questions.

Calculation of doubling period T: $y(T) = 2 \cdot y_0$.

Calculation of double period T:

use: $\text{solve}(y(t) = 2 \cdot y_0, t)$

store the right side of the answer as Th:

$\text{right}(\text{ans}) \rightarrow Th$

delete variable y_0 and a

$\text{delvar}(y_0) :: \text{delvar}(i)$

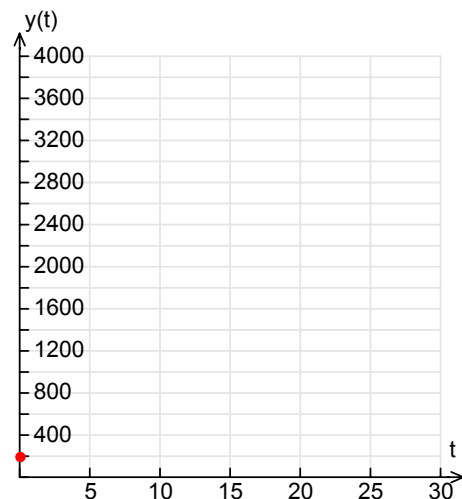
- b. trebling period:

$\text{solve}(y(t) = 3 \cdot y_0, t)$

- d. factor n period:

$\text{solve}(y(t) = n \cdot y_0, t)$

$\text{solve}(y(t) = n \cdot y_0, t) | y_0 = 100 \text{ and } i = 0.1$



a. $y(3) = 25$
 $y(4) = 31.5$

absolute increment: $y(4) - y(3)$

relative increment: $\frac{y(4) - y(3)}{y(3)}$

relative increment in %: $\frac{y(4) - y(3)}{y(3)} \cdot 100 \%$

$\frac{\Delta y}{\Delta t} = \frac{y(4) - y(3)}{1}$

b. $y(2) = 25$
 $y(7) = 31.5$

absolute increment: $y(7) - y(2)$

relative increment: $\frac{y(7) - y(2)}{y(2)}$

relative increment in %: $\frac{y(7) - y(2)}{y(2)} \cdot 100 \%$

$\frac{\Delta y}{\Delta t} = \frac{y(7) - y(2)}{1}$

$y(t) := y_0 \cdot (1 + i)^t$ Define the formula for exponential growth.
 $y_0 := 300$ Define the population at the beginning of exploration.
 $i := 0.1$ Define the relative growth rate.

c. Define the formula of the absolute increment:
 $\text{deltay}(t) := y(t) - y(t - 1)$.

Enter *deltay(t)* in a new Math Box to show the formula of the function.

Notice the exponential growth of the absolute increment!

Define the formula of the relative increment:

$\text{rely}(t) := \frac{y(t) - y(t - 1)}{y(t - 1)}$ relative increment

Enter *rely(t)* in a new Math Box to show the formula of the function.

Define the formula of the relative increment in %:

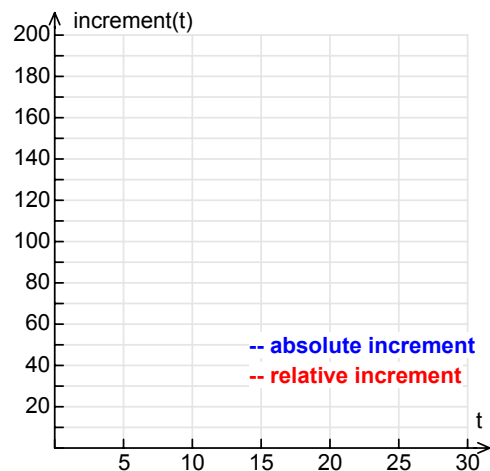
$\text{rely}(t) := \frac{\text{deltay}(t)}{y(t - 1)} \cdot 100$ relative increment in %

Enter *rely(t)* in a new Math Box to show the formula of the function.

$\text{rely}(t) \%$ relative increment in %

d/e. With a slider bar you can change the values of y_0 and i .

Watch the influence of the changes on the graph.



- a. Define the formula for exponential growth.

$$y(t) := y_0 \cdot (1+i)^t$$

$$y_0 := 0.9$$

$$y_{80} := 8.5$$

Calculate the absolute increment and the relative rate of growth for 80 years.

$$y_{80} - y_0 = 8.5 - y_0 \quad \text{absolute increment}$$

$$\frac{y_{80} - y_0}{y_0} \quad \text{relative increment}$$

$$\frac{y_{80} - y_0}{y_0} \cdot 100 \quad \% \quad \text{relative increment in \%}$$

- b. Open a new MathBox and solve the equation $y(80)=8.5$ with $i > 0$:

$$\text{solve}(y(80) = 8.5, i) \mid i > 0.$$

Write in the next MathBox: $\text{right}(ans) \rightarrow i$.

Plot the graph of the function $y(t)$.

Define $y(t) = y_0 \cdot (1+i)^t$ and $y_0 = 100$.

$$y(t) := y_0 \cdot (1+i)^t$$

$$y_0 := 100$$

Define $Th = 5730$.

$$Th := 5730$$

Solve the equation $y(Th) = y_0/2$ with $i > -1$.

$$\text{solve}\left(y(Th) = \frac{y_0}{2}, i\right) \mid i > -1$$

$\text{right}(ans) \rightarrow i$

$$y(t)$$

Solve the equation $y(t) = 10$.

Define $ya(t) = y_0 \cdot (1+i)^t$.

$$ya(t) := y_0 \cdot (1+i)^t$$

$$Ta := Th \cdot 1.01$$

Calculation see above!

Calculation with $T = 5730 + 1\% = 5787.3$ years.

Define $ya(t) = y_0 \cdot (1+i)^t$

$$ya(t) := y_0 \cdot (1+i)^t$$

$$Ta := Th \cdot 1.01$$

Calculation see above!

- a. Define y_0 , M and i with slider bars. Answer the questions using the sliders.



Maximum, saturation of logistic growth.

This means $i \cdot 100\%$ growth in the first time interval.

$$y(t) := \frac{M}{1 + b \cdot c^t}$$

Define the function of logistic growth $y(t)$.

Important: Start with the calculation of b , using the solve command.
Then continue with the calculation of c .

- b. Graph the function and use the slider bar to change the value of M. Examine the effects.
- c. Graph the function and use the slider bar to change the value of i. Examine the effects.
- d. If T is the duplication time, $y(T)=2*y(0)$.

hint 9

- a. Define y_0 , M and i.

Define $y(t)$: $y(t) := \frac{M}{1 + b \cdot c^t}$.

First calculate b and c. If the initial value y_0 is doubled, $y(t) = 2*y_0$.

- b. Follow the approach of the previous assignment, but this time notice that $y(t) = 3*y_0$.

hint 10

- a. First calculate the value of c using the given value of $y(1)$.
- b. Change the value of $y(1)$ and examine the effect this has on the graph of delay.

hint 11

- a. Define y_0 and M.

Define $y(t)$: $y(t) := \frac{M}{1 + b \cdot c^t}$.

Use y_0 to calculate the value of b.
Use $y(4)$ to calculate the value of c.

- b. The absolute increment function is defined by $\text{deltay}(t) := y(t) - y(t - 1)$.
- c. Use Calculate Maximum for finding the maximum of the absolute increment.

hint 12

- a. The general model for exponential growth is $\text{ex}(t) := y_0 \cdot (1 + i)^t$.
Use the data to calculate y_0 and i.

- b. The general model for logistic growth is $y(t) := \frac{M}{1 + b \cdot c^t}$.
Use the data points and $M=15$ to calculate b and c.

hint 13

First define y_0 with a slider and i, M and a with:



y_0

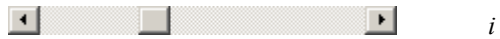
$i := 0.1$

$M := 400$

$a := i \cdot y_0$

Then enter the three growth models.

First define i with a slider and y_0 , a and M with:



i

$$y_0 := 100$$

$$M := 400$$

$$a := i \cdot y_0$$

Then enter the three growth models.

First define M with a slider and y_0 , i and a with:



M

$$y_0 := 100$$

$$i := 0.1$$

$$a := i \cdot y_0$$

Then enter the three growth models.

a. Start with an example:

$$y_0 := 500$$

$$i := -0.1$$

$$M := 300$$

Find the formulas for the three types of growth.

b. Use the above example to investigate the three graphs.

4. Parameter

In this unit you will discover how an 'extra letter' or parameter in a formula may lead to studying families of curves or dynamic curves instead of one static graph. The unit consists of three sub-units, entitled The shifting parabola, Rotating lines, and Lines touching a parabola.

4.1 Shifting parabola

teacher guide

Goal of the task

This task is part of the Parameter Unit, which includes the tasks 'The shifting parabola', 'Rotating lines', and 'Lines touching a parabola'.

The goal of this task is that students realise that the value of the parameter, in this case c , determines the position and/or the form of the graph. Furthermore, the task aims to help students understand that an additional criterion concerning the distance of the zeros 'filters out' one or more parameter values. The first part of the task concerns the effect that parameter changes have on the parabola which contains this parameter in its equation. The second question requires students to identify the parameter value that fulfils a condition concerning the zeros of the parabola. The final question requires a mental shift from students as the parameter takes the character of an unknown.

Students can approach these tasks graphically and the TII slider bar allows for a dynamic view of the situation. However, the aim is that students support their findings using algebra and the algebraic features of TII can be useful.

Target group and required time

The target group for this task are tenth-grade students of average to high ability in mathematics. Eventually, ninth-grade might be considered. The task requires approximately two lessons in the computer lab.

Preliminary TI InterActive! skills

The students need basic skills in using TII, in particular those related to working with graphs (trace) and for applying commands from the algebra menu such as 'solve' and 'approximate'.

Preliminary mathematical skills

The required preliminary mathematical knowledge is limited to a basic knowledge of parabolas, in particular the role of the parameter a in $y = a \cdot x^2 + b \cdot x + c$. The task might fit well after a chapter in which parabolas have been taught.

File organization

The task consists of the following linked TII files:

- ParabNotebook.tii: The file in which the student writes the solutions and also the starting point which contains hyperlinks to the other files.
- ParabMain_Task.tii: The file that describes the main task.
- ParabHintA.tii: A file that suggests a question to start with if the student doesn't know what to do with the main task A.

- ParabHintB.tii: A file that suggests a question to start with if the student doesn't know what to do with the main task B.
- ParabControlA.tii: Contains control questions for the main tasks, in order to provoke some reflection on the results.
- ParabAdditional.tii: Contains an additional task for the brighter / faster students.
- ParabSolution.tii: Contains the solutions.

The hyperlinks only work if the files are installed in the map c:\TII\Parameter\ShiftPar.

The file StartParameter in c:\TII\Parameter provides access to the three notebook files of the three units that together form the Parameter Unit.

Classroom organisation

We suggest a short plenary at the start of about 10 minutes to introduce the problem and demonstrate some important TII skills. The slider bar can be demonstrated, as well as some other TII skills such as the solve command. The equation of a parabola might need some practice.

Then students can work in pairs and explore the problem by substituting different values for the parameter c . By means of the slider bar in HintA students can get a more dynamic view of the situation and this enables them to develop an understanding of the relationship between the parameter value and the form of the parabola, at least at a phenomenological level.

The second lesson can also start with a short plenary demonstration or classroom discussion so that the findings of the first lesson are summarised. Eventually, a pair of students may be requested to demonstrate their findings to date.

Technical hints

Students start with opening the notebook file in TII. They then open the main task in a separate window using the hyperlink. Two versions of TII are now activated. By means of a right mouse click in the Windows menu bar the option 'Windows Cascade' offers opportunities for switching easily between the files. However, if the same hyperlink is used twice two versions of the same file will be opened! This may be a source of confusion and therefore requires some attention by the teacher. As TII only supports absolute hyperlinks and no relative links, the links need to be adjusted after installation of the files, for example on the school network.

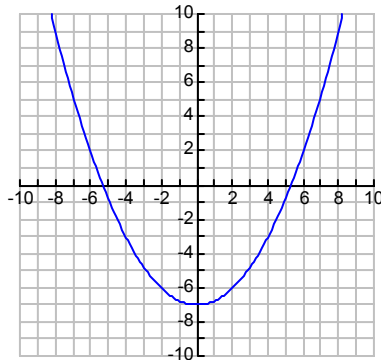
Didactical suggestions

Many students will see that the parabola narrows, but may not be able to explain this from the formula. To focus students' view on the vertical movement the hintA graph contains a point with a fixed x -coordinate.

Many students will describe the dynamics as 'the smaller the value of c , the bigger the distance between the zeros'. They probably will not search for algebraic expressions for the zeros. A suggestion by the teacher may be needed. There is also a difficulty in what students perceive as 'an exact answer'. It may be that students will say that 1.41 is more exact than $\sqrt{2}$, because it has more decimals. This is a useful issue for classroom discussion.

We consider graphs of the function f that is defined by: define $f(x) = \frac{x^2}{c} - 7$.

The graph of f for the case where the value of c equals 4 is shown below:



Main task

- a. How does the value of c influence the form of the parabola?

Hint 1

Explain this using the formula.

- b. The value of c also affects the position of the zeros.

Hint 2

For what value of c is the distance between the zeros exactly 6 units?

Control task

Please read the three descriptions below.

- (i) If the value of c gets bigger, the parabola is pushed outwards.
- (ii) If the value of c gets bigger, the parabola is pulled towards the x -axis.
- (iii) If the value of c gets bigger, the parabola is pulled towards the line $y = -7$.

Use the formula to explain which of the above descriptions you think most properly describes the situation.

Did you find an exact answer (a fraction)? If not, find out where you approximated and find a means to calculate the exact answer too.

Additional task

- a. For which value of c is the distance between the two zeros equal to 50000 units?
Draw the graph of f for this particular case.

- b. We now consider the graphs of the function: Define $g(x) = \frac{x^2}{4} - c$.

Investigate the effect of the value of c on the parabola.

Calculate the exact value of c so that the distance between the zeros is exactly 10 units.

4.2 Rotating line

Goal of the task

This task is part of the Parameter Unit, which includes the tasks ‘The shifting parabola’, ‘Rotating lines’, and ‘Lines touching a parabola’. In this task, the Rotating Lines, which follows the shifting parabola task, students gain an understanding of a ‘sliding parameter’ and its effect on lines and their intersection points.

The initial focus for this task is that students study the effect of changing a parameter on lines that contain this parameter in their equation. Students are then required to identify parameter values that fulfil specific conditions. Finally students need to make a mental shift as the parameter takes on the characteristics of an unknown.

Students can approach these tasks graphically with the TII slider bar allowing for a dynamic view of the situation. However, the aim is that students also support their findings using algebra and therefore the algebraic features of TII can be useful.

The main task consists of two questions, one which considers the influence of parameters on the intersection point of two lines and the other which requires students to find values of the parameter which result in an intersection point to the right of the vertical line with equation $x = 7$.

Target group and required time

The target group for this task is tenth-grade students who are of average to high ability in mathematics. Eventually, ninth-grade might be considered.

The task requires approximately two lessons in the computer lab.

Preliminary TI InterActive! skills

Students need some basic skills in using TII, in particular for using graphs and for applying algebraic commands such as the solve command.

Preliminary mathematical skills

The required preliminary mathematical knowledge is limited to a basic knowledge of linear functions.

File organization

The task consists of the following linked TII files:

- RotLineNotebook.tii: The file where the student writes the solutions and this is also the starting point as it contains links to the other files.
- RotLineMain_Task.tii: The file that briefly describes the task.
- RotlineHintA.tii: A file that suggests a question to start with if the student doesn't know what to do with the main task.
- RotlineHintB.tii: A file that suggests a question to start with if the student doesn't know how to proceed further after hintA.
- RotlineControlA.tii: Contains a control question on the main tasks, in order to provoke some reflection on the results.
- RotLineAdditional.tii: Contains an additional task for the brighter / faster students.
- RotLineSolution.tii: Contains the solutions.

The hyperlinks only work if the files are installed in the map `c:\TII\Parameter\RotLine`. The file StartParameter in `c:\TII\Parameter` provides access to the three notebook files of the three units that together form the Parameter Unit.

Classroom organisation

This task can be done after ‘the sliding parabola’. We suggest that teachers start the session with a short plenary of about 10 minutes to introduce the problem and to demonstrate some important TII skills.

Students can work in pairs to explore the problem by substituting different values for the parameter c . By means of the slider bar in HintA students will have a dynamic view of the situation. This will allow students to develop an understanding of the relationship between the parameter value and the lines, at least at a phenomenological level.

The second lesson can also start with a short plenary demonstration or classroom discussion so that the findings of the first lesson are summarised. The teacher may have a pair of students demonstrate their findings.

Technical hints

Students start with opening the notebook file in TII. They then open the main task in a separate window using the hyperlink. Two versions of TII are now activated. By means of a right mouse click in the Windows menu bar the option ‘Windows Cascade’ offers opportunities for switching easily between the files. However, if the same hyperlink is used twice two versions of the same file will be opened! This may be a source of confusion and therefore requires some attention by the teacher. As TII only supports absolute hyperlinks and no relative links, the links need to be adjusted after installation of the files, for example on the school network.

Didactical suggestions

Most students are happy with the numerical answers to the task which can be found graphically. The task of the teacher is to suggest an algebraic approach, by means of questioning the certainty and exactness of the approximated results.

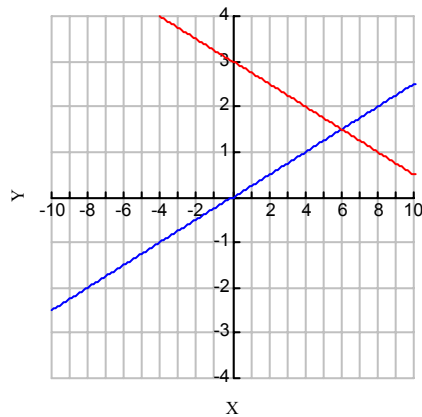
In our experience the control question in RotLineControl part B was not often answered by the students. This is something to keep track of as this question is intended to guide students towards an algebraic approach.

We consider graphs of functions f and g that are defined by:

$$\text{define } f(x) = c \cdot x$$

$$\text{define } g(x) = 3 - c \cdot x$$

Below you see the graphs of f and g in case the value of c equals $1/4$.



task

Main task

- What is the effect of the value of c on the positions of the intersection of the two lines?
Explain by means of the formulas. **Hint 3**
- What values of c result in the intersection point being to the right of the line $x = 7$?
Give an exact answer. **Hint 4**

task

Control task

- Explain what happens to the intersection point when the value of c is negative.
- You will probably have noticed that the value of c does not affect the height of the intersection point.
Explain why this is the case using the formulas.
- Did you find an exact answer (a fraction)? If not, then please find one.
- Did you consider in your answer the possibility that the value of c is smaller than zero?

task

Additional task

- What values of c lead to an intersection point that is on the left of the line $x = 1/100$? Provide an exact answer.
- Find two linear functions which both contain a parameter c in their formulas and have the following property: If the value of c gets smaller, the intersection point of the two graphs goes straight upwards.

4.3 Lines touching a parabola

teacher guide

Goal of the task

This task is part of the Parameter Unit which includes the tasks ‘The shifting parabola’, ‘Rotating lines’ and ‘Lines touching a parabola’.

Following from tasks on the shifting parabola and the rotating lines this unit integrates the skills, experiences and conceptual understanding that were developed. This task aims to deal with parameters in a flexible and versatile way in order to solve a ‘difficult’ problem. For this task not many hints are provided to guide the student, but instead the aim is for students to use their experience to find their own way through the problem. Students are supposed to develop a problem solving strategy and carry out their plan to solve the problem.

The central focus of the task is to find lines through the point $(0, -5)$ that touch the parabola with equations $y = x^2 - 3x - 2$. Please note that the students of the target group are not familiar with derivatives so they cannot use this in their strategy.

The TII slider bar allows for a dynamic view of the situation and an estimation of the right slope. It does accurate and quick graphing for the students allowing them to visualise the problem and convincing students that there is a solution. However, the exact answer will not be found using this approach and the task aims to have students find the exact answer using algebra. Therefore the algebraic power of TII which guarantees correct performance of hard calculations is useful. After this main task the additional task presents an even harder but more fascinating problem. The additional task, ‘how to find the equation of a line that touches two given parabolas’ is borrowed from Luc Trouche (see reference below).

Target group and required time

The target group for this task are tenth-grade students who are of average to high ability for mathematics. Eventually, eleventh-grade might be considered.

The task requires approximately 4 to 7 lessons in the computerlab, depending on whether the additional task is attempted.

Preliminary TI InterActive! skills

The students need some basic skills in using TII for using graphs and for applying algebraic commands such as the solve command.

Preliminary mathematical skills

The required preliminary mathematical knowledge concerns some experience with using a parameter to represent a sheaf of lines, and with solving quadratic equations. Knowledge of differential calculus is not expected.

File organization

The task consists of the following linked TII files:

- LineParNotebook.tii: The file in which the student writes solutions and the starting point containing hyperlinks to the other files.
- LineParMainTask.tii: The file that briefly describes the task.
- LineParHint.tii: A file that suggests a question to start with if the student doesn't know what to do with the main task.
- LineParControl.tii: Contains a control question on the task in hintA, in order to provoke some reflection on the results.

- LineParAdditional.tii: Contains an additional task for the brighter / faster students.
- LineParSolution.tii: Contains the solutions.

The hyperlinks only work if the files are installed in the map c:\TII\Parameter\LinePar. The file StartParameter in c:\TII\Parameter provides access to the three notebook files of the three units that together form the Parameter Unit.

Classroom organisation

This task can be done after ‘the shifting parabola’ and the ‘rotating lines’. We suggest that teachers start the session with a short plenary of about 10 minutes, to introduce the problem and reinforce some important TII skills. In particular, the concept of ‘touching’ lines or of tangent lines may need some illustration as students are not yet familiar with differential calculus. Also, it may be worthwhile to remind students that intersection points can be found with the solve command.

Students can then work in pairs to explore the problem graphically or by substituting different values for the parameter c . By means of the slider bar in the Hint-file students have a dynamic view of the situation. This will allow them to develop an understanding of the relationship between the parameter value and the lines, at least at a phenomenological level. The teacher, while discussing with pairs of students, can play ‘the devil’s advocate’ to make sure that the students understand what they have to do and why.

The second lesson can also start with a short plenary demonstration or classroom discussion so to summarise the findings of the first lesson. The teacher may also have a pair of students demonstrate their findings.

Technical hints

Students start by opening the notebook file in TII. It is recommended that the main task is opened in a separate window so that two versions of TII are activated. By means of a right mouse click in the Windows menu bar the option ‘Windows Cascade’ offers opportunities for switching easily between the files. However, if the same hyperlink is used twice two versions of the same file will be opened! This may be a source of confusion and therefore requires some attention by the teacher. As TII only supports absolute hyperlinks and no relative links, the links need to be adjusted after installation of the files, for example on the school network.

Didactical suggestions

Most students may be happy with the numerical answers for the task which were found graphically. The task of the teacher is to suggest an algebraic approach through questioning the certainty and exactness of the approximated results.

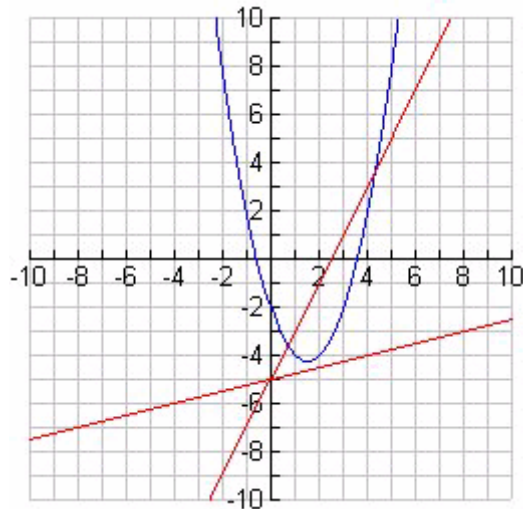
The additional task is ‘the ultimate challenge’ that would be a shame to miss.

Reference

Trouche L. (1998). *Expérimenter et prouver, 38 variations sur un thème imposé.*

[Experiment and prove, 38 variations to an imposed theme.] Montpellier, France: IREM, Université Montpellier II.

Below is a coordinate plane with a parabola and two lines.



The parabola is the graph of the function f : define $f(x) = x^2 - 3x - 2$.

Both lines go through the point $(0, -5)$. One line intersects the parabola at two points while the other line does not intersect the parabola. This suggests that there is a line through $(0, -5)$ that intersects the parabola at exactly one point. In this case we say that the line *touches* the parabola.

task

Main task

Find the exact equation of the line through $(0, -5)$ that touches the parabola.

Hint 5

task

Control task

- Did you find two solutions for a ? If not, find the second one. Draw the graph with the two 'touching' lines.
- Did you find approximate solutions? If you did then find the exact solutions.
- So far the lines we wanted to touch the parabola went through the point $(0, -5)$. As a generalisation we now consider lines through the point $(0, b)$.

Find the values of b for which there exists lines that touch the parabola with equation $y = x^2 - 3x - 2$.

task

Additional task

The Ultimate Challenge

Below you see a coordinate plane with two parabolas and one line.

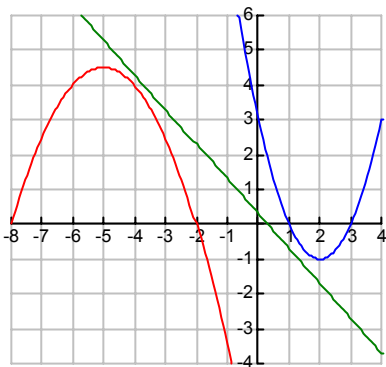
The parabolas are graphs of the functions f and g :

$$\text{define } f(x) = x^2 - 4x + 3$$

$$\text{define } f(x) = \frac{-1}{2}x^2 - 5x - 8$$

Find the exact equation of the line that touches both parabolas.

$$\text{define } l(x) = a \cdot x + b$$



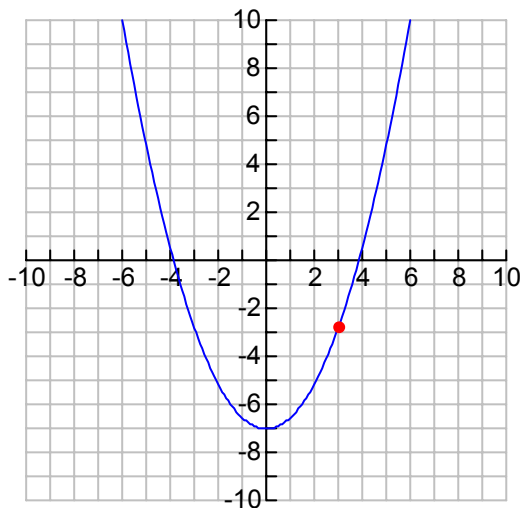
$$\text{solve}(f(x) = l(x), x) \quad \text{solve}(g(x) = l(x), x)$$

4.4 Hints

hint 1

We considered the function: define $f(x) = \frac{x^2}{c} - 7$.

Below is the graph of f . Using the slider bar you can change the value of c .



The following question may guide you towards the solution.

- (i) Investigate what happens to the parabola when you change the value of c .
- (ii) In the graph the point A with x -coordinate 3 is shown.

$$A := [3. \quad f(3.)] = [3. \quad -2.71429]$$

What happens to point A if the value of c changes?
Explain this using the formula for f .

- (iii) There is one point that does not move. Which one? Explain why, using the formula for f .

hint 2

We considered the function: $f(x) = \frac{x^2}{c} - 7$.

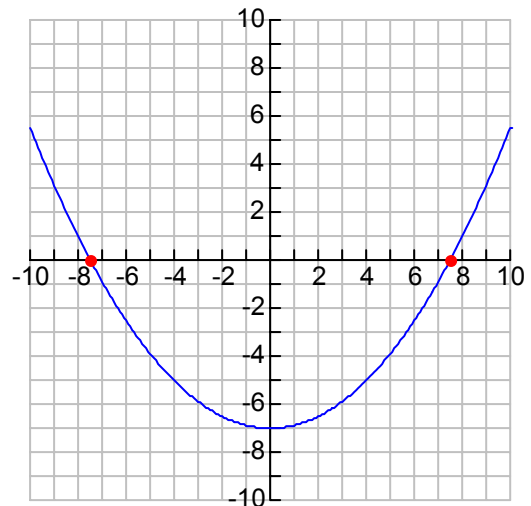
Alongside is the graph of f .

Using the slider bar you can change the value of c .



The following questions may guide you towards the solution.

- (i) Using the command **solve**(you can calculate the coordinates of the intersection point of the parabola with the x -axis. Consult the TII help if you don't remember the syntax for **solve**(.



- (ii) Calculate the coordinates of the intersection point(s) of the parabola and the x -axis in general. The answer contains the letter c . Explain why.
- (iii) Two solutions are found. State which solution corresponds to each intersection point.
- (iv) The condition that the distance between the intersection points equals 6 units, implies that the zeros are $x = 3$ and $x = -3$. Find out how you can use the result of 2. to calculate the value of c .

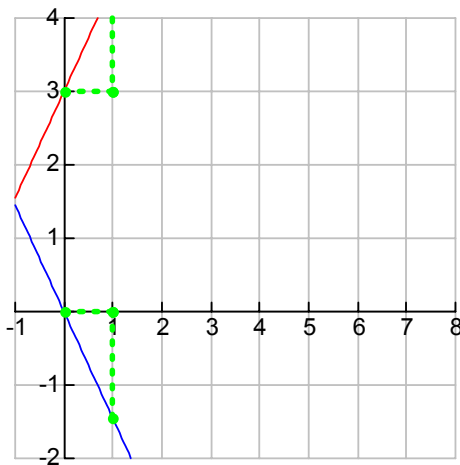
hint 3

We have considered the following functions:

$$\text{define } f(x) = c \cdot x$$

$$\text{define } g(x) = 3 - c \cdot x$$

Below you see the graphs of f and g . Using the slider bar you can change the value of c .



The following questions may guide you towards the solution.

- (i) Investigate what happens with the lines and the intersection points when you change the value of c .
- (ii) Complete: If the value of c gets smaller, the intersection point ...
- (iii) Try to explain your findings using the formulas.
Think about what happens with the y -intercept of the lines and with the slopes.

hint 4

We have considered the following functions:

$$\text{define } f(x) = c \cdot x$$

$$\text{define } g(x) = 3 - c \cdot x$$

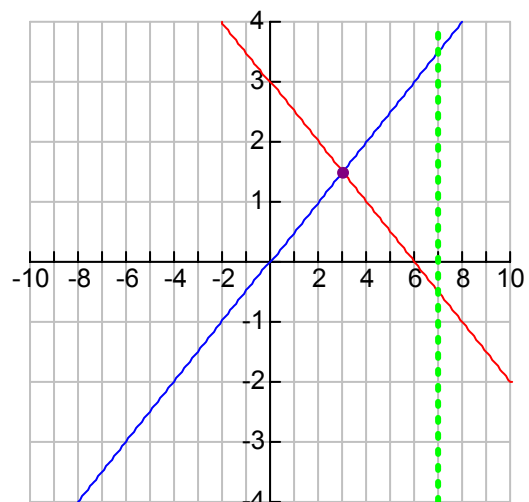
Alongside you see the graphs of f and g .

Using the slider bar you can change the value of c .



The following questions may guide you towards the solution.

- (i) Using the command **solve**(you can calculate the coordinates of the intersection point of the two lines. Consult the manual if you don't remember the syntax for **solve**(.



- (ii) Calculate the coordinates of the intersection point in general. The answer contains the letter c . Explain why.
- (iii) Find out how you could use the results from (ii) to answer the question when the intersection point is to the right of the line $x = 7$.

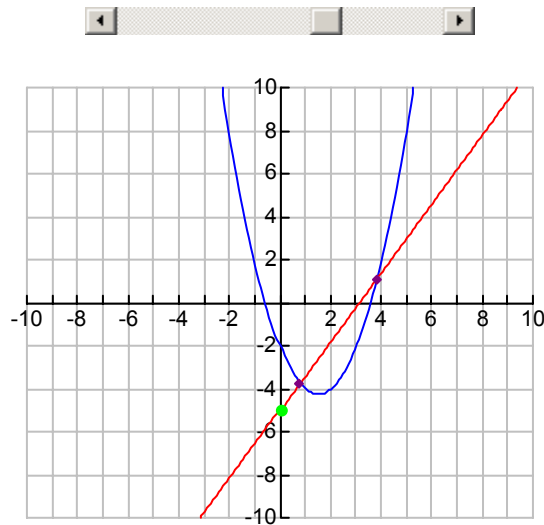
hint 5

The coordinate plane below shows the graphs of the functions f and g :

$$\text{define } f(x) = x^2 - 3x - 2$$

$$\text{define } g(x) = a \cdot x - 5$$

The lines all go through the point $(0, -5)$. (Think about how you can see that in the formula for g). With the slider bar below you can change the value of a and thereby the slope of the line.



In the working below the x -coordinates of the intersection points of the graphs of f and g are calculated twice using the **solve** command, with the first calculation being for the value of a that is chosen in the slider bar. In the second calculation no value has been chosen for a and the solutions that are provided depend on a .

$$\text{solve}(f(x) = g(x), x) \quad x = 3.81327 \text{ or } x = .786725$$

$$\text{delvar}(a)$$

$$\text{solve}(f(x) = g(x), x)$$

$$x = \frac{-\left(\sqrt{a^2 + 6a - 3} - a - 3\right)}{2} \text{ or } x = \frac{\sqrt{a^2 + 6a - 3} + a + 3}{2}$$

The following questions may guide you towards the solution.

- (i) Substitute $a = 1$ in the second set of solutions. Verify that this yields the same solutions as those obtained from the first method of solving.
- (ii) Find a value of a for which the line and the parabola do not intersect. Substitute this value into the solutions obtained using the second method for solving. Explain why this does not yield a solution.
- (iii) Use the general solution of the equation $f(x) = g(x)$ to find out for which value of a there is exactly one intersection point.

5. Modelling a basketball throw

Goal of the task

The main aim is to bring a familiar “real-life” application into the mathematics classroom and provide the need for the use of linear and quadratic functions. The task also helps to relate skills of data-handling (statistics) to those of algebra.

Target group and required time

All these prerequisites are generally (more or less) well known by students of 11th grade after the first semester or 10th grade at the end of the academic year. The task requires approximately 3 to 5 lessons in the computerlab, depending on whether the additional task is completed.

Preliminary TI InterActive! skills

The students need some basic skills in using TI InterActive!, in particular the use of the Data Editor, Statistical plots (scattergraphs), formatting graphs (axes and labels), superimposing functions on graphs and performing statistical calculations such as linear and quadratic regressions. Students familiar with the underlying theory of motion under gravity may use the Math Box to solve equations and/or compute derivatives.

Preliminary mathematical skills

Students should be familiar with the notion of a displacement-time graph.

Students need to appreciate how the shape of a scattergraph can suggest the form of an equation of a curve of good fit. Students need to understand the effect of the parameters a , b in the linear function $x = a + bt$. They should also have met transformations of the quadratic function in the form: $y = c + d.(t - e)^2$ and be able to relate these to translations and change of scale.

Students should have met the basic idea that statistical software can compute “best-fit” functions to represent data but they need not have knowledge of the algorithms used (or what “least-squares” means).

Students who have studied some physics or applied mathematics should be able to relate the linear and quadratic functions to the equations of motion under constant acceleration and may be in a position to use simple methods of differential calculus.

File organization

The task consists of the following linked TII files:

- Notebook.tii: The file where the student writes the solutions and the starting point containing hyperlinks to the other files.
- MainTask.tii: Describes the task and contains hyperlinks to Hint files and Help files.
- HelpA1.tii and HelpA3.tii: contain some technical help to perform some of the required tasks of the Main Task.
- HintA2.tii and HintB2.tii: Contain mathematical suggestions on fitting functions “by eye”.
- AdditionalTask.tii: Contains an additional task for the brighter or faster student.
- Solution.tii: file Contains possible solutions for each of the tasks.
- GeomScreen.tii: A file containing two screen dumps to illustrate possible approaches using Dynamic Geometry Systems.

The hyperlinks only work if the files are installed in the map c:\TII\Basketball. Ideally, the teacher should delete the link that exists from the Notebook file to the Solution file before the students start to work!

Classroom organisation

The lesson can be introduced by the teacher using a computer displayed through a data-projector. The follow-up work involves the students working either individually or in pairs at computers during the normal mathematics classes or in their own time. The above files are organized and detailed in such a way that the students should be able to work without the presence of their mathematics teacher.

Didactical suggestions

In order to set the scene it might be useful to arrange to have a basketball in the class to give the idea of a “trajectory”. Then using software such as Windows Media Player the teacher can project the video clip “Basket.avi” onto a whiteboard and students can mark positions of the ball in flight.

Technical hints

Students start by opening the notebook file in TII. It is recommended that the main task is opened in a separate window so that two versions of TII are activated. By means of a right mouse click in the Windows menu bar the option ‘Windows Cascade’ offers opportunities for switching easily between the files. However, if the same hyperlink is used twice two versions of the same file will be opened! This may be a source of confusion and therefore requires some attention by the teacher.


In case the data in the table in the Main Task file are different from the ones on page 63, please consider changing you Windows regional setttings to English (US) and install the TII student units files again.

As TII only supports absolute hyperlinks and no relative links, the links need to be adjusted after installation of the files, for example on the school network.

5.1 Main task

intro

The main task consists of three exercises (A, B, C) in which you will have to use, among others features, the stat plot tool, function plotting and statistical calculations. If you do not feel familiar with one of the above tools then

consult the help links provided in this document or use the TI InterActive! Help icon .

The mathematical skills you need to do the exercises are related to distance-time graphs, velocity and acceleration, linear and quadratic functions and their graphs. Some links to mathematical hints are also provided.

task

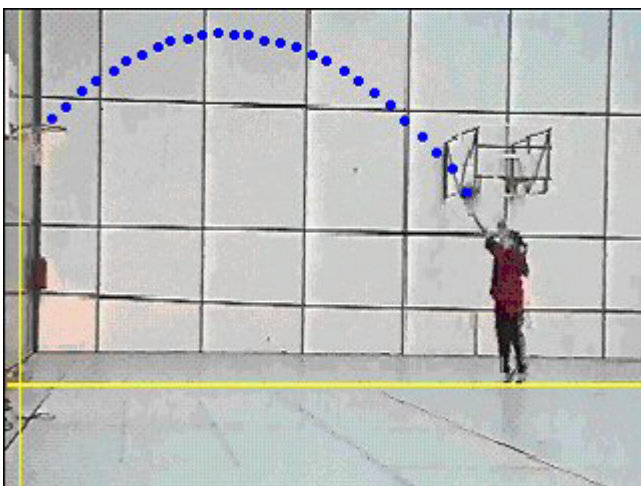
Description

The figure below shows some of the positions of a basketball during a shot at the basket.

Using the floor as the x -axis and the wall as the y -axis the co-ordinates of the ball, together with the elapsed time were captured using the Vidshell 2000 software by Doyle V. Davis which can be downloaded from:

<http://webphysics.tec.nh.us/vidshell/vidshell.html>

The video clip is in the file [Basket.avi](#) and comes from the DAVID project at the University of Munich, Germany.



x	y	t
5.15	2.22	.00
4.99	2.50	.04
4.81	2.68	.08
4.64	2.87	.12
4.44	3.05	.16
4.27	3.23	.20
4.09	3.37	.24
3.90	3.51	.28
3.74	3.63	.32
3.54	3.74	.36
3.37	3.81	.40
3.19	3.90	.44
3.00	3.95	.48
2.82	3.97	.52
2.63	4.04	.56

The x - and y -coordinates are measured in metres from the origin which is below the basket. The time values are measured in seconds (the video has 25 frames per second.)

Task A

- Draw the scattergram of horizontal displacement x against time t .
Choose suitable scales for the axes and include labels.
- Fit a linear function of the form $p(t) = a + b.t$ to the data "by eye".
- Find an appropriate regression model for x against t .

Help 1

Hint 1

Help 2

Task B

- Draw the scattergram of vertical displacement y against time t .
Choose suitable scales for the axes and include labels.
- Fit a quadratic function of the form $q(t) = c + d(t - e)^2$ to the data "by eye".
- Find an appropriate regression model for y against t .

Hint 2

Task C

- a. Draw the scattergram of vertical displacement y against horizontal displacement x .
Choose suitable scales for the axes and include labels.
- b. Fit a quadratic function of the form $r(x) = f + g(x - h)^2$ to the data "by eye".
- c. Find an appropriate regression model for y against x .

5.2 Additional task

task

Velocity and Acceleration

If you ignore air-resistance, spin etc., the only force acting on the ball during its flight is the Earth's gravitational attraction which is in the vertical direction. Now you will see what information you can draw from the linear and quadratic mathematical models you constructed in the Main Task.

Remember that with the co-ordinate axes used in this task positive x is measured to the right and positive y is measured upwards. Take care to give the correct units for velocities and accelerations and to make sure that they have the correct sign (positive or negative).

a. Horizontal velocity

From the linear regression model for the horizontal motion:
estimate the value of the (constant) horizontal velocity.

b. Vertical velocity and acceleration

From the quadratic regression model for the vertical motion:

- (i) find its derivative to give a linear model for the vertical velocity,
- (ii) estimate the value of the vertical velocity at time $t = 0$,
- (iii) estimate the value of the vertical velocity when the ball enters the basket,
- (iv) estimate the value of the (constant) vertical acceleration g .

c. Velocity vectors

Using any of the three models and the information derived above, find:

- (i) the velocity vector u of the ball at the point of release ($t = 0$),
- (ii) the angle a that the vector u makes with the horizontal,
- (iii) the magnitude of the vector u ,
- (iv) the velocity vector v of the ball as it enters the basket,
- (v) the angle b that the vector v makes with the horizontal,
- (vi) the magnitude of the vector v .

d. Validation of angles

Using geometric instruments, such as a ruler and protractor,
check your answers to c(ii) and c(v) against the picture on page 1 of the Main Task.

5.3 Hints

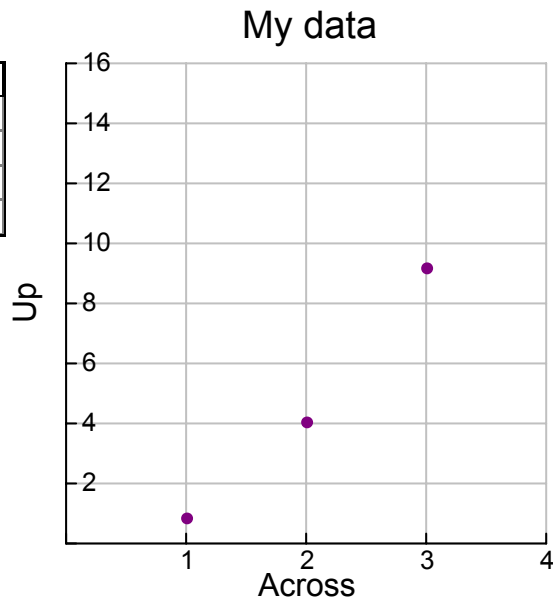
Fitting a linear function to data "by eye"

Suppose you have some data in lists and have plotted a scattergraph from them.

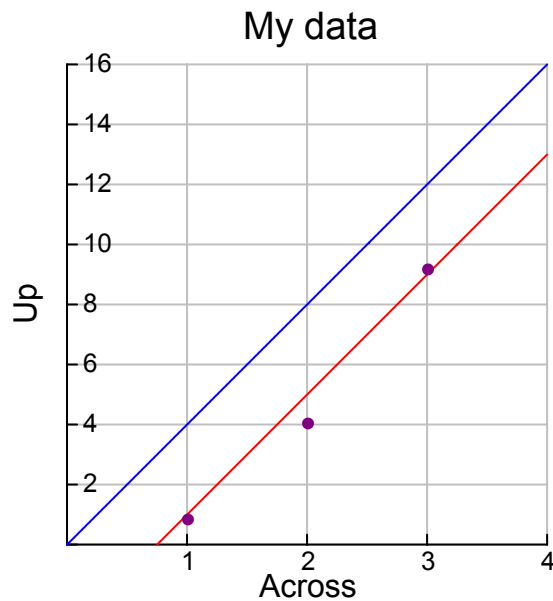
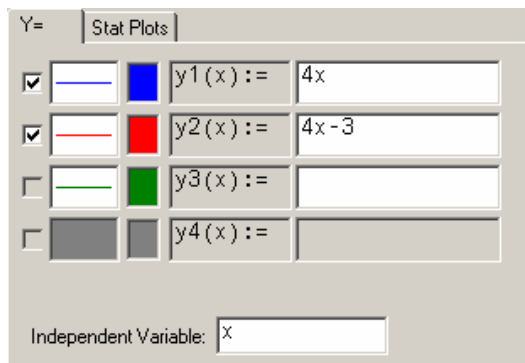
L1	L2
1	0.9
2	4.1
3	9.2

You can use the f(x) Tab from the Functions dialog box to enter the equation of a function whose graph you want to display over the scattergraph.

For example my first estimate of a suitable linear function might be $y = 4x$ which is entered in the first box, and plotted as a blue line.



I might think that a better model would be a line which was closer to the points, and I can enter another choice e.g. $y = 4x - 3$ in the second box and plot it as a red line. In this way you can build up better estimates of functions which give a "good fit" to the data.



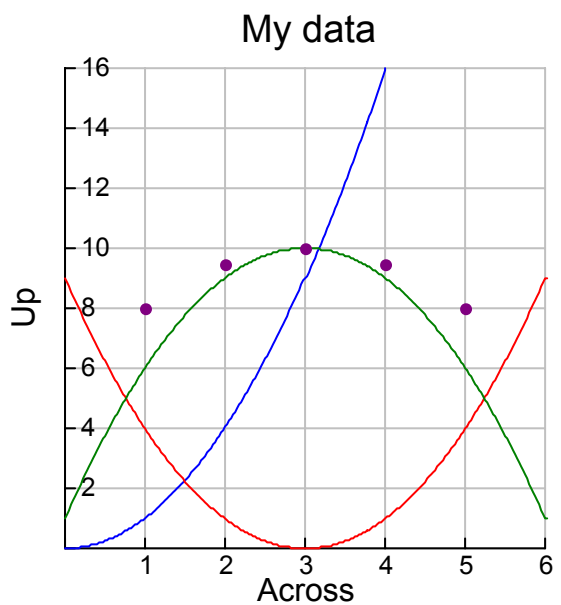
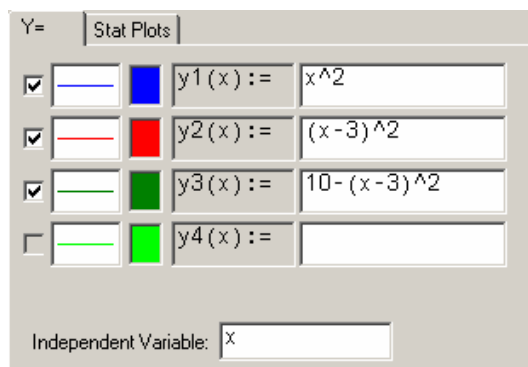
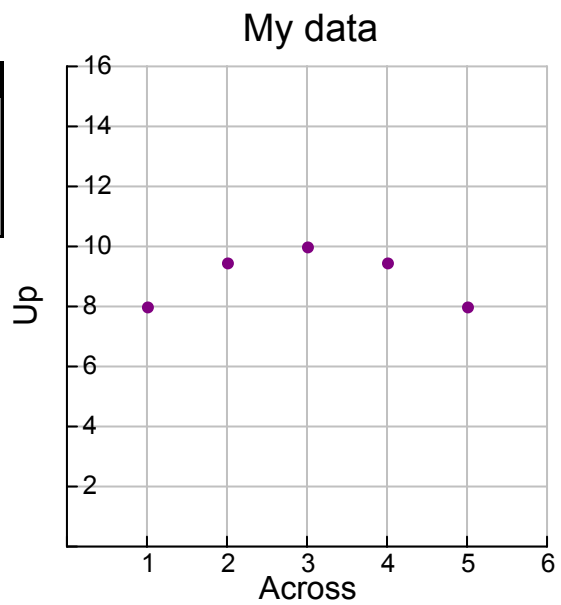
Fitting a quadratic function to data "by eye"

Suppose you have some data in lists and have plotted a scattergraph from them.

L1	L2
1	8
2	9.5
3	10
4	9.5
5	8

You can use the $f(x)$ Tab from the Functions dialog box to enter the equation of a function whose graph you want to display over the scattergraph.

In this case it looks as if a quadratic function might give a good fit. The vertex is at (3,10) and the curve slopes downwards so we need to find transformations of the basic quadratic function $y = x^2$ to give a curve with the desired characteristics.



The blue graph is for $y = x^2$.

The red graph is for $y = (x - 3)^2$.

The green graph is for $y = 10 - (x - 3)^2$.


Can you find the equation of a quadratic function which passes through all 5 data points?

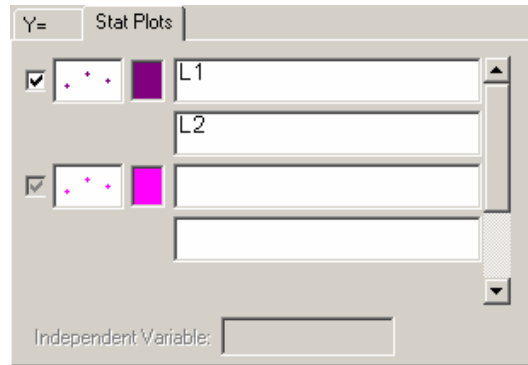
5.3 TI InterActive! help

Drawing a scattergraph from data in lists

With some data entered in lists, such as L1 and L2, highlight the list you want plotted on the x-axis by clicking on the name e.g. L1 at the top. To select the list for the y-axis, hold down the CTRL key while clicking the name, for example L2, at the top.

L1	L2
1	0.9
2	4.1
3	9.2

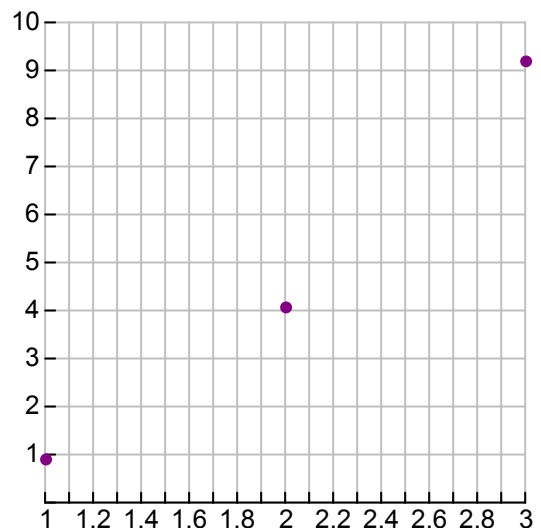
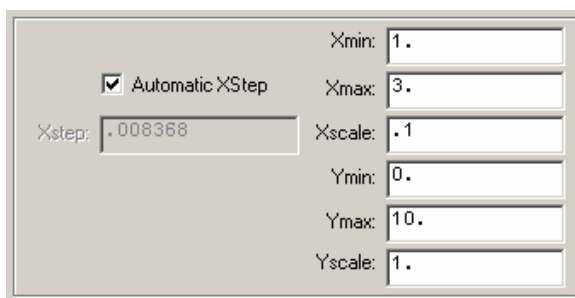
Then double click  on the Graph icon in the toolbar.



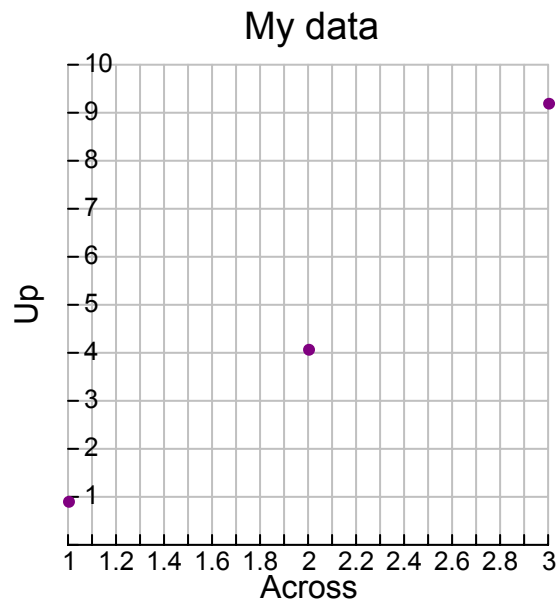
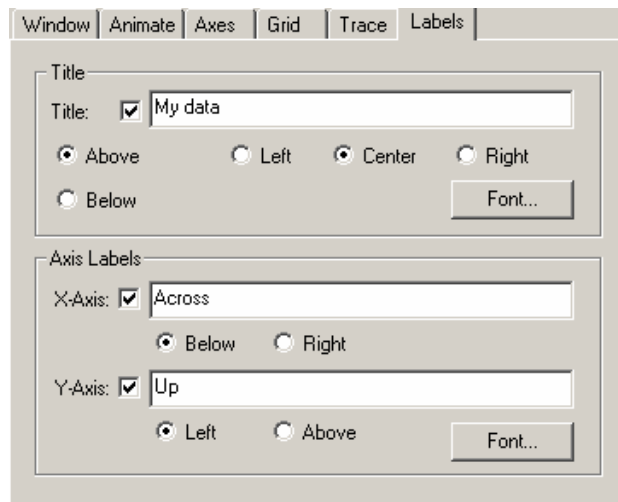
This brings up the Functions window which shows that the lists L1 and L2 have been selected and that a scattergraph will be drawn. The scales for the axes have been chosen by the software.

Click on the brown FORMAT button to bring up the dialog box for you to enter your own preferences.

From the Window tab you can enter your own choices for the Minimum, Maximum and Step values for each of the axes.



Clicking on the Labels tab you can give the graph a title and label each of the axes.



help 2

Fitting a regression model to data in lists

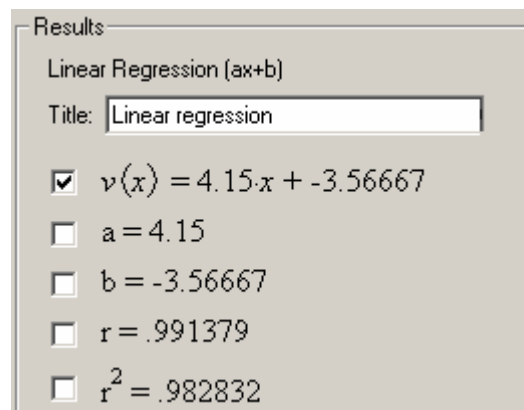
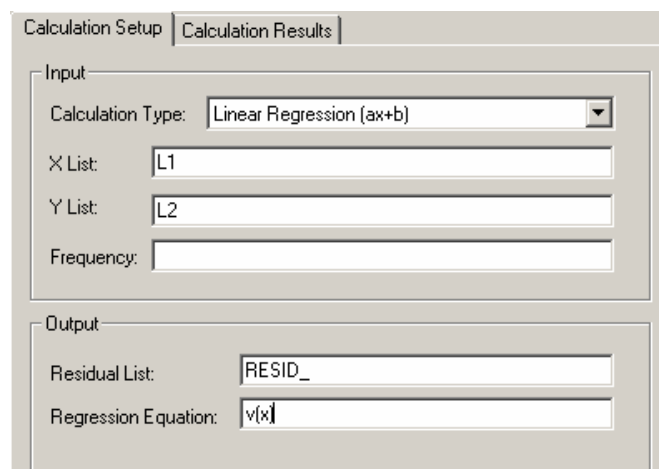
With some data entered in lists, like in L1 and L2, highlight the list you want for the independent variable by clicking on the name e.g. L1 at the top. To select the list for the dependent variable hold down the CTRL key while clicking in the name e.g. L2 at the top.

L1	L2
1	0.9
2	4.1
3	9.2

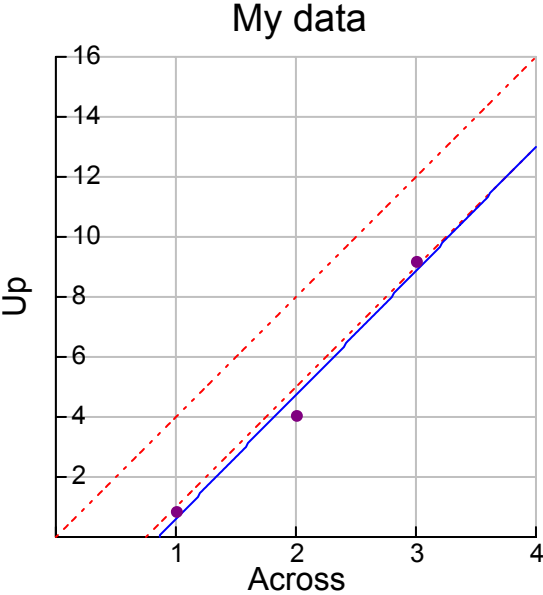
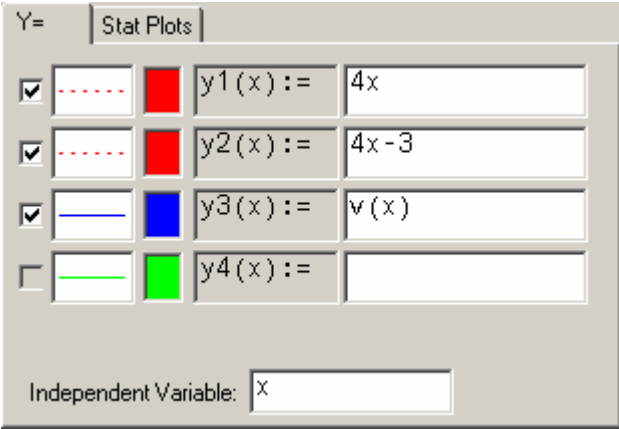


Then double click on the StatCalc icon, , in the toolbar.

Click on the arrow alongside the "Calculation Type" box and select from the list, which in this case is Linear Regression. The names of your selected lists should appear in the X-list and Y-list boxes, but you can type them in or edit them if you prefer. In the last box type a sensible name to be used for the regression function. Then click the Calculate button to see what the software computes as the regression ("best fit") function. You can choose which parts of the computation will be saved to your document.



Now you can open the Grapher and enter the name of the calculated regression function e.g. $v(x)$ to plot its graph. Here the two "by eye" functions from previous work are shown as dotted lines and the regression function by the blue graph.



6. Periodic waves

Goal of the task

In physics students need some mathematical skills to understand the phenomenon of periodic waves. To understand waves is essential in physics, so the mathematical model for waves ought to be a central part of a study in physics. The goal of this task is to give students some associations between the mathematical model and the phenomenon in physics they are studying.

The main task is to study the mathematical model for the wave function and the effect on the graphical picture when we are changing the values for the parameters in the function. Using the hints should enable students to find the connection between the values of the parameters and the shape of the graph.

Target group and required time

The target group for this task are eleventh-grade mathematics and physics students (17 years old) who are unfamiliar with both the effect of changing parameter values in a parametrised function and mathematical modelling.

The task requires approximately 4 to 6 lessons in the computerlab with additional time at home. The work could contribute to the students' understanding of mechanical waves, particularly if done after an introduction into the topic.

Preliminary TI InterActive! skills

The students need some basic skills in using TII for graphing and for applying algebraic commands such as the solve command.

Preliminary mathematical skills

The students need some knowledge about graphing and about trigonometric functions.

File organization

The task consists of the following linked TII files:

- Notebook.tii: The file in which the student writes solutions and the starting point containing hyperlinks to the other files.
- MainTask.tii: The file that briefly describes the task.
- Hint1.tii, Hint2.tii, ..., Hint5.tii: Files that suggest a question to start with if the student doesn't know what to do with the task.
- Addition_Physics.tii: Contains an additional task which focuses on the relationships between the representations in mathematics and physics.
- Definition.tii: File containing some basic knowledge concerning periodic functions
- Meaning_phys.tii: File containing the different representations in mathematics and physics
- Solution.tii: Contains the solutions.

The hyperlinks only work if the files are installed in the map c:\TII\PeriodicWaves.

Technical hints

Students start by opening the notebook file in TII. It is recommended that the main task is opened in a separate window so that two versions of TII are activated. By means of a right mouse click in the Windows menu bar the option 'Windows Cascade' offers opportunities for switching easily between the files. However, if the same hyperlink is used twice two versions of the same file will be opened! This may be a source of confusion and therefore requires some attention by the teacher.

As TII only supports absolute hyperlinks and no relative links, the links need to be adjusted after installation of the files, for example on the school network.

Physical Meaning

description

In accordance with one of the theses in quantum mechanics, all particles in nature can be associated with a wave. Thus it is of great importance that we understand the mechanism which influences the movement of the waves.

With a PERIOD we mean a movement that repeats itself after a constant time lag T .

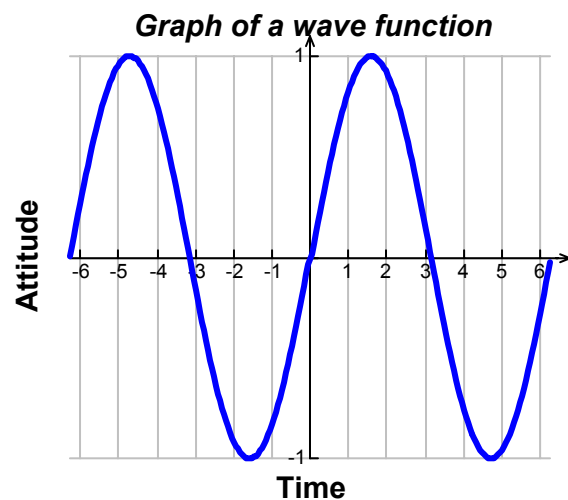
We say the movement is periodic with T as period.

The wave function f is given by
 $f(x) := A \cdot \sin(B \cdot x + C) + D$.

With given values:

$A := 1$, $B := 1$, $C := 0$ and $D := 0$.

We get the graph shown at the right.



This task is a study of wave functions, which are also called functions for harmonic waves. The investigation is connected to the vital importance that values of the parameters A , B , C and D have on the shape of the graph.

The additional task concerns phenomena in physics which are influenced by the parameter values in the general formula of the wave function.

period

Period: If any quantity is a function of the time, and this function repeats itself exactly after constant time intervals T , the quantity is said to be periodic, and T is called the period of the function.

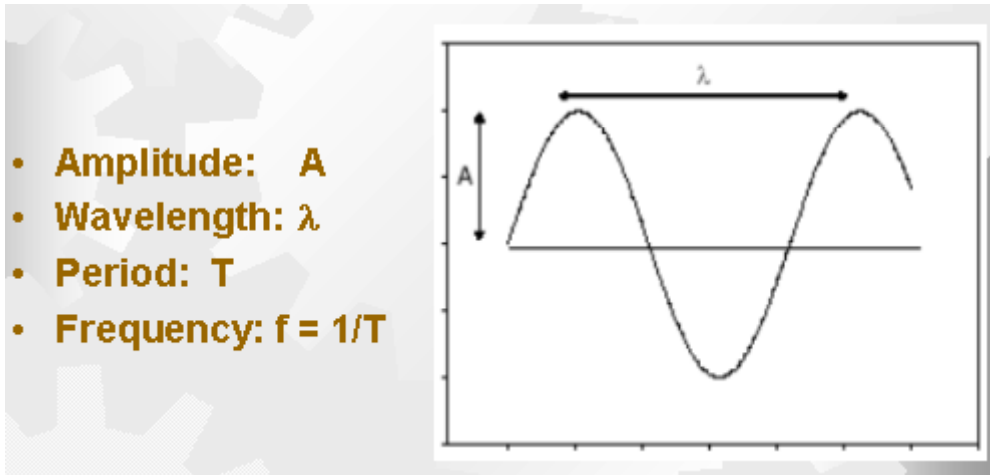
interval

Time interval T : The time a movement is using to swing a period.

wavelength

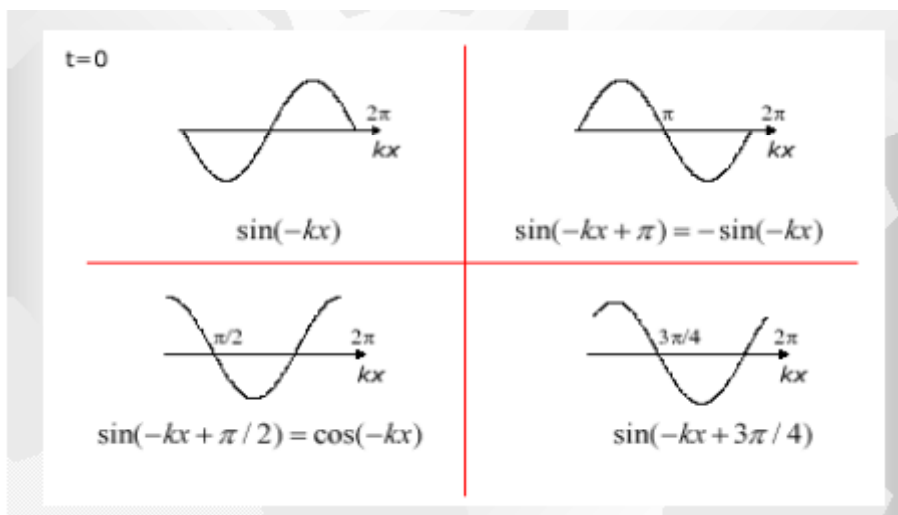
Wavelength λ : The distance between successive points of equal phase of a wave.

Phase: Points in the path of a wave motion are said to be points of equal phase if the displacements at those points at any instant are exactly similar; i.e. of the same magnitude and varying in the same manner.



The **velocity** of the wave: $v = \lambda \cdot f = \frac{\lambda}{T}$.

The following graphs illustrate the phase shift:



6.1 Main task

task

The general wave function f is given by $f(x) := A \cdot \sin(B \cdot x + C) + D$.

In what way is the graph of f influenced by the values each of the following parameters?

- a. Parameter A
- b. Parameter B
- c. Parameter C
- d. Parameter D

Hint 1

Hint 2

Hint 3

Hint 4

6.2 Additional physics task

intro

The wave function in mathematics is often represented as follows: $f(x) = A \cdot \sin(B \cdot x + C) + D$.

In physics, the following representation is most common: $f(x) = A \cdot \sin(k \cdot (x - v))$ with $v = \lambda \cdot f = \frac{\lambda}{T}$.

Here v is the wave velocity, λ is the wavelength, f is the frequency (and not the name of the function anymore!) and T is the period.

task

- a. Which parameter determines the amplitude?
Give an example which shows the physical effect.
- b. How do we decide on the wave length for the model?
Give an example which shows the physical effect.
- c. In what way is the graph of the wave function influenced by the wave's frequency and velocity?

Hint 5

6.3 Hints

hint 1

The value of the parameter A

Ensure that TI InterActive! is set to radians!

In what way is the graph influenced by the values of the parameter A ?

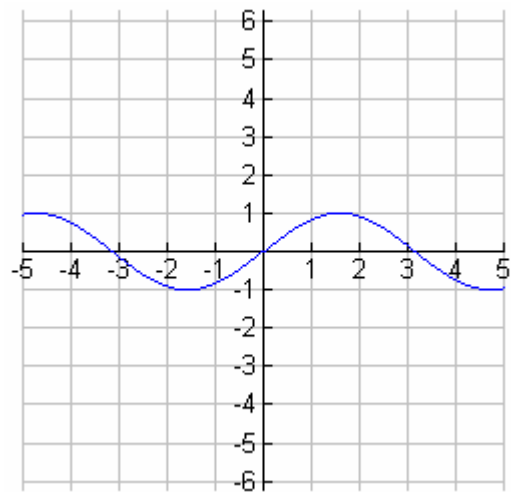
The function f is given by $f(x) := A \cdot \sin(B \cdot x + C) + D$.

Given values: $B := 1$, $C := 0$ and $D := 0$.

The value to A can be changed using the slider.



$$f(x) = \sin(x)$$



hint 2

The value of the parameter B

In what way is the graphical picture influenced by the values given to the parameter B ?

Ensure that TI InterActive! is set to radians!

We are using a reference curve $g(x) := \sin(x)$.

The function f is given by $f(x) := A \cdot \sin(B \cdot x + C) + D$.

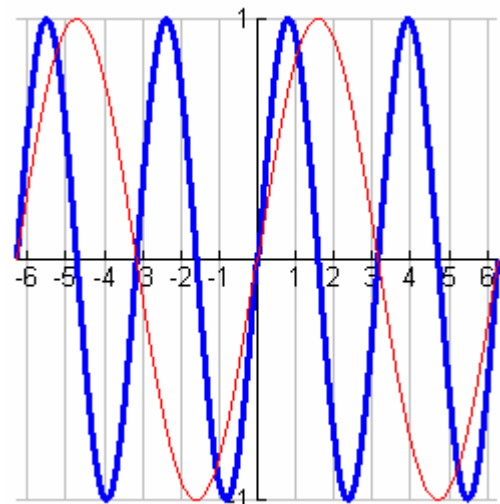
Given values: $A := 1$, $C := 0$ and $D := 0$.

The value of B can be changed using the slider.



$$f(x) = \sin(2x)$$

You can calculate the zeros with the solve command.



hint 3

The value of the parameter C

Ensure that TI InterActive! is set to radians!

In what way is the graph influenced by the values of the parameter C ?

We are using a reference curve: $g(x) := \sin(x)$.

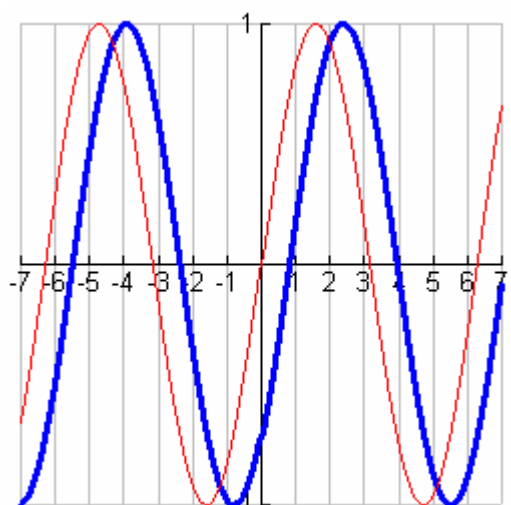
The function f is given by $f(x) := A \cdot \sin(B \cdot x + C) + D$.

Given values: $A := 1$, $B := 1$ and $D := 0$.

The value for the parameter C can be changed using the slider.



$$f(x) = \sin(x - 0.8)$$



The value of the parameter D

Ensure that TI InterActive! is set to radians!

In what way is the graphical picture influenced by the values given to the parameter D ?

The function f is given by $f(x) := A \cdot \sin(B \cdot x + C) + D$.

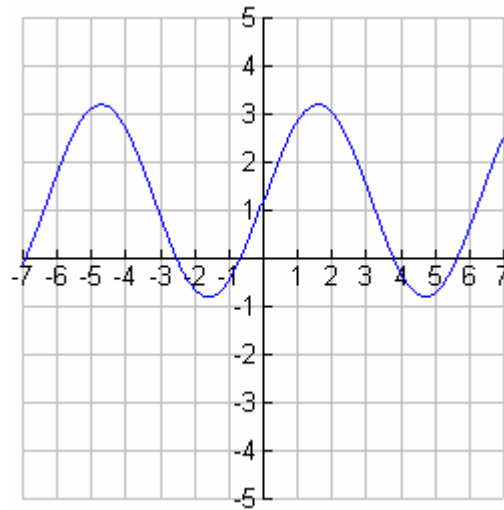
Given values: $A := 2$, $B := 1$ and $C := 0$.

The value to D can be changed using a slider.



$D = 1.2$

$$f(x) = 2 \cdot \sin(x) + 1.2$$



The value of the parameter A

We graph a reference curve: $g(t) := A \cdot \sin(t)$.

Instead of changing the parameter's value directly, we can by use a slider to study the dynamics.

The function f is given by $f(t) := A \cdot \sin(k \cdot t + C)$.

The value of parameter A :



$A = 2.5$

The value of parameter k :



$k = 2$

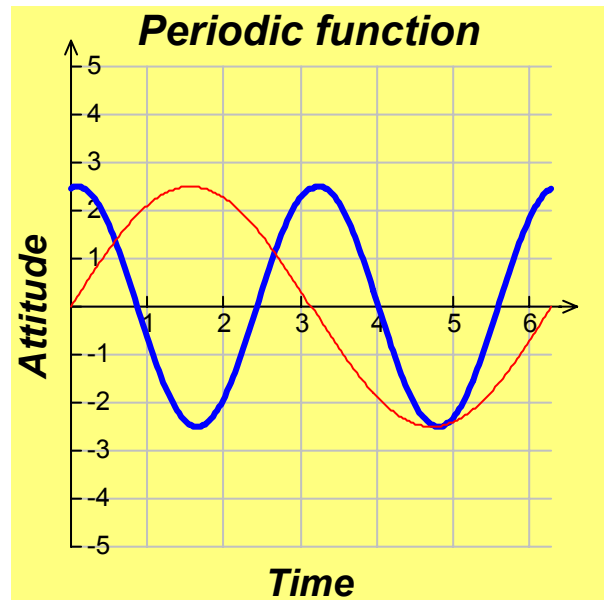
The value of parameter C :



$C = 1.4$

This is the graph of the function

$$f(t) = 2.5 \sin(2t + 1.4)$$



7. Plant evapo-transpiration

Goal of the task

This task deals with a specific aspect of plant physiology, together with clear physical implications. Students require access to a Texas Instruments CBL 2™ datalogger and Biology gas pressure, managed by a PC employing TII functionality.

A small sample branch with leaves (250-300 mm length, base diameter 5-6 mm) is connected to a small rubber tube, almost completely filled with water. The other end of the tube is inserted into the pressure sensor. The water that evaporates and transpires from the leaf surface produces a pressure drop and this is collected by the pressure sensor for a specified time period.

Through use of samples from different species or samples in different vegetative stages, or by simulating different environmental conditions (temperature, humidity, wind) it is possible to point out significant aspects of water balance in vegetables.

Target group and required time

The task is mainly targeted at students of 15-16 years of age (secondary school, 2^o-3^o year) who have the required prerequisite knowledge which is given below. The required time for the unit is:

- Two hours classroom lessons
 - Before the experiments: Task description, prerequisite knowledge check and remedial work
 - After the experiments: Discussion of the results and consideration of the issue.
- One/two hours biology lab session
 - Collection of data
- Two hours computer lab sessions
 - Processing and analysis of data
 - Lab report draft

Lab report draft and preliminary TI InterActive! skills

The students need basic computer skills, such as file management, word processing, printing and spread sheet skills. As far as TI InterActive! is concerned, students should be able to:

- use the Text Editor tools,
- use Quick Data tools,
- use the List Editor,
- calculate a regression equation,
- produce graphs of data and equations.

Specific skills for this unit include the ability to use the CBL 2, a TI-83 Plus and the Datamate application.

Preliminary knowledge of biology, physics and mathematics

The required preliminary biological knowledge includes:

- vegetal cell structure,
- foliar structure,
- functions of water in plants,
- structure and stomatic function,
- plant nutrition.

The required preliminary physical knowledge includes:

- changes of phase: evaporation,
- gas pressure.

The required preliminary mathematical knowledge includes:

- Understanding the meaning of a regression equation in a data series,
- Being able to interpret the graphs of a data series and regression equation.

File organization

The task consists of the following linked TII files:

- Notebook: the file in which the student writes the solutions. This is the starting point and contains hyperlinks to the other files.
- Main_Task: Contains the task for the students.
- Main_Task_Hint: Provides suggestions for the development Lab_Activity as well as information about the procedure to perform.
- Main_Task_Solution: Provides the Main_Task results and answers to the questions.
- Lab_Activity: Contains lists of requested materials and procedures.
- Lab_Activity_Hint: Provides suggestions for each stage of the lab activities.
- Control: Proposes some elements of reflection to verify if the task has been carried out correctly.
- Control_Hint: Provides support/help for the Control_Task.
- Control_Solution: Contains the Control task results and answers to the questions.
- Additional: Provides optional supplementary questions for students.
- Additional_Task_Hints: Provides suggestions for the Additional_Task.
- Additional_Task_Solution: Contains the Additional task results and questions.

The hyperlinks only work if the files are installed in the map c:\TII\Evapotranspiration. Students can start with the Notebook file in c:\TII\Evapotranspiration.

Classroom organization

To understand the meaning of the activity students have to know the role of evapo-transpiration with respect to water metabolism and mineral nutrition.

The task concerns an aspect of plant physiology and therefore it should be carried out in that particular disciplinary context. Initially it is necessary to discuss:

- the difference between evaporation and transpiration, and,
- conditions that influence evapo-transpiration process intensity.

During the experimental activity it is useful to set up groups of 3-4 students with different samples. Each group investigates how the assigned sample reacts to environmental conditions. Each group acquires data at room temperature and at a temperature of about 30°C (lit by a spotlight for a video camera).

The students' final product is a lab report, which can be evaluated by the teacher according to:

- Procedural correctness in experimental activity and data process,
- Completeness,
- Quality of answers to questions.

Technical hints

To install the task, double click on EVAPOTRANSPIRATION.TASK.exe file. Students can start with the Notebook file in the folder c:\TII\Evapotranspiration.

During the experimental activity it is possible to come up against the following difficulties:

- The sensor does not measure any pressure variation. The cause of this error can be:
 - an air bubble developed in the tube that links the sample to the pressure sensor.
 - an air bubble developed in the insertion point of the sample in the tube that links the sample to the pressure sensor.
- After several tests of data collection it is possible that the system pressure decreases and it can contrast with the sample “suction force”. In this case it is useful to disconnect the little tube from the sensor for a few seconds and then link it again.
- When you start data collection with QUICK DATA TOOL, an error message is often shown: “COMMUNICATION ERROR. VERIFY CONNECTION AND COMMUNICATION SETTING THEN RETRY”. When this happens it is often useful to select “Retry” a number of times to start data collection.

Didactical suggestions

Important aspects to point out during the results discussion:

- The process trend is the same in all groups independent of the type of sample. The samples have offered resistance to dehydration by starting a biological process of defence.
- The intensity of the evapo-transpiration process depends on the vegetative stage of the sample. In spring/summer time the process is generally more intense than autumn/winter time.

As a development activity students could test different environmental conditions to point out particular reactions of some vegetable species.

7.1 Main task

intro

Water metabolism is a fundamental part of plant physiology. Water, by cell's turgidity, performs structural functions as well. All plants wither when water metabolism is lacking. When this variation occurs for a long time plants shrivel up and die. In addition, plants satisfy the need for mineral nutrition by water metabolism. Root apparatus, absorbing the circulating solution of the soil, allows plants to soak up macro elements (N, P, K, S, Ca, Mg, Fe) and microelements (B, Mn, Cu, Zn, I).

The term Evapo-transpiration indicates two concomitant processes: the evaporation of a physical type and the transpiration of a biological type.

Transpiration is a complex process founded on delicate self-control systems. The excessive transpiration of the plant damages the cellular turgidity while insufficient transpiration depresses the mineral nutrition and therefore the growth of the plant.

Stomata opening, the basis of the process of transpiration, is also justified by the need of the plants to absorb and metabolize CO₂ through the photosynthetic process.

Transpiration can take place through the cuticle, the stomata and the lenticelles. Stomatic transpiration is the most important process because it assures the plant of necessary water flow.



Picture 1: *Pothos aureus* L - Stoma with chloroplastis.

In this experiment we want to compare the necessity of water for the different plant species, Laurel (*Laurus nobilis* L.), Lemon (*Citrus lemon* L.), Olive (*Olea europaea* L.) and Orange (*Citrus sinensis* L.) under constant environmental conditions.

The deduction is indirect. The sample evapo-transpiration is evaluated through the measurement of the drop in pressure and so evapo-transpiration is collected and measured by a pressure sensor.

It would be correct to correlate pressure variation with leaf surfaces actually transpiring; but, to simplify the experiment we will correlate pressure variation – due to the water loss – with the sample mass.

task

Hint 1

- a. Complete the lab activity. For a description click on the Lab procedure link at the bottom of this page.
- b. Process the data:
 - (i) For each data set calculate the absolute value of pressure with reference to the starting value.
 - (ii) Produce and label a graph for each new data set.
 - (iii) Create a new list to relate the pressure to the sample mass.
 - (iv) Create a new list to relate the pressure to 100 cm² of leaf surface.
 - (v) Graph the results.
 - (vi) Calculate regression equations for the new data sets (related to the sample mass). To do this you will need to select an appropriate regression model, insert the results into the opened document and modify the equations so that the curves pass through the origin. Graph the regression equations.
- c. The process of evapo-transpiration shows the same trend in both conditions? How can we define it? Is it stationary, increasing or decreasing? Explain your comments.
- e. In which condition has the highest evapo-transpiration activity been recorded? Explain why you believe this is the case.
- f. How does the evapo-transpiration process evolve in the two tested conditions?
- g. Calculate the ratio between the two data sets of drop in pressure, relative to the highest and lowest intensity recorded for the evapo-transpiration.
- h. Give an explanation of the results both for the physics and biological aspects.

7.2 Additional task

task

Hint 2

- a. Copy data collected by another group into your Notebook file .
- b. Process the other group's data as requested in the Main Task file (Data processing).
- c. Compare the results of the two samples (your own data and the data of the other group) and write a reflection on the similarities and differences.

7.3 Control task

task

Hint 3

- a. Describe the trend in the graph (point 1 of “Process data” in Main_task) without using the absolute value of pressure.
- b. In this case what is the relationship between pressure and loss of water due to the evapo-transpiration process?
- c. How does the trend observed in the graph change (points 2 and 4 of “Process data” in the Main Task) after you have correlated the sample mass to the pressure? Compare your results with the findings of other groups.
- d. Explain how you can modify the regression equation so that it will pass through the origin. Find this regression equation. Compare your results with other groups' findings.

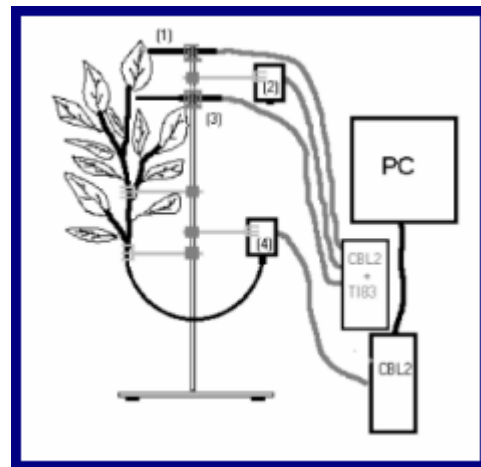
7.4 Lab activity

lab

Requested materials

- Gas Pressure Sensor
- Light Sensor Vernier – Range 6000 Lux
- Relative Humidity Sensor Vernier
- 1 ring stand with rod
- 2 adjustable clamps
- a PVC wash bottle with distilled water
- silicone grease for dryers
- spot for video (IANIRO 800 W Mod. 190MK1)
- 2 CBL 2 data loggers
- 1 TI-83 Plus SE
- PC with TI InterActive! installed

Experimental apparatus



1 - Light Sensor 2 - Relative Humidity Sensor
3 - Temperature Sensor 4 - Gas Pressure Sensor

- Prepare experimental apparatus as represented in picture.
- Start DATAMATE (software to control CBL 2 system) on your TI-83 Plus graphic calculator. and set the Relative Humidity Sensor (not auto ID). Read and record the values for brightness, relative humidity and temperature.
- Set TI InterActive! for data acquisition in QUICK DATA TOOL mode by selecting:
 - * sensor: **Pressure sensor**
 - * number of samples: **60**
 - * collection interval: **5 s**
 - * x-List: **Time**
 - * y-List:
 - test 1: **Pressure**
 - test 2: **Pressure1**
- Collect data. Environmental conditions:
 - * room temperature, lowest brightness conditions,
 - * temperature not over 30°C, brightness about 5000 Lux, (using spot for video, placed at right distance from sample).
- Transfer collected data to Notebook file.
- At the end of measurement defoliate the sample, weigh leaves, record their mass and estimate the total leaf surface area (using a piece of graph paper).

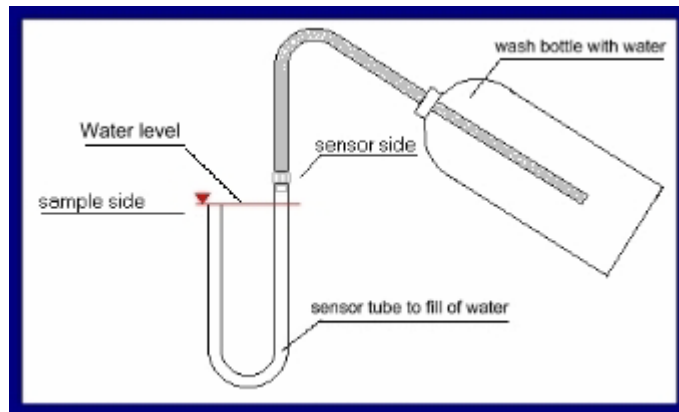
7.5 Hints

hint 1

a. Do the lab activity

Preparing Experimental Apparatus

- To ensure correct water flow to the sample, make a clean, oblique cut on the sample bottom, and place the sample into the little sensor tube, previously filled with water.
- When filling the sensor tube with water be careful to position the tube as shown in the picture below. It is important to avoid air bubbles forming while filling the tube.



- On the side of the sensor the tube must be empty for a length of 10 mm. You can do this by pressing the end of the tube (sensor side) with your fingers after having inserted the sample in the other side of the tube.
- Check that there aren't air bubble along the tube.

Data collection

- In the Notebook file open LIST EDITOR.
- If it is necessary to repeat the data collection test several times then it is advisable to disconnect the little tube from the sensor for a number of seconds, and then link it again.
- When you start data collection with the QUICK DATA TOOL, an error message is often shown: "COMMUNICATION ERROR. VERIFY CONNECTION AND COMMUNICATION SETTING THEN RETRY". If this happens, it is often useful to click "Retry" a number of times to start data collection.
- To measure the leaves surface area group (by superimposition) those with the same dimensions. For each group trace the contour of a leaf on a piece of graph paper. Estimate the leaf surface using the drawing. Calculate the leaves surface for the group's sample.

b. (i) Calculate the absolute value

Use the function **abs()**.

b (ii) Show the graph of each new data set

Use the the Graph Editor and Label option in the Draw menu. Refer to the Manual for more information.

b (iii)-(iv) Create a new calculated list

To calculate pressure variation per gram or to relate the pressure to 100 cm² of leaf surface, label a new list and insert a suitable formula.

b (v) Show the graph of results

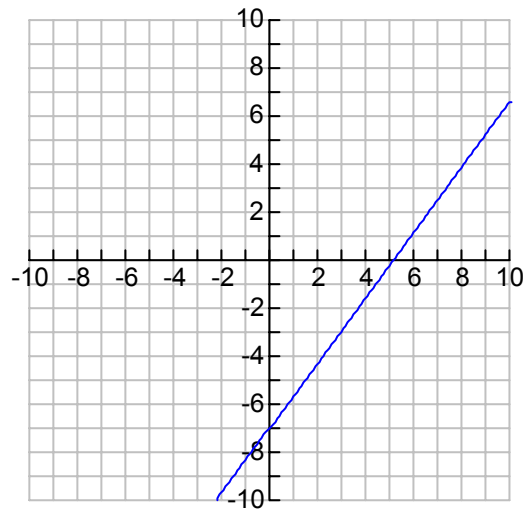
Refer to step 2.

b (vi) Calculate regression equations

To calculate the regression equation use the Stat Calculation Tool and choose the appropriate regression model (look at the graph before selecting a model). To modify the equations to pass through the origin follow the steps in the following example. You can modify the linear graph by changing the value of the coefficients **m** and **n**.

define $f(x) = m \cdot x + n$

m: [input field] n: [input field]



- c. Consider the trend in the graph.
- d. The Stat Calculation tool offers many statistical variables useful to evaluate the results.
- e. Compare the trend of pressure variation in the final graph.
- f. Use the two angular coefficients of the regression equations.
- g. For physics aspects you can correlate the experimental results to the evaporation phenomenon. To explain the biological aspects you can refer to the stomata activity and working of stomata opening.

hint 2

- a. Describe the trend that you expect to appear in the first graph if the pressure variation is negative.
- b. Reread the introduction section in the Main Task file. It explains the way in which the pressure sensor measures the sample loss of water. If the phenomenon increases, how does the pressure measured by the sensor change?
- c. Compare the graphs that were produced for points 2 and 4 of the data process. Describe any significant changes observed. Can a different mass for the sample (greater or less) modify the trend of the second graph with reference to the first? Is it important to correlate the pressure variation to the sample mass? In order to compare the results of different samples, do you think that it is important to take into consideration the mass difference?
- d. In the Main_Task file it is suggested that you may find it necessary to verify the meaning of the coefficients **m** and **n** into the equation $f(x)=mx+n$. If you have difficulty to answer, you can reread this point and then try to represent a graph using the regression equations without arrangement.

- a. Describe the trend that you expect to appear in the first graph if the pressure variation is negative.
- b. Reread the introduction section in the Main Task file. It explains the way in which the pressure sensor measures the sample loss of water. If the phenomenon increases, how does the pressure measured by the sensor change?
- c. Compare the graphs that were produced for points 2 and 4 of the data process. Describe any significant changes observed. Can a different mass for the sample (greater or less) modify the trend of the second graph with reference to the first? Is it important to correlate the pressure variation to the sample mass? In order to compare the results of different samples, do you think that it is important to take into consideration the mass difference?
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8. Electrolytes and conductivity

teacher guide

Goal of the task

This unit is unlike most of the other units in that it concerns science teaching rather than mathematics. The activity considers some aspects of properties of electrolytes. Two different phases can be clearly distinguished: the experimental stage in which students acquire data using data loggers, and the processing stage where students elaborate results and use TI InterActive! to answer the questions. The main intention of the unit is to understand the relationship between electrolytic solution conductivity and the properties of the ions present.

The experimental activity consists of:

- Use of a conductivity probe to collect preliminary data relating to the electrolytic conductivity of distilled water and other equimolar solutions (NaCl, CaCl₂, AlCl₃),
- Consideration of an acid-base reaction of neutralization, to measure the electrolytic conductivity variation through use of two titres with different solubility.

Students should understand the effect of ionic charges on the electrolytic conductivity of a solution and be able to determine the equivalence point through a conductimetric titration. Furthermore students should understand how the electrolytic conductivity of solutions is influenced by the substance's solubility.

TI InterActive! will be used as a tool to import data from a CBL 2/TI-83 Plus system, as a text editing environment where students produce their written reports, as a data list managing tool and as a data graphing tool.

Globally speaking, the task has an experimental approach and is structured as follows:

- Introductory phase,
- Experimental lab activity,
- Analysis and processing of data,
- Report phase.

Target group and required time

The target group of this unit is 15-18 year old students who have the required prerequisite knowledge. The required time for the unit is:

- 2 hours classroom teaching which includes the description of the task, checking of prerequisite knowledge, remedial work and discussion of the results,
- 2 hours in the chemistry lab to carry out the experiment and gather data,
- 2 hours in the computer lab for data processing, data analysis and for writing the final report.

Preliminary TI InterActive! skills

Students require basic computer skills in file management, word processing, spread sheets and printing documents. Students should also be able to do the following using TII:

- produce graphs (trace),
- apply commands from the algebra menu such as 'solve' and 'approximate',
- Use TII text editor functionalities,
- Import data lists from TI-83 Plus,

- Use the List Editor,
- Calculate regression equation,
- Display data graphs and equations.

Specific skills for this unit include the ability to use a CBL 2, TI-83 Plus and the Datamate application.

Preliminary knowledge of chemistry and mathematics

The required preliminary chemical knowledge includes:

- Periodic system of elements,
- Chemical nomenclature,
- Equations and chemical reactions,
- Ionic dissociation,
- Solution concentration,
- Conductivity.

The required preliminary mathematical knowledge includes:

- Understanding the meaning of a regression equation in a data series,
- Ability to interpret the graphs of a data series and regression equation.

File organization

The task consists of the following linked TII files:

- Notebook: The file in which the student writes the solutions. This is the starting point and contains hyperlinks to the other files.
- Main_Task: Contains the task for the students.
- Main_Task_Hint: Provides suggestions on the development Lab_Activity and information on the procedure to perform.
- Main_Task_Solution: Provides Main_Task results and answers to the questions.
- Lab_Activity: Contains lists of required materials and procedures.
- Lab_Activity_Hint: Provides step by step suggestions for the lab activities.
- Control: Proposes some elements of reflection to verify if the task has been carried out correctly.
- Control_Hint: Provides support/help for the Control_Task.
- Control_Solution: Contains Control task results and answers to the questions.
- Additional: Provides some supplementary questions for interested students.
- Additional_Task_Hints: Provides suggestions about how to carry out the Additional_Task.
- Additional_Task_Solution: Contains Additional task results and questions.

The hyperlinks only work if the files are installed in the map c:\TII\Conductivity. Students can start with the Notebook file in c:\TII\Conductivity

Classroom organisation

We recommend that the unit is introduced to the students by pointing out the ionic nature of electrolytes and the way in which electric charges affect the concentration of the substances.

Topics to address are:

- Concentration of solutions,
- Electrolytes and conductivity,
- Volumetric titrations and equivalence point,
- Conductimetric titration.

The experimental activity can be performed by groups of 2/3 students while analysis of data using TI InterActive! can be performed individually or by student groups depending on PC availability.

For homework it can be useful to give students exercises on molarity and normality and to revise concepts of electrolytic dissociation and solubility.

The students' final product is a lab report which can be evaluated by the teacher on the issues of:

- Correctness of experimental procedure and data processing,
- Completeness,
- Quality of answers to questions.

Technical hints

To install the task, double click on Conductivity.exe file. Students can start with Notebook file in c:\TII\Conductivity.

The experimental stage of the task consists of two lab activities: Lab Activity 1 and Lab Activity 2. The first one is of an introductory nature and involves measuring the conductivity of equimolar substances with different ionic charges (Cl^- , 2Cl^- , 3Cl^-). The second activity consists of conductimetric titration of two hydroxides (NaOH and $\text{Ba}(\text{OH})_2$ about 0.1N) with H_2SO_4 0.1N (?). In order to measure the required volume of hydroxide to titrate it can be useful to fill a burette from which every group takes their sample.

Titration can be carried out using two methods:

- a. Conventional, by using a 25-50 mL glass burette with support and a CBL 2/TI-83 Plus to run conductivity probe and to acquire data;
- b. Using a CBL 2/TI-83 Plus system to manage the acquisition of data with a conductivity probe and a step motor burette managed through Vernier DCU (Digital Control Unit).

It is useful to have a magnetic stirrer.

The file Lab_Activity contains a complete list of materials needed. Step_Motor_Burette_T_H contains the connection diagram for the step motor burette to the DCU and CBL 2. The control of the step motor burette through the DCU and CBL 2/TI-83 Plus requires use of a program for TI-83 Plus. Enclosed in the task folder you will find, as example, the file program "Titratio.8XP" which is useful to control the CMA Step Motor Burette. The entire program listing is included as Titration_Progr.txt.

By using a TI-GRAPH LINK it is possible to open a new program file and to insert through copy/paste the entire file listing. The program can be copied into the calculator RAM and used to control the Step motor burette.

Probe calibration is very easy using an auto-ID sensor and Datamate. It is advisable to set the range to 20.000 μS .

If you use the Step motor burette, it is also helpful to initially calibrate the conductivity probe using Datamate and then exit from this software before starting the program for DCU/Step motor burette control (ex. Titratio.8XP).

In order to prepare a $\text{Ba}(\text{OH})_2 \cong 0.1\text{N}$ solution, dissolve about 2 g of barium hydroxide octahydrate into 100 mL of distilled water. Wait for two days and then filter the solution.

Didactical suggestions

Crucial steps in the unit are the collection of experimental data, analysis of the data and discussion of results. The teacher should stress the role of ions' electric charges in the electrolytic conductivity of solutions, the meaning of titration and equivalence point, as well as the meaning of solubility product constant.

Electrolytic conductivity

The conductivity (Λ) of a solution is the reciprocal value of the resistance that an electrical current met to go through the volume of solution between two electrodes.

This resistance depends both on the volume of solution between the electrodes and the specific resistance (resistivity “ ρ ”) of the solution, so conductivity may be better defined as

$$\Lambda = \frac{1}{\rho * \frac{l}{S}} \text{ or } \Lambda = \frac{1}{\rho} * \frac{S}{l}$$

where:

l = conductor length (cm),

S = section area (cm²)

Fixing: χ (specific conductivity) = $\frac{1}{\rho}$ and $K = \frac{S}{l}$.

Conductimetry main law is: $\Lambda = \chi * K$.

As: $1 S$ (Siemens) = $\frac{1}{\Omega}$ and this is a very large measure, in conductimetry we use μS ($1 S \times 10^{-6}$).

When K has a unitary value, then Λ (conductivity) becomes numerically equal to χ (the specific conductivity). Several conductivity sensors (Vernier, CMA) have $K = 1.0 \text{ cm}^{-1}$ and these sensors actually measure specific conductivity rather than conductivity.

If we require conductivity then we may express our measure as μS and if we want specific conductivity we express our measure as $\mu S/\text{cm}$.

Electrolytic conductivity of a solution depends on:

- ions' concentrations,
- ionic charges,
- ion's migration speed,
- temperature.

Solubility product constant

A generic salt $A^+ B^-$ into solution will make the following equilibrium between the dissociated part and the undissociated part: $A^+ B^- \rightleftharpoons A^+ + B^-$

The equilibrium constant will be expressed as: $K = \frac{[A^+][B^-]}{[AB]}$.

$[AB]$ (undissociated solute concentration) can be considered constant and unitary, so it can be included in K . If we indicate as K_{ps} (solubility product constant) the term: $k * [AB]$, the equilibrium constant expression becomes: $K_{ps} = [A^+][B^-]$.

8.1 Main Task

intro

Read the information sheet for a summary of the formulas on electrolytic conductivity and solubility.

task

- a.** Do Lab Activity 1. For a description click on the Lab Activity link at the bottom of this page. **Hint 1**
- b.** Do Lab Activity 2. Use the Lab Activity link once more.
- c.** Process the data:
- (i)** Import the experimental data from the two lab activities into the Notebook file.
 - (ii)** Display the data related to Lab Activity 1 in a histogram. Label the graph “Graph 1”.
 - (iii)** Display the data related to Lab Activity 2 in a x-y line graph. Label the graphs “Graph2” and “Graph3”.
 - (iv)** Use Graph2 and Graph3 to determine the regression equation of the data and use this to calculate the equivalent point of the two titrated substances: barium hydroxide and sodium hydroxide.
- d.** Order the samples displayed in Graph1 (H₂O, NaCl, CaCl₂, AlCl₃) from 1 to 4 as conductivity changes. Can you see a relation between the measured conductivity and the number of chloride ions into the three tested saline solutions?
- e.** What does the equivalence point mean?
- f.** How can you recognize the equivalence point in a conductimetric titration?
- g.** For tested solutions, which value does conductivity approach at the equivalence point?
- h.** How can you explain the differences between Graph2 and Graph3?

8.2 Additional Task

task

Now 10 mL of barium hydroxide reacts exactly with 10.63 mL of sulphuric acid 0.1N. 100 mL of distilled water are also added.

Hint 2

- a.** Calculate barium hydroxide normality and number of moles held in 10 mL of its solution.
- b.** Predict whether barium sulphate, the salt produced from the reaction of acid-base neutralization, could form a precipitate ($K_{sp}(\text{BaSO}_4, 298 \text{ K}) = 1.1 \times 10^{-10}$).

8.3 Control Task

task

Consider the titration of sulphuric acid/sodium hydroxide.

Hint 3

- a.** What salt is formed during the neutralization reaction?
- b.** Is a precipitate formation visible?
- c.** Try to write the solubility equation for the salt produced during the titration and an expression for the product solubility constant.
- d.** If the concentration of the salt produced through the reaction is 290 g/L (add 10 H₂O molecules to the molecular mass of the salt), how much should the solubility product constant be?

8.4 Lab Activity

intro

This lab activities includes two experiments:

Lab Activity 1 concerns measuring the electrolytic conductivity of different substances.

Lab Activity 2 concerns barium hydroxide and sodium hydroxide conductimetric titration with sulphuric acid.

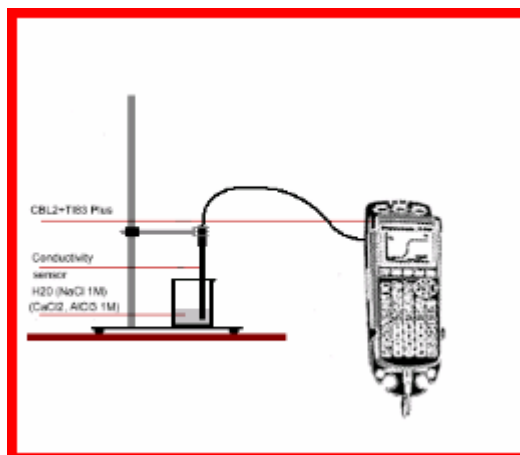
lab1

Lab Activity 1

Hint 4

Materials needed:

- CBL 2/TI-83 Plus system
- Conductivity probe
- Distilled water
- 1 M NaCl solution
- 1 M CaCl₂ solution
- 1 M AlCl₃ solution
- 4 100 mL beakers
- PVC wash bottle with distilled water
- dropper
- Paper towel



Procedure:

- Add 50 mL of distilled water into each labelled beaker.
- Add to three beakers respectively 10 drops of each solution to test: 1 M NaCl, 1 M CaCl₂, 1 M AlCl₃.
- Start Datamate.
- Calibrate the conductivity probe (range 20,000 μ S).
- Set the system choosing "Event with entry" mode.
- Dip the probe into the beaker containing only distilled water. When the reading is steady, press the ENTER button and insert "1" to refer to the distilled water sample.
- Extract the probe, dry it and then dip it into the second sample. When the reading is steady, press the ENTER button and insert "2" for the NaCl sample.
- Continue with the other samples, inserting 3 for CaCl₂ and 4 for AlCl₃.
- Save data lists, L1 as *test* and L2 as *con*.

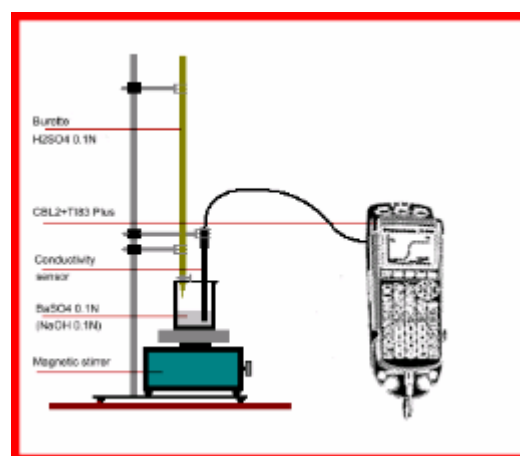
lab2

Lab Activity 2

Hint 5

Materials needed:

- CBL 2/TI-83 Plus system
- Conductivity probe
- Distilled water
- Ba(OH)₂ roughly 0.1 N
- NaOH roughly 0.1 N
- H₂SO₄ 0.1N
- N° 1 200 mL beaker
- Magnetic stirrer
- 25 mL burette or CMA Step motor Burette with DCU Vernier
- 50 mL burette with Ba(OH)₂ (to dispense 10 mL to each work group)
- 50 mL burette with NaOH (to dispense 10 mL to each work group)
- 100 mL graduated cylinder
- PVC wash bottle with distilled water
- Paper towel



Procedure using 25 mL glass burette:

- a. Using an appropriate burette measure 10 mL $\text{Ba}(\text{OH})_2$ in the 200 mL beaker.
- b. Add 100 mL of distilled water.
- c. Fill the 25 mL burette with H_2SO_4 0.1N.
- d. Start Datamate.
- e. Calibrate the conductivity probe (range 20,000 μS).
- f. Set the system choosing "Event with entry" mode.
- g. Dip the conductivity probe into the sample.
- h. Turn on the magnetic stirrer to regulate the speed.
- i. When the reading is steady press the ENTER button and insert "0".
- j. Add 1 mL H_2SO_4 0.1N.
- k. When the reading is steady, press the ENTER button. Insert the titrant volume added as "1".
- l. Continue to add H_2SO_4 0.1N as conductivity. After the conductivity reaches a minimum value, it increases and will approach the starting value.
- m. Save the data lists. For the barium hydroxide titration save L1 as *vol1* and L2 as *C_Ba*. and for sodium hydroxide save L1 as *vol2* and L2 as *C_Na*.
- n. Repeat this procedure with NaOH instead of $\text{Ba}(\text{OH})_2$.

Procedure using CMA Step motor Burette and Vernier DCU:

- a. Using an appropriate burette measure 10 mL $\text{Ba}(\text{OH})_2$ 0.1N in the 200 mL beaker.
- b. Add 100 mL of distilled water.
- c. Link the DCU to the digital CBL 2 channel.
- d. Link the DCU to the Step motor Burette.
- e. Fill the burette syringe with H_2SO_4 0.1N.
- f. Start Datamate.
- g. Calibrate the conductivity probe (range 20,000 μS).
- h. Close Datamate.
- i. Dip the conductivity probe into the sample.
- j. Start the Titration program or another program to control the Step motor Burette.
- k. Turn on the magnetic stirrer to regulate the speed.
- l. Add 1 mL H_2SO_4 0.1N.
- m. When the reading is steady press the ENTER button and insert "1".
- n. Continue to add H_2SO_4 0.1N as conductivity. After the conductivity reaches a minimum value, it increases and will approach the starting value.
- o. Save the data lists. For the barium hydroxide titration save L1 as *vol1* and L2 as *C_Ba*. and for sodium hydroxide save L1 as *vol2* and L2 as *C_Na*.
- p. Repeat this procedure with NaOH instead of $\text{Ba}(\text{OH})_2$.

8.5 Hints

hint 1

- a.**
- Write the chemical equation for the acid-base neutralization reaction.
 - Calculate the equivalent amount of sulphuric acid held into 10.63mL of 0.1 N reacted solution. Remember to express the volume of the substance in litres.
 - Comment on the mean of the chemical equivalent and look for the existing ratio between the reacted equivalents of sulphuric acid and the corresponding number of barium hydroxide's equivalents.
 - After you have found the barium hydroxide's equivalents, calculate the normality value.
 - Comment on the difference between normality and molarity. Use the barium hydroxide number of equivalents to calculate the corresponding number of moles.
- b.**
- Use:
 - The chemical equation of the acid-base neutralization reaction between sulphuric acid and barium hydroxide
 - The number of moles of reacted barium hydroxide (already calculated)
 - The total solution volume (as indicated in the text of the question)
 - The barium sulphate constant of solubility product
 - Calculate the barium sulphate molarity (referring to total solution volume).
 - Write the barium sulphate solubility equation.
 - Use the stoichiometric coefficients to help determine the concentration of ionic species (g-ions/L) which is present relative to the barium sulphate molarity (already calculated).
 - Write the relationship for the barium sulphate constant of solubility product.
 - Calculate the value of the ionic product of the ionic species and compare the result with the barium sulphate K_{sp} .
 - If the calculated result is greater than K_{sp} this means that the precipitate will be barium sulphate.

hint 2

- a.**
- Write the chemical equation for the acid-base neutralization reaction.
 - Calculate the equivalent amount of sulphuric acid held into 10.63mL of 0.1 N reacted solution. Remember to express the volume of the substance in litres.
 - Comment on the mean of the chemical equivalent and look for the existing ratio between the reacted equivalents of sulphuric acid and the corresponding number of barium hydroxide's equivalents.
 - After you have found the barium hydroxide's equivalents, calculate the normality value.
 - Comment on the difference between normality and molarity. Use the barium hydroxide number of equivalents to calculate the corresponding number of moles.
- b.**
- Use:
 - The chemical equation of the acid-base neutralization reaction between sulphuric acid and barium hydroxide
 - The number of moles of reacted barium hydroxide (already calculated)
 - The total solution volume (as indicated in the text of the question)
 - The barium sulphate constant of solubility product
 - Calculate the barium sulphate molarity (referring to total solution volume).
 - Write the barium sulphate solubility equation.
 - Use the stoichiometric coefficients to help determine the concentration of ionic species (g-ions/L) which is present relative to the barium sulphate molarity (already calculated).
 - Write the relationship for the barium sulphate constant of solubility product.
 - Calculate the value of the ionic product of the ionic species and compare the result with the barium sulphate K_{sp} .
 - If the calculated result is greater than K_{sp} this means that the precipitate will be barium sulphate.

hint 3

- a.** Write the chemical equation for the neutralization reaction between sodium hydroxide and sulphuric acid.
- b.** Observe the water solution during titration to verify that the solution is clear.

- c. The solubility equation symbolizes the process through which the salt liberates its ions due to water dissolution. The expression of solubility product refers to the equilibrium constant (solubility product constant) of the salt solubility equation.
- d. Calculate the number of moles corresponding to salt quantity (290 g).
Determine the number of g-ions/L formed by salt dissolution.
Substitute the the ions' concentration into the expression for the solubility product constant.

hint 4

- (i) Acquire data with Datamate
- a. Link the TI-83 Plus to the CBL 2.
 - b. Link the Conductivity probe to CH1 channel on CBL 2.
 - c. Start Datamate.
 - d. Calibrate the Conductivity Probe:
 - On the probe select the medium range: 0–2000 μS
 - Press:
 - 1: SETUP
 - 2: CALIBRATE
 - 2: CALIBRATE NOW
 - With the sensor dry (in the air) wait until the tension reading becomes stable and then press ENTER.
 - Insert “0” as the first value. Press ENTER
 - Dip the probe tip into the calibration standard solution (1000 μS).
Wait until the reading is stable and then press ENTER.
 - Type “1000” and press ENTER.
 - Return to the SETUP menu by pressing 1:OK.
 - e. Sampling mode setup:
 - Press 1:SETUP.
 - Select MODE and then press ENTER.
 - Choose 3: EVENTS WITH ENTRY.
 - Return to the main menu by pressing 1:OK.
 - f. Collect data
 - Dip the probe into the first sample (distilled water).
 - Press 2:START.
 - Wait for a few minutes. When the reading is steady, press ENTER.
 - Type “1” and press ENTER.
 - Rinse the probe with distilled water, dry the probe and then dip it in the second sample.
 - When the reading is steady, press ENTER.
 - Type “2” and press ENTER.
 - Repeat this procedure for the other samples. At the end press STO➔.
 - Quit Datamate.
- (ii) To save data lists
- a. Press the CLEAR button.
 - b. Type:
 - L1 STO ➔ test
 - L2 STO ➔ cond

hint 5

- (i) Acquire data with Datamate:
- a. Link the TI-83 Plus to the CBL 2.
 - b. Link the Conductivity probe to CH1 channel on the CBL 2.
 - c. Start Datamate.

- d. Calibrate the Conductivity Probe:
- On the probe select the medium range: 0–2000 μS .
 - Press:
 - 1: SETUP
 - 2: CALIBRATE
 - 2: CALIBRATE NOW
 - With the sensor dry (in the air) wait until the tension reading becomes stable and then press ENTER.
 - Insert “0” as first value. Press ENTER.
 - To dip the probe tip in the calibration standard solution (1000 μS). Wait the stable reading and then press ENTER
 - Type “1000” and press ENTER.
 - Return to the SETUP menu by pressing 1:OK.
- e. Sampling mode setup:
- Press 1:SETUP.
 - Select MODE and then press ENTER.
 - Choose 3: EVENTS WITH ENTRY
 - Return to the main menu by pressing 1:OK.
- f. Collect data
- Start the magnetic stirrer and regulate the speed.
 - Dip the probe in to the $\text{Ba}(\text{OH})_2$ solution.
 - Press 2:START.
 - Wait for a few minutes.
 - When the reading is stable note the conductivity value and press ENTER.
 - Type “0” (mL of added titrant) and press ENTER.
 - Add 0.5 mL of 0.1N H_2SO_4 .
 - When the reading is stable press ENTER.
 - Type “0.5” and press ENTER.
 - Repeat the procedure adding 0.5 mL of 0.1N H_2SO_4 from time to time.
 - Continue until the conductivity approaches the starting value.
 - At the end press STO \times key.
 - Close Datamate.
- (ii) To save data lists
- a. Press the CLEAR button.
- b. Type:
- For $\text{Ba}(\text{OH})_2$ titration:
 - L1 STO \rightarrow vol_1
 - L2 STO \rightarrow C_Ba
 - For NaOH titration:
 - L1 STO \rightarrow vol_2
 - L2 STO \rightarrow C_Na

Part 2: Teaching Ideas

Part 2 contains ‘Teaching Ideas’, which are less elaborated than the Part 1 student units. These ‘Teaching Ideas’ can be used as an inspiring source for teaching, but may need some customizing for your educational situation. Some items are meant to help the teacher to demonstrate relevant TI InterActive! features to students or colleagues. Others contain concrete tasks for students, but in a less complete way compared to the Part 1 units. Again, the table of contents provides an overview.

The format of the items in Part 2 is less uniform than is the case for the Part I units. Some of the Part 2 items, such as the ‘Fourier Teaching Idea’, consists of one single TI InterActive! document. In this case the installation is quite simple: just download the zip file and extract it into the folder you want. In other cases, however, the ‘Teaching Idea’ is a small unit with hyperlinked files. In these cases the installation becomes more subtle, as the files need to be extracted to a folder with a specific name. The procedure described in the ‘Introduction to Part 1’ should be followed. Examples of this category of ‘Teaching Ideas’ are the ‘Course Notes’ materials and the ‘Plenary Demo’ materials.

9. Plenary demonstrations

introduction

Introduction

For a mathematics teacher there are several ways to use TI Interactive!:

- as a "dynamic" tool, to prepare lessons at home, or for your students to perform well adapted exercises in the class or at home,
- as a "super blackboard" to be handled by the teacher in front of the class to illustrate small lesson sequences in which the interactivity of the software provides additional value compared to normal blackboard and chalk.

The subjects treated in the following files are supposed to fit into the last category.

content

Content

Demonstrations for two topics are offered:

- | | |
|--|----------------|
| 9.1 The slope as a characteristic property of the line | Grades 9 or 10 |
| 9.2 Matrices and beetles | Grade 11 |

procedure

Working procedure

For both topics there is a teacher guide file and one or more demonstration files. You can open them by clicking on the blue hyperlinks below. Each of these files contains links to the other files of the group. Once different files are open, do not use the links between them any more, simply switch from one to another using the buttons on the Status Bar.

If you want to keep the initial content of the present files unchanged, before beginning to work you must either put the files into "read only" mode so that it is not possible to erase or change parts by mistake (in Windows Explorer, right-click on the name of the file, choose Properties and select read only) or save the files under a different name.

9.1 The slope, a characteristic property of the line

introduction

Introduction

The aim of this demo is to illustrate graphically the fact that the slope is a characteristic property of the line, in the sense that if you consider the quotient of the difference of the ordinates and of the difference of the abscissas of any two distinct points of a curve, this quotient is a constant if and only if the curve is a straight line non parallel to the Oy axis.

instructions

Instructions

The slope demo file contains 2 pages separated by a Math Section Break.

On the first page, you should define a function $f1(x) = m \cdot x + 1$ where m is a variable parameter controlled with a slider bar (we call SBm).

You should also define a fixed reference point A (square, blue colored) with coordinates $(a, f1(a))$ and a variable point B (round and red) with coordinates $(b, f1(b))$ where b is controlled by a second slider bar (we call SBb).

The left graph shows the two points A and B on the graph of the function $f1(x)$ as well as the segments representing the differences of their abscissas and of their ordinates (*).

The right graph shows a round and green point W with coordinates $(b, w1(b))$ where

$w1(b) = \frac{f1(b) - f1(a)}{b - a}$ is the slope calculated with respect to the actual points A and B.

a. Leave SBm and the variable a unchanged and animate SBb.

On the left graph : Point B moves on the graph of $f1(x)$ and the construction (*) follows.

On the right graph : Point W moves on a horizontal line which means that $w1(b)$ is a constant.

b. Using SBm, change the value of m while leaving the variable a unchanged. Animate SBb again.

This results in another straight line with a constant slope that has a value equal to the value of m .

c. Repeat the previous question for a different value of a .

This affects the choice of the fixed reference point but the result is always the same.

On the second page, you find exactly the same scenario but with a function $f1(x)$ that you choose and define. For this example choose a different polynomial function which is not of degree 1.

It is easy to see that the value of $w1(b) = \frac{f1(b) - f1(a)}{b - a}$ is no longer a constant!

Remark : the graph of $w1(b)$ (green dotted line) is NOT the graph of the derivative of function $f1(x)$!

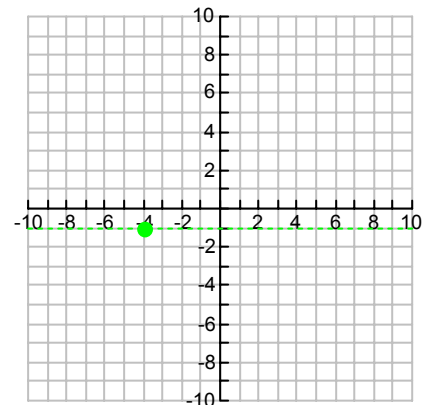
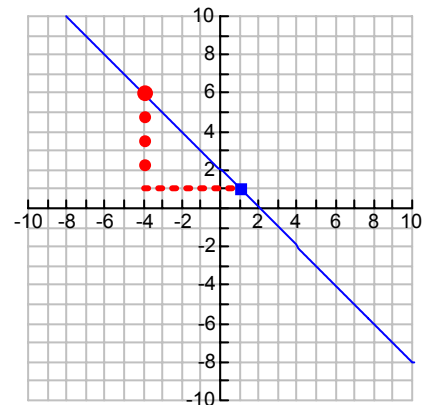
Demo 1The value of m

define $f1(x) = m \cdot x + 2$
 $a := 1$

The value of b

Graph of $f1(x)$: blue graphBlue square point: reference point $(a, f1(a))$ Thick red round point: $(b, f1(b))$

$$w1(b) = \frac{f1(b) - f1(a)}{b - a}$$

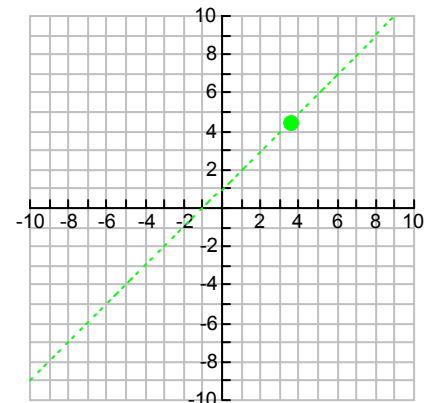
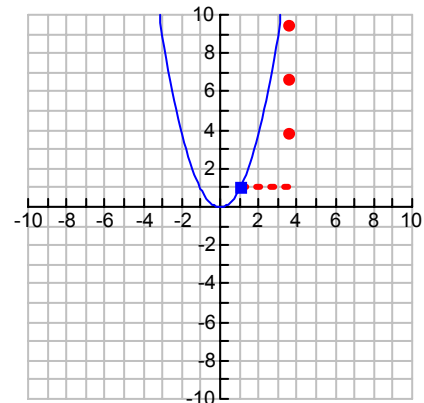
Green round point: $(b, w1(b))$ **Demo 2**

define $f1(x) = x^2$
 $a := 1$

The value of b

Graph of $f1(x)$: blue graphBlue square point: reference point $(a, f1(a))$ Red round point: $(b, f1(b))$

$$w1(b) = \frac{f1(b) - f1(a)}{b - a}$$

Green round point: $(b, w1(b))$ 

9.2 Matrices and beetles

introduction

Introduction

The starting point of this demo is an application task on matrices and matrix operations. It consists of an exercise relative to a population of insects.

The demo contains several steps, taking into account the interactivity within a TII document and the possibility to introduce sliders.

A possible scenario is described in the instructions.

starting task

Starting Task

A population of beetles is developing according to the following rules.

- None of the beetles reaches the age of 3 years. During their first year, only one quarter of the beetles survive. Among those that survive, only 50% are alive at the end of the second year.
- Let us call adults (A) the beetles that reach the age of two, young (Y) the beetles that are one year old and new (N) the new born beetles.
- The number of males is always equal to the number of females.
- Each young female beetle (Y) can have four baby beetles, each adult female beetle (A) can have eight babies.
- Initially the population of beetles is composed of 200 adults, 400 young and 800 new.

Find the matrix M which enables you to determine the population in a given year if the population in the preceding year is known. Use M to calculate the number of new, young and adult beetles after two years.

instructions

Instructions

We recommend the following scenario for demonstration.

- Ask the students to write down the answers to the exercise on paper.
- Use the file M&B1 to explain and show the correct answer.
- Use the interactivity within a TII document to investigate what happens to the population if we look further than two years ahead. Change the value of t (take t equal to 20) and observe the result (also in M&B2).
- Consider a new question:

How does the total population of beetles develop during a period of 20 years?


To investigate this, the first idea is to calculate the sum $N + Y + A$ when t equals 20 (see M&B3).

It is clear that the total number of beetles is bigger than in the beginning (initially the total number of insects was 1400) but the way the population is developing is not so clear ... unless we make a graph of the total number of beetles in function of t ! (see M&B4).

- Make use, once more, of the interactivity of TII and ask a new question:

What is the incidence on this population of the use of pesticides whose effect is to divide by two the birth rate of the beetles?

To investigate this, it is sufficient to change the values of some elements of the matrix M of the file M&B4, as has been done in file M&B5.

Hint : if you change elements of matrix M in file M&B4, it will be necessary to click on  in the Graph window, in order to have the graph updated.

f. Let us go still further and use the fact that TII provides the possibility to insert sliders ...

Suggest a last exercise:

Suppose that the impact of pesticides on the rate of birth rate is sufficiently well known to be able to predict that each female beetle (Y) will have $4a$ baby beetles and each female beetle (A) $8a$ babies with parameter a varying between 1.02 and 0.98 according to the quantity of pesticides used.

Study the influence of a (and hence of the quantity of pesticides) on the growth of the population.

Take file M&B4, insert a slider corresponding to a parameter a whose values range from 0.98 to 1.02 and change the elements of the first row of M into 0, $2a$, $4a$ or load file M&B6. Observe the effect on the population of beetles when scrolling the slider button.

demo

Demo 1

Taking into account that the number of females is equal to the half of the total number of beetles, after one year, the number of new beetles (N) is given by:

$$N = 4 \cdot \left(\frac{400}{2}\right) + 8 \cdot \left(\frac{200}{2}\right) = 2 * 400 + 4 * 800$$

The number of young beetles (Y) is given by: $Y = \left(\frac{1}{4}\right) \cdot 800$

The number of adult beetles (A) is given by: $A = \left(\frac{1}{2}\right) \cdot 400$

Let us call P the initial population vector: $P := \begin{bmatrix} 800 \\ 400 \\ 200 \end{bmatrix}$

The population growth matrix M is then: $M := \begin{bmatrix} 0 & 2 & 4 \\ \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$

And the population vector after t years is given by $R(t)$ with $R(t) := M^t \cdot P$.

So, when $t := 2$: $R(t) = \begin{bmatrix} 1200 \\ 400 \\ 100 \end{bmatrix}$

Demo 2

Let us call P the initial population vector:

$$P := \begin{bmatrix} 800 \\ 400 \\ 200 \end{bmatrix}$$

The population growth matrix M is then:

$$M := \begin{bmatrix} 0 & 2 & 4 \\ \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$$

And the population vector after t years is given by $R(t)$ with $R(t) := M^t \cdot P$.

So, when $t := 20$:

$$R(t) = \begin{bmatrix} 40975 \\ 32 \\ 20475 \\ 64 \\ 20475 \\ 128 \end{bmatrix} \quad \text{and} \quad \text{approx}(R(t)) = \begin{bmatrix} 1280. \\ 320. \\ 160. \end{bmatrix}$$

Demo 3

Let us call P the initial population vector:

$$P := \begin{bmatrix} 800 \\ 400 \\ 200 \end{bmatrix}$$

The population growth matrix M is then:

$$M := \begin{bmatrix} 0 & 2 & 4 \\ \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$$

And the population vector after t years is given by $R(t)$ with $R(t) := M^t \cdot P$.

So, when $t := 20$:

$$R(t) \begin{bmatrix} 40975 \\ 32 \\ 20475 \\ 64 \\ 20475 \\ 128 \end{bmatrix} \quad \text{and} \quad \text{approx}(R(t)) \begin{bmatrix} 1280. \\ 320. \\ 160. \end{bmatrix}$$

The **total population Tot(t) after t years** is the sum of the elements of $R(t)$.

This number is the last element of the cumulative sum of $R(t)$.

Let us define: $C(t) := \text{cumSum}(R(t))$

$$\text{Tot}(t) := (C(t))_{[3, 1]}$$

Finally, the total number of beetles after t years is: $\text{approx}(\text{Tot}(t)) \quad 1760.$

demo

Demo 4

Let us call P the **initial population vector**:

$$P := \begin{bmatrix} 800 \\ 400 \\ 200 \end{bmatrix}$$

The **population growth matrix M** is then:

$$M := \begin{bmatrix} 0 & 2 & 4 \\ \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$$

And the **population vector after t years** is given by $R(t)$ with $R(t) := M^t \cdot P$.

So, when $t := 20$:

$$R(t) \begin{bmatrix} 40975 \\ 32 \\ 20475 \\ 64 \\ 20475 \\ 128 \end{bmatrix} \quad \text{and} \quad \text{approx}(R(t)) \begin{bmatrix} 1280. \\ 320. \\ 160. \end{bmatrix}$$

The **total population Tot(t) after t years** is the sum of the elements of $R(t)$.

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Let us define: $C(t) := \text{cumSum}(R(t))$

$$\text{Tot}(t) := (C(t))_{[3, 1]}$$

Finally, the total number of beetles after t years is: $\text{approx}(\text{Tot}(t)) \quad 1760$.

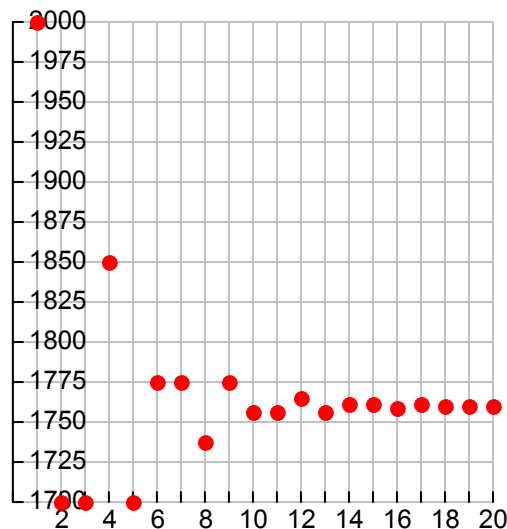
Graph of the total number of beetles Tot(t) in function of the year t (1 ≤ t ≤ 20)

Let us define

$$u := \text{seq}(t, t, 1, 20, 1) \quad \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$$

$$v := \text{seq}(\text{Tot}(t), t, 1, 20, 1)$$

to obtain the following graph:



demo

Demo 5

Let us call P the initial population vector:

$$P := \begin{bmatrix} 800 \\ 400 \\ 200 \end{bmatrix}$$

The population growth matrix M is then:

$$M := \begin{bmatrix} 0 & 1 & 2 \\ \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$$

And the population vector after t years is given by $R(t)$ with $R(t) := M^t \cdot P$.

So, when $t := 20$:

$$R(t) \begin{bmatrix} 136575 \\ 32768 \\ 22425 \\ 16384 \\ 59075 \\ 65536 \end{bmatrix} \quad \text{and} \quad \text{approx}(R(t)) \begin{bmatrix} 4. \\ 1. \\ 1. \end{bmatrix}$$

The **total population Tot(t) after t years** is the sum of the elements of $R(t)$.

This number is the last element of the cumulative sum of $R(t)$.

Let us define: $C(t) := \text{cumSum}(R(t))$

$$\text{Tot}(t) := (C(t))_{[3, 1]}$$

Finally, the **total number of beetles after t years** is: $\text{approx}(\text{Tot}(t)) \quad 6.$

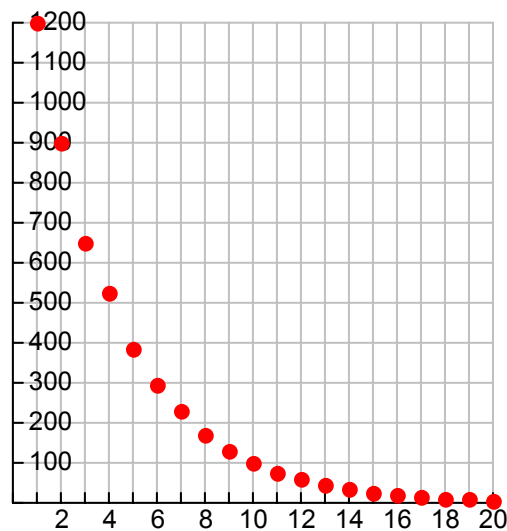
Graph of the total number of beetles Tot(t) in function of the year t (1 ≤ t ≤ 20)

Let us define

$$u := \text{seq}(t, t, 1, 20, 1) \quad \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$$

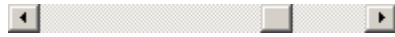
$$v := \text{seq}(\text{Tot}(t), t, 1, 20, 1)$$

to obtain the following graph:



Demo 6

$a:=1.01$



Let us call P the initial population vector:

$$P := \begin{bmatrix} 800 \\ 400 \\ 200 \end{bmatrix}$$

Suppose the population growth matrix M is:

$$M := \begin{bmatrix} 0 & 2 \cdot a & 4 \cdot a \\ \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$$

and the population vector after t years is given by $R(t)$ with $R(t) := M^t \cdot P$

So, when $t := 20$:

$$R(t) = \begin{bmatrix} 178172270882201755492239 \\ 1280000000000000000000 \\ 44339996305727797282851 \\ 1280000000000000000000 \\ 176650229492529966707 \\ 1024000000000000000000 \end{bmatrix} \quad \text{and} \quad \text{approx}(R(t)) = \begin{bmatrix} 1392. \\ 346. \\ 173. \end{bmatrix}$$

The total population $\text{Tot}(t)$ after t years is the sum of the elements of $R(t)$.

This number is the last element of the cumulative sum of $R(t)$.

Let us define : $C(t) := \text{cumSum}(R(t))$

$$\text{Tot}(t) := (C(t))_{[3, 1]}$$

Finally, the total number of beetles after t years

is: $\text{approx}(\text{Tot}(t)) = 1911.$

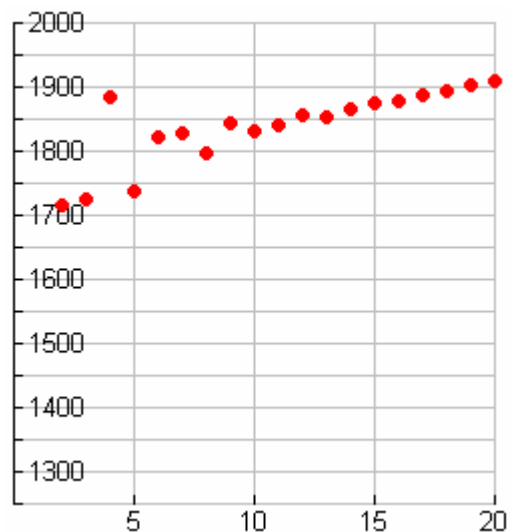
Graph of the total number of beetles $\text{Tot}(t)$ in function of the year t ($1 \leq t \leq 20$).

Let us define

$$u := \text{seq}(t, t, 1, 20, 1)$$

$$v := \text{seq}(\text{Tot}(t), t, 1, 20, 1)$$

to obtain the following graph:



10. Programming with TI InterActive!

introduction

This file is in a different format from the other materials. Instead of having a number of student tasks, help, hint and other files there is just a single document explaining the way in which programs can be developed and used within TI InterActive!

function definitions

Review of function definitions



The basis of programming is the use of the Math Box to define a function. Here are a few examples of simple ways to define and evaluate a function.

$x^2 \rightarrow f(x)$	$f(2)$	$f(p)$
"Done"	4	p^2
$g(x) := x^3$	$g(2)$	$g(n)$
"Done"	8	n^3
Define $h(x) = x^4$	$h(2)$	$h(g(f(y)))$
"Done"	16	y^{24}

A function can also be 'piecewise' defined. Here are a few examples.

The first uses the 'when' function and the input is entered as $p(x) := \text{when}(x > 0, x, -x)$.

$p(x) := \begin{cases} x & x > 0 \\ -x & \text{e l s e} \end{cases}$	$p(2)$	$p(-2)$
"Done"	2	2

This can also be entered using the  menu symbol from the Math Palette and selecting the  symbol. The dialog box allows you to select the number of pieces.

$q(x) := \begin{cases} 0 & x < 1 \\ 1 & x < 2 \text{ and } x \geq 1 \\ 2 & x > 2 \end{cases}$	$q(-2)$	$q(1.5)$	$q(3)$
"Done"	0	1	2

Another way to produce multi-part definitions is to add a number of 'when' statements together:

$$r(x) := \begin{cases} -1 & x < 0 \\ 0 & \text{e l s e} \end{cases} + \begin{cases} 1 & x \geq 0 \text{ and } x < 1 \\ 0 & \text{e l s e} \end{cases} + \begin{cases} -1 & x > 1 \\ 0 & \text{e l s e} \end{cases}$$

"Done"

The graph of $r(x)$ clearly shows the 3 pieces.

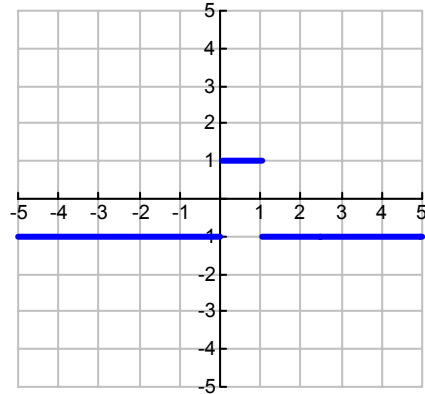
The piecewise structure can also be inserted using the menus of the Math Palette:

Math → Algebra → Piecewise function.

If you follow a different menu path you will find the program structure commands:

Tools → Program Logic → Control.

Before we can use these commands we need to learn about an alternative way to define a function.



multi-statements

Multi-statement functions

The following function expects a positive integer x as input and returns different values depending on whether x is positive ($x/2$) or negative ($3x+1$).

This example illustrates the use of the If..Then..Else..EndIf structure.

Note that:

- the function can only be set up using the 'Define' structure,
- the Input must be set to Plain Text,
- statements are separated by two colon symbols (::),
- the body of the definition is enclosed in a Func...EndFunc structure,
- a value to be assigned to the function is set using Return.

Define $s(x) = \text{Func} :: \text{If } \text{int}\left(\frac{x}{2}\right) = \frac{x}{2} \text{ Then} :: \text{Return } x/2 :: \text{Else} :: \text{Return } 3 \cdot x + 1 :: \text{EndIf} :: \text{EndFunc}$

$$\begin{array}{ccc} s(5) & s(4) & s(s(s(s(117)))) \\ 16 & 2 & 22 \end{array}$$

loops

Functions with loops

In order to produce a tidy looking program it is helpful to break the program into separate lines.

Remember to use the Plain Text mode of input and to use CTRL-Enter to begin a new line. You can create variables within a program as well as access previously defined variables. If you do not need the internal variables to be accessible outside the program then define them to be 'Local'.

The next example computes the factorial of a given positive integer.

```
Define fac(x)=Func
::Local f,c
::1→f
::For c,1,x
::f*c→f
::EndFor
::Return f
::EndFunc

"Done"
```

fac(5)
120

using programs _____

Using programs to extend TII's functionality

This short introduction concludes with the development of a tool to help with data-handling using lists. It will be used to take a list of data of a variate recorded at regular intervals and compute a new list of data containing the moving averages of the data over a given number of intervals. List L1 contains data on 36 monthly house sales for an estate agent (realtor) taken from the book "Statistics" by Roger Fentem, Collins Educational ISBN 0-00-322371-X.

```
Define movave(list1,m) = Func
::Local n,c,r
::dim(list1)→n
::seq(c,c,1,n)→list2
::{0}→list3::{0}→list4
::For c,1,n-m+1
::mean(seq(list1[r],r,c,m+c-1))→list3[c]
::((m-1)/2+c)→list4[c]
::EndFor
::Return approx(list3)
::EndFunc

"Done"
```

L1
15.
14.
13.
14.
13.
14.
17.
15.
18.
19.
16.
15.
16.
13.
14.

A MathBox is used to create a program to define a function called 'movave'. This function has two arguments; the first argument 'list1' is the original data list while the second argument 'm' defines the number of months to use when making the moving averages.

The function returns the list 'list3' of moving averages of data from 'list1' taken 'm' at a time. As a side effect lists 'list2' and 'list4' are created - 'list2' is a count of the number of elements for the data set in 'list1', and 'list4' contains the adjusted count numbers for corresponding moving averages.

The program is entered in plain text input mode.

Programming commands are found in the MathBox's Tool menu. Each line of the program ends with Ctrl+Enter. Each line begins with a pair of colons "::".

```
movave(L1, 5)
```

```
{ 13.8, 13.6, 14.2, 14.6, 15.4, 16.6, 17., 16.6, 16.8, 15.8, 14.8, 14.4, 14.2, 14.4, 15.6, 16., 16.8, 17.8, 18., 17.6, 17.8, 17.2, 16.6, 16., 16., 16.4, 17.4, 18.6, 19.6, 20., 20., 18.8 }
```

Since 'list2' and 'list4' were not declared as 'Local' they are still available:

list2

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36}

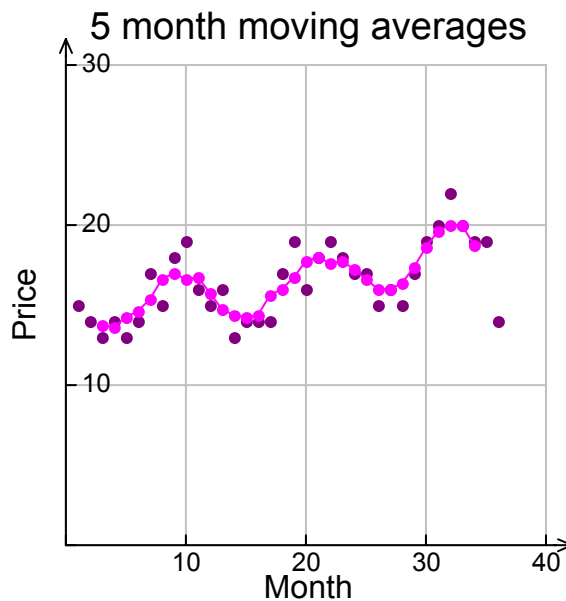
list4

{3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34}

This function and lists can be used to define list L2, L3, L4 from which we can plot the original data and the moving averages.

L1	L2 _a	L3 _a	L4 _a
15	1	13.8	3
14	2	13.6	4
13	3	14.2	5
14	4	14.6	6
13	5	15.4	7
14	6	16.6	8
17	7	17	9
15	8	16.6	10
18	9	16.8	11
19	10	15.8	12
16	11	14.8	13
15	12	14.4	14
16	13	14.2	15
13	14	14.4	16
14	15	15.6	17

Now we can draw the scatterplot of L1 against L2, together with the xyline of L3 against L4. By changing the value of $m = 5$ we can see how the moving averages smooth data as m increases.



Note: It would be good for the program to be able to set up the statistical plots directly, but we do not have access to the Xmin, Xmax etc. variables. It would also be beneficial to have the option of smaller size dots for data points.

11. Fourier analysis with TI InterActive!

introduction

If you are familiar with Fourier analysis, you might know that (most) functions can be expanded into a Fourier series. This is a sum of sines and cosines that approximate a given function on a given interval. In this example we use *TI InterActive!* to (interactively) explore the connection between a function and its Fourier series. I must stress that this is just a light introduction, as the inner workings of Fourier analysis can be of quite a subtle nature (see note at the bottom).

example

We first define the function we will use. For example, let $f(x) := x$. This function can be altered at a later stage. We will use the interval $[-\pi, \pi]$ throughout. You may try to use other intervals in one of the exercises.

The so-called Fourier coefficients are defined by, respectively,

$$a(n) := \frac{1}{\pi} \int_{-\pi}^{\pi} (f(x) \cdot \cos(nx)) dx \quad \text{and} \quad b(n) := \frac{1}{\pi} \int_{-\pi}^{\pi} (f(x) \cdot \sin(nx)) dx.$$

Any text on differential equations or Fourier analysis will provide a definition of these coefficients. There are minor differences between textbooks, often through the use of different intervals or perhaps complex notation.

We can see that both of the Fourier coefficients depend on the function f . In Fourier theory, the intention is to add the coefficients in an attempt to reconstruct the function f . Before we write the partial sum of the Fourier series, we need to agree on the degree of the sum. The higher the degree, the more accurately the partial sum will approximate the given function. We use a variable for this, so let $p := 3$. This can, of course, also be altered later on.

The partial sum $s(x)$ in f 's Fourier series is given by the expression:

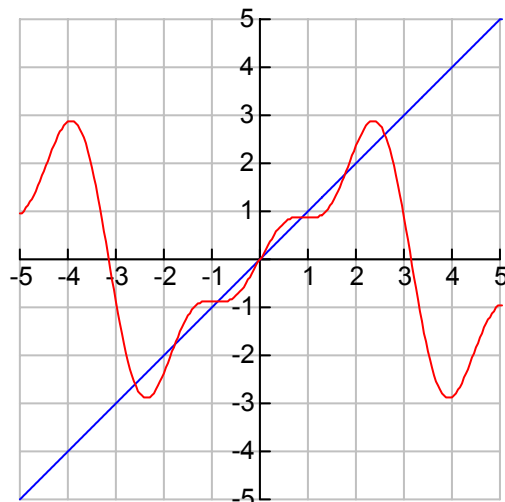
$$s(x) := \frac{1}{2} a(0) + \sum_{i=1}^p (a(i) \cdot \cos(ix) + b(i) \cdot \sin(ix)).$$

This is where the variable p comes into play.

One important point, in terms of the theory is: does the sum converge to f if we use an infinite number of terms?

We can investigate this question through the use of *TI InterActive!*. We can plot the given function f with the corresponding Fourier series - partial sum in the same coordinate system.

The blue curve is the graph of the function f and the red curve is the graph of the partial sum.



This document will automatically be updated when you alter the function f or the degree of the partial sum.

Calculating Fourier series might be rather taxing on your computer's processor, so don't choose too high a value for the degree p .

As we see, the partial sum of f 's Fourier series seems to closely resemble f on the given interval. You can also use other intervals as Fourier series is not restricted to $[-\pi, \pi]$.

This interval is normally chosen to ease calculations.

Exercise 1: Change the function f and the degree p to see how different functions correspond to different Fourier series.

Exercise 2: Consult a text on calculus and find expressions for a general Fourier series on the interval from $-L$ to L . Make a TI InterActive! document to illustrate the situation.

Note: Fully understanding Fourier series is quite a difficult task. One can use a functional analysis perspective, vector analysis or plain trigonometry. Rather difficult theorems on convergence and divergence may be required, as well as measure theory and Lebesgue integration. Fourier theory is actually responsible for changing the way one looks at subjects as convergence, function notion and the meaning of integration. Fourier's series made all these notions change some two hundred years ago and it took the leading mathematicians fifty years to settle matters. These theories are beyond the level at which one is usually introduced to this kind of series. As a rule of thumb the functions that we use in practice are well suited for Fourier analysis. Functions that won't work are usually pathologic examples constructed to be counter-examples. Annoying mathematicians can find examples of Fourier series that do not converge and also those that converge to something other than the expected function. It is also possible to find trigonometric series that are not Fourier series for any function at all.

12. Polar curves with TI InterActive!

introduction

This file contains a teaching idea for studying polar curves with TI InterActive!. It illustrates that the use of TI InterActive! is valuable in discovering mathematics, and shows a way of resequencing mathematics teaching. Polar curves is an exciting subject in math that can be animated in a relatively simple manner using technological tools. These tools also contribute to the development of students' understanding .

assignment

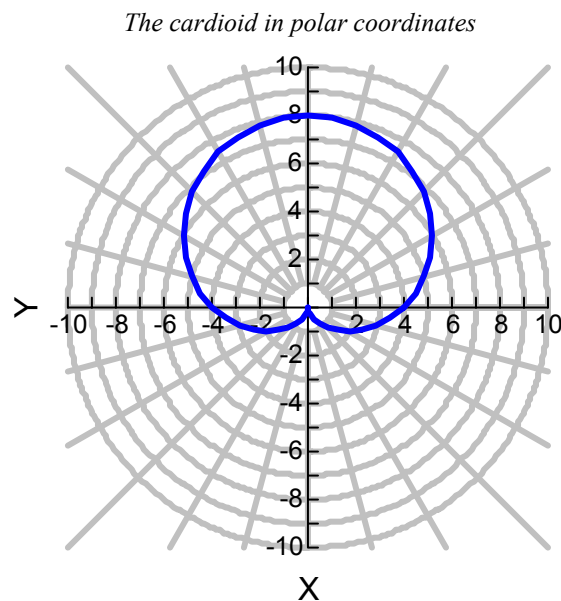
Assignment

The following assignment stems from a recent national examination in Norway. We will use this to illustrate how TI InterActive! can be used to solve problems.

The curve to $r = a + a \cdot \sin(\theta)$ is called a Cardioide due to its heart-shaped form.

This assignment concerns the curve given by $r = 4 + 4 \cdot \sin(\theta)$ for $\theta \in [0, 2\pi[$.

a. Draw the curve in a polar coordinate system.



b. Read the coordinates of the intersection points of the curve and the coordinate axes. Then calculate the coordinates.

Reading from the curve with TRACE: $\theta = 0 \Rightarrow r = 4$

$$\theta = \frac{\pi}{2} \Rightarrow r = 8$$

$$\theta = \pi \Rightarrow r = 4$$

$$\theta = \frac{3\pi}{2} \Rightarrow r = 0$$

Using calculation: $\theta = 0 \Rightarrow 4 + 4 \cdot \sin(0) = 4$

$\theta = \frac{\pi}{2} \Rightarrow 4 + 4 \cdot \sin\left(\frac{\pi}{2}\right) = 8$

$\theta = \pi \Rightarrow 4 + 4 \cdot \sin(\pi) = 4$

$\theta = \frac{3\pi}{2} \Rightarrow 4 + 4 \cdot \sin\left(\frac{3\pi}{2}\right) = 0$

c) Find the area defined by the curve.

The radius vector runs over a complete circulation [CYCLE or interval?], so the area is

$$\frac{1}{2} \int_0^{2\pi} r^2 d\theta = 24\pi$$

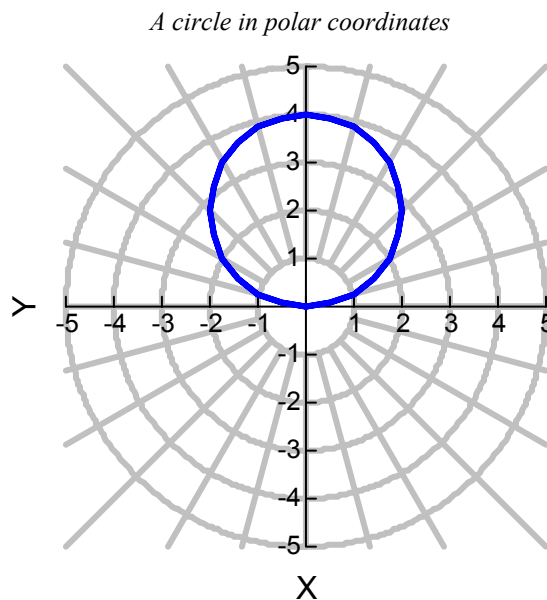
$$24\pi \cong 75.3982$$

variation

Variation

What if we drop the first a in $r = a + a \cdot \sin(\theta)$ and study the curve given by $r = 4 \cdot \sin(\theta)$ for $\theta \in [0, 2\pi[$?

Once more, we start with the graph:



The curve seems to be a circle with centre $(0, 2)$ in rectangular coordinates and radius 2. Can we prove that this is indeed the case?

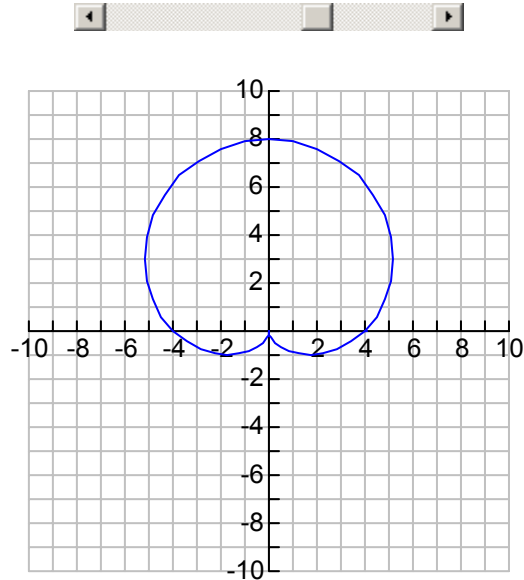
The equation of the circle is $x^2 + (y - 2)^2 = 2^2$. In this case, $x = r \cdot \cos(\theta)$ and $y = r \cdot \sin(\theta)$.

Substitution into the equation gives: $x^2 + (y - 2)^2 = 2^2 \mid x = r \cdot \cos(\theta)$ and $y = r \cdot \sin(\theta)$.

Exploration

Now what if we consider the cardioid as a member of the family of curves given by $s(\theta) := 4 + a \cdot \sin(\theta)$?

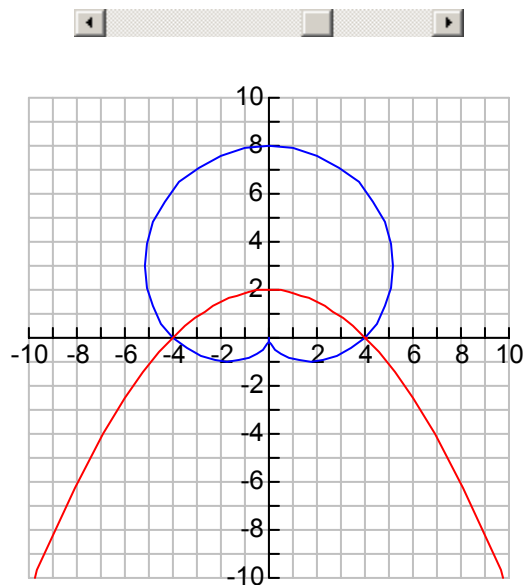
Using the slider bar we can investigate this family as the value of the parameter a changes.



Extension

Another fascinating activity is to consider the inverses of these curves: define $v(\theta) = \frac{16}{s(\theta)}$.

Once more, the slider bar provides insight into the dynamics:



It seems that the inverses are the conic sections; ellipse, parabola and hyperbola.

How can we prove this using TII?

As an example, we consider the case $a = 4$.

The graph suggests that the inverse is a parabola with equation $y = 2 - \frac{x^2}{8}$.

In that case, $v(\theta) = \frac{4}{\sin(\theta) + 1}$.

As a consequence, $v(\theta) \cdot \cos(\theta) = \frac{4 \cdot \cos(\theta)}{\sin(\theta) + 1}$ and $v(\theta) \cdot \sin(\theta) = \frac{4 \cdot \sin(\theta)}{\sin(\theta) + 1}$.

$$y = 2 - \frac{x^2}{8} \mid x = \frac{4 \cdot \cos(\theta)}{\sin(\theta) + 1} \text{ and } y = \frac{4 \cdot \sin(\theta)}{\sin(\theta) + 1}$$

Substitution gives:

$$\frac{4 \cdot \sin(\theta)}{\sin(\theta) + 1} = \frac{-2 \cdot ((\cos(\theta))^2 - (\sin(\theta) + 1)^2)}{(\sin(\theta) + 1)^2}$$

And this is true for all values of θ : $\text{solve}(ans, \theta)$.

13. Course notes

introduction

This file is in a different format to the other material. Instead of having a number of student tasks, help, hints and other files there is just a single Word document containing an introductory demonstration of TI InterActive! in the form of a teacher training course. It does contain, however, a few hyperlinks to TI InterActive! files. In case the instructor wishes to carry out the operations in TI InterActive!, hyperlinks to the TII files are provided.

data handling

Data handling

The first activity requires you to fit a model to some data and could provide the basis for a data handling coursework project. The following data are extracted from the Highway Code: www.highwaycode.gov.uk/09.shtml#97.

The TI InterActive! file which contains the data is called Braking.tii.

Shortest stopping distances

Speed		Thinking distance		Braking distance		Stopping distance	
kph	mph	m	ft	m	ft	m	ft
32	20	6	20	6	20	12	40
48	30	9	30	14	45	23	75
64	40	12	40	24	80	36	120
80	50	15	50	38	125	53	175
96	60	18	60	55	180	73	240
112	70	21	70	75	245	96	315

When you open TI InterActive! most of the toolbars should look familiar. Use the word-processing facilities to start your report with a title and date. Practice changing the format of the text using different styles, fonts, sizes, alignment etc.

The last row in the toolbars contains the mathematical icons. We want to enter data into lists, so select the Data Editor icon which is the 5th from the left. If you are familiar with graphical calculators you will recognise the list-based 'Stat Edit' layout.



Double click on the header for list L1 and rename it "Speed". Then enter the speeds into the first 6 cells of L1.

Double-click in the header for list L2 and rename it "Think". The values in the "Think" column are exactly the same, numerically, as the speeds, so enter "Speed" for the formula so that "Think" becomes a copy of "Speed".

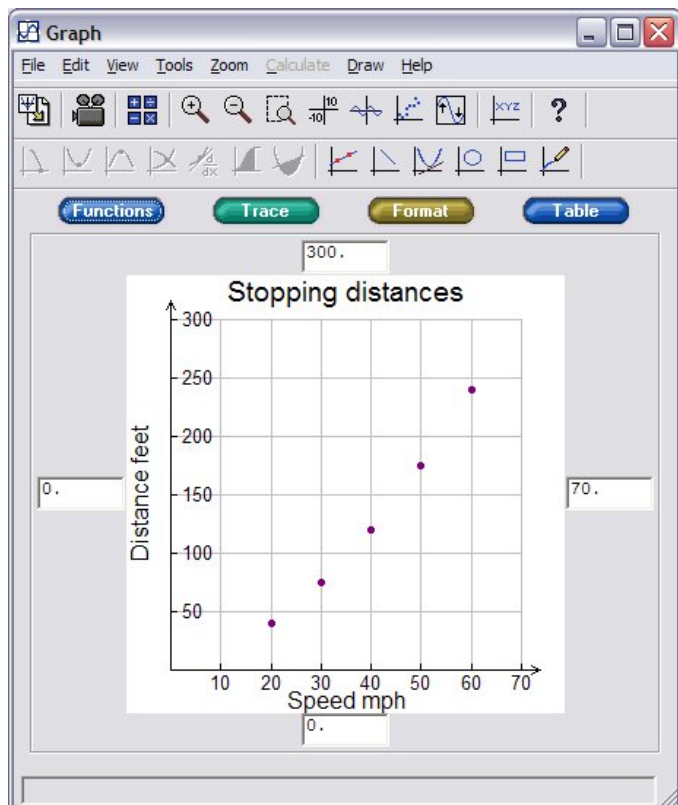
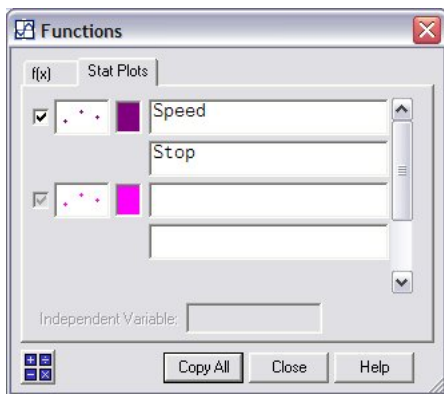
Rename L3 as "Brake" and key in the corresponding 6 data items (as shown below).

Rename L4 as "Stop" and enter "Brake + Think" as the formula for "Stop".

Finally, select columns L5 and L6 and use 'Edit' and 'Delete' to remove them.

listname formula	Speed {...}	Think :=Speed	Brake {...}	Stop :=Brake+Think
1	20	20	20	40
2	30	30	45	75
3	40	40	80	120
4	50	50	125	175
5	60	60	180	240
6	70	70	245	315
7				

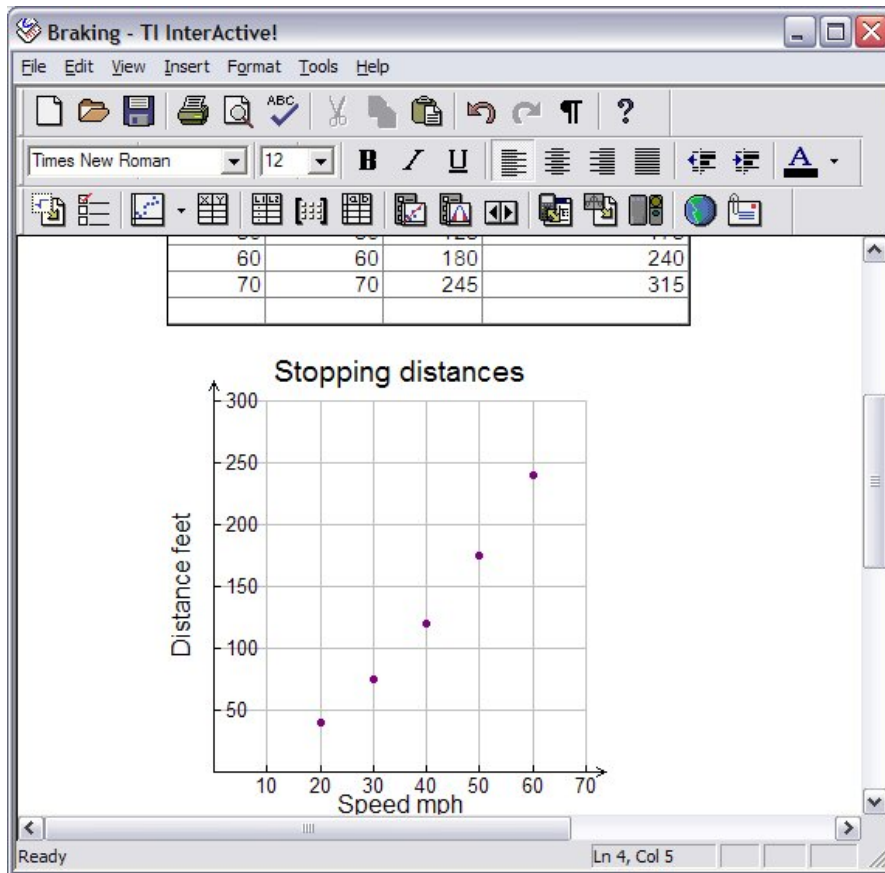
As you see, there are a lot of tools to use for data-handling with lists! We will start by drawing a scattergram for the data in the lists 'Speed' and 'Stop'. To produce a graph, select the Graph icon which is the 7th icon from the right in the upper toolbar.



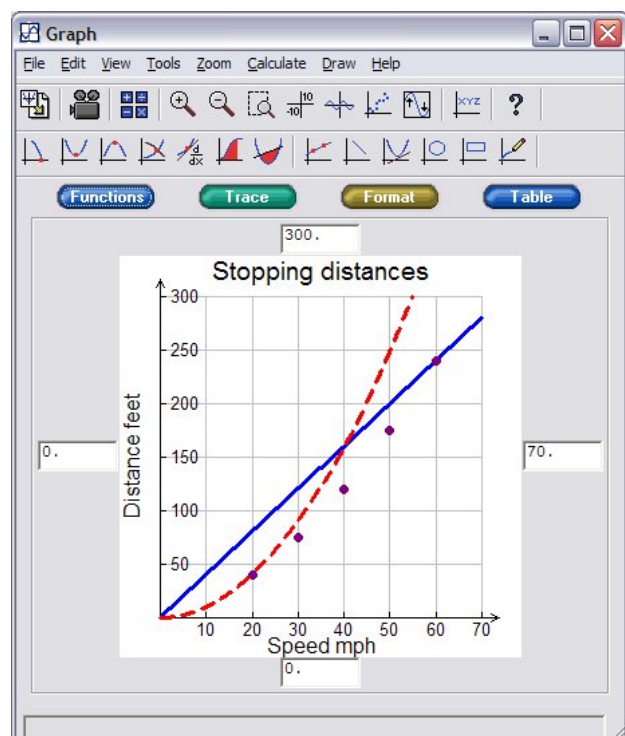
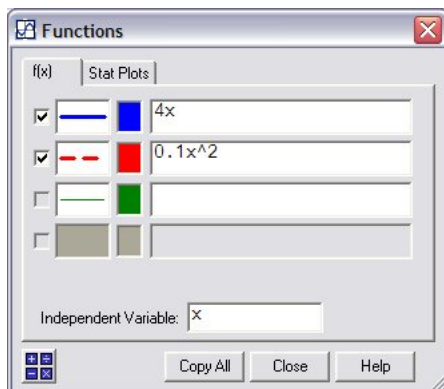
In the "Functions" window the "Stat Plots" tab should show empty boxes. Enter "Speed" and "Stop" for the names of the lists to be used for the scatterplot, as shown above. You can change the appearance of the scatterplot by double clicking in the type and/or colour icons next to the lists' names. Press "Copy All" to plot the data. You will now need to adjust the settings for the "Graph" window.

Change the settings for the graph by entering new values for the minimum and maximum on each axis. Use the "Format" button to change the step sizes on the axes, add a title, label each axis and to add arrows.

When you close the “Graph” window you will be asked if you want to save the graph in your current document – reply “Yes”! Similarly when you close the “Data Editor” window the data table is stored in the document. These items are “live” – just double-click the table or graph to re-open the “Data Editor” or “Grapher”.



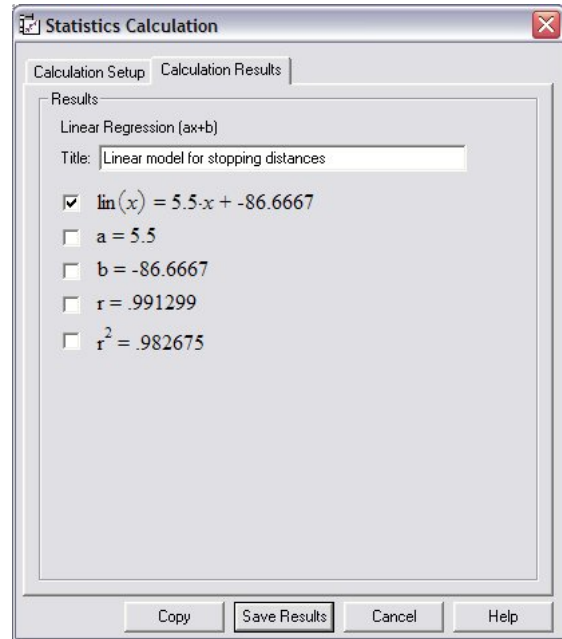
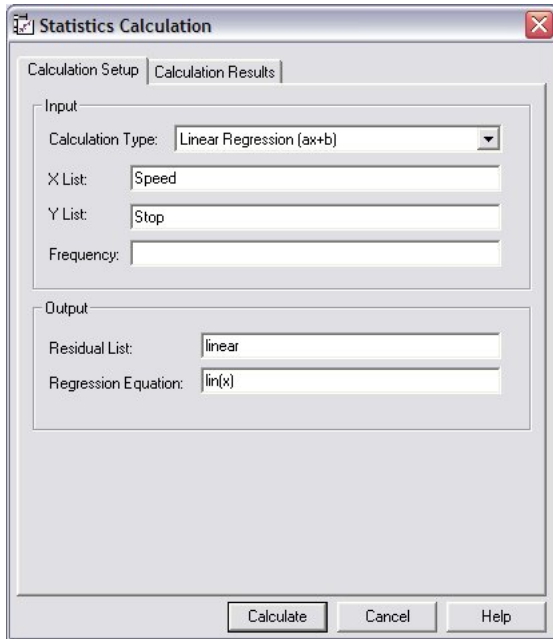
We’re now well on the way to making our report but haven’t as yet tried modelling the data, that is, experimenting by graphing different functions $f(x)$ to see whether we can get a good fit to the points in the scattergram. This graphing is easy using the tab called “ $f(x)$ ” on the “Functions” dialog box for the “Grapher”. Double-click on your graph and enter a function to graph over the scattergram.



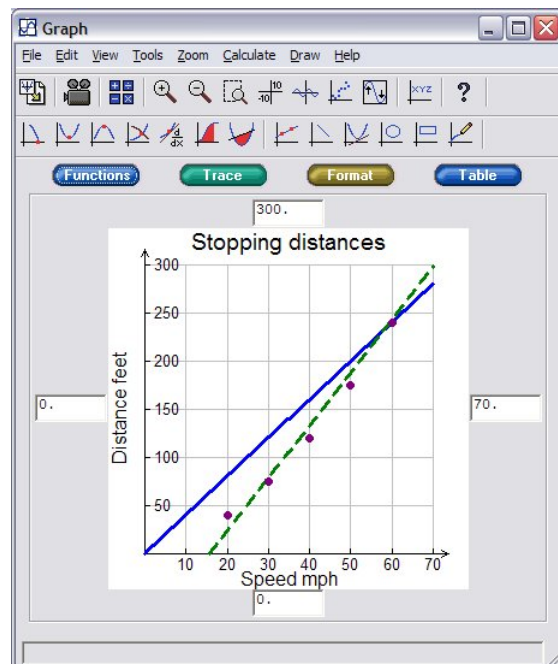
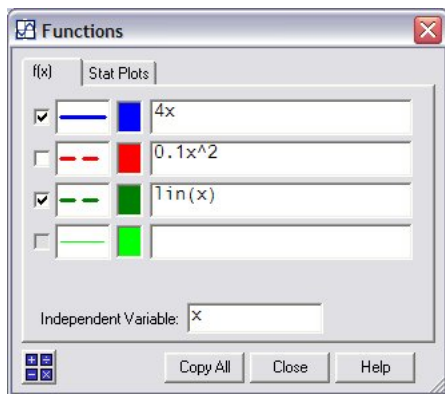
The figure shows a linear and a quadratic model – both fitted “by eye”. For these two functions we have assumed that $f(0) = 0$. We can now investigate the “best-fit” models produced by the software called “regression models”. To do this you will need to re-open the “Data Editor” and select the 5th icon from the right on the upper toolbar for “Statistics Calculation”.



Select “Linear Regression” as the “Calculation Type”, and specify the names of the data lists to use. Give the “Regression Equation” a sensible name, like $\text{lin}(x)$. Click the “Calculate” button to see the result. TI InterActive! also gives a value for r , the “correlation coefficient”.

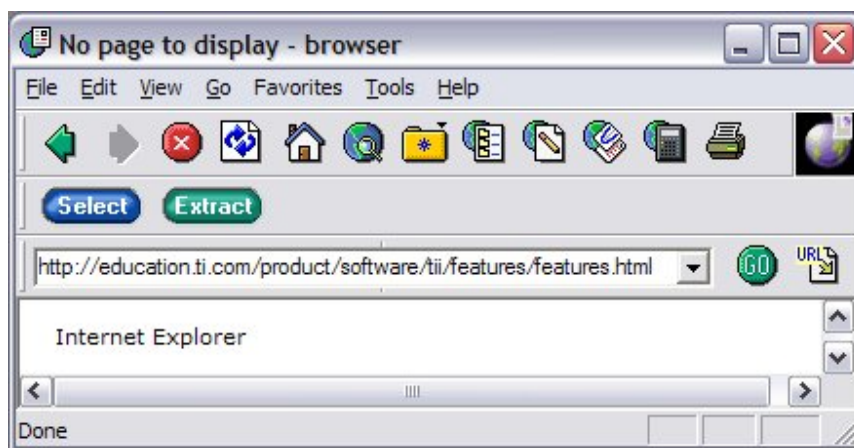


In order to see the effect on the graph, double click on the graph and use the “ $f(x)$ ” tab for the “Functions” box. Here the quadratic graph has been ‘unchecked’ so it isn’t drawn, and $\text{lin}(x)$ has been entered as the third equation.



Now see what the “best-fit” quadratic model looks like. Can you justify why a quadratic model might fit the data?

This exercise used “secondary data” obtained from the Web. One nice feature of TI InterActive! is the incorporation of a web browser (similar to MS *Internet Explorer*) with features that allow you to capture data from a web-page into the Data Editor’s lists, or into the *Excel*-style spreadsheet version – using the “Select” and “Extract” buttons.

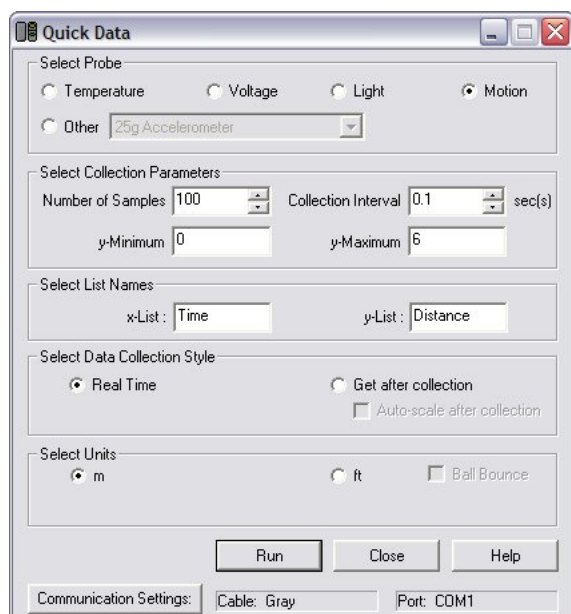


Data can be easily exchanged between TI InterActive! and other software programs such as *Excel*, or between TI InterActive! and TI graphical calculators using the appropriate link cable.

Experimental data can be captured directly using compatible dataloggers such as the CBR motion detector and the CBL versatile logger. The 4-th icon from the right on the upper toolbar of the Data Editor allows you to easily set up experiments such as those where distance-time graphs are produced by pupils walking in front of the CBR.



An example of such a data set is provided in the TI InterActive! file CBR.tii.

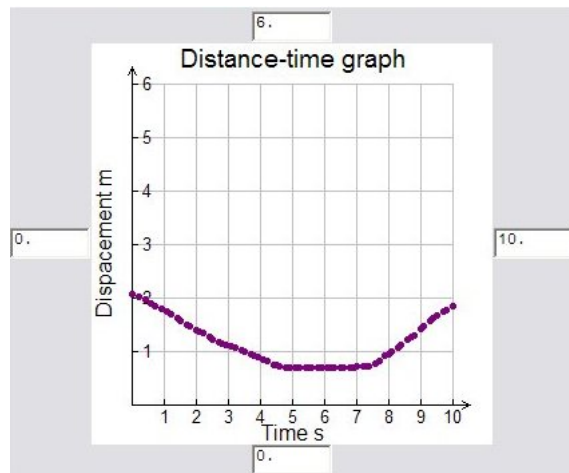
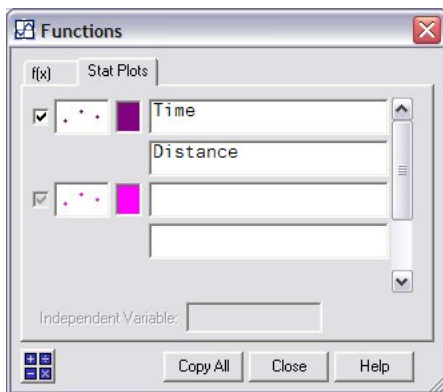


The ‘Quick Data’ screen lets you select the kind of probe or sensor to use. For the motion detector (CBR) select ‘Motion’, and set up the number of samples, the sampling rate and the names of the lists into which the data will be collected.

The CBR can be connected to the PC using a grey- or black-link cable via a serial port. An upgrade of TI InterActive!, version 1.2, is available and this will enable use of a USB cable to connect to a CBR.

Connect the CBR and perform a simple experiment such as walking in the path of the CBR for 10 seconds.

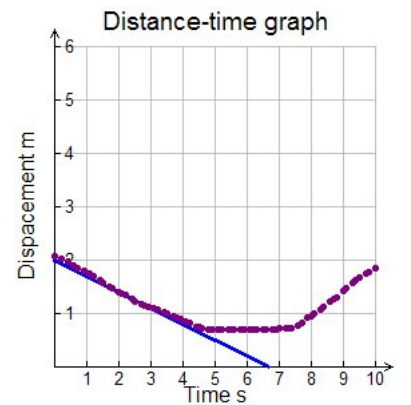
The corresponding values of time and displacement are plotted in real time as a scattergram in the 'Grapher'.



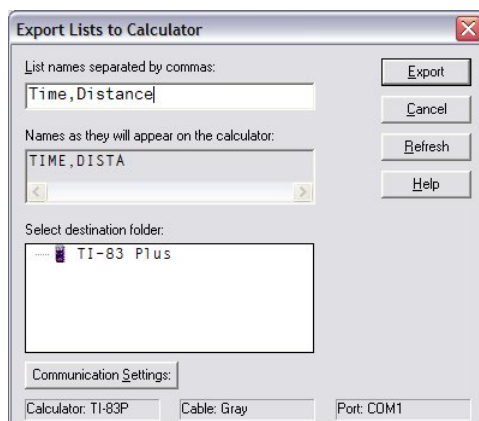
Can you find functions that model each part of the simple motion illustrated here?

This kind of experiment using a CBR enables teachers to provide contexts for constant and linear functions.

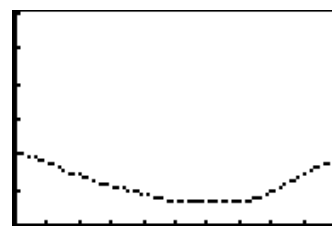
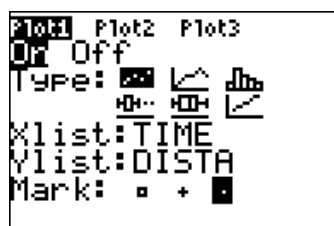
Students could be asked to walk in such a way that the graph obtained is (roughly) quadratic. What sorts of practical experiments can one perform which should produce (theoretically) a quadratic function?



If you want to export the data directly to a TI-83 Plus graphical calculator, or similar, it is very easy to do.



listname formula	Time {...}	Distance {...}
1	0	2.06835
2	0.19988	2.03608
3	0.39977	1.97471
4	0.59965	1.89508
5	0.69959	1.86131
6	0.89948	1.80378
7	1.09936	1.71721



That concludes this little tour of some data handling facilities of TI InterActive!

The TI InterActive! Math Box

We will conclude this short exploration of TI InterActive! by looking at some functions of the 'Math Box'. To follow this session in TI InterActive!, open the file Maths.tii.

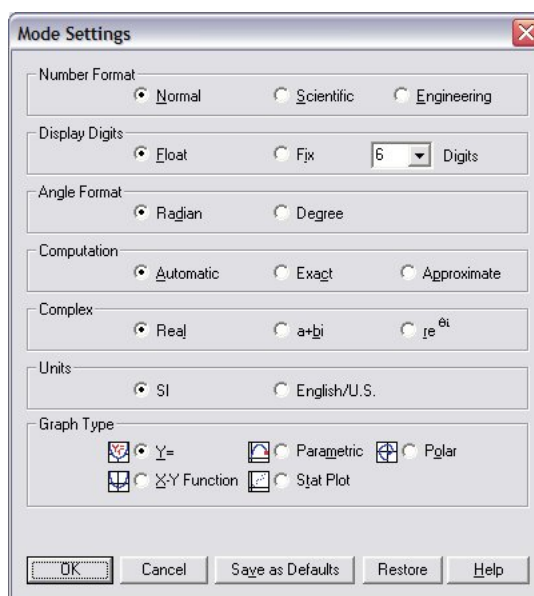
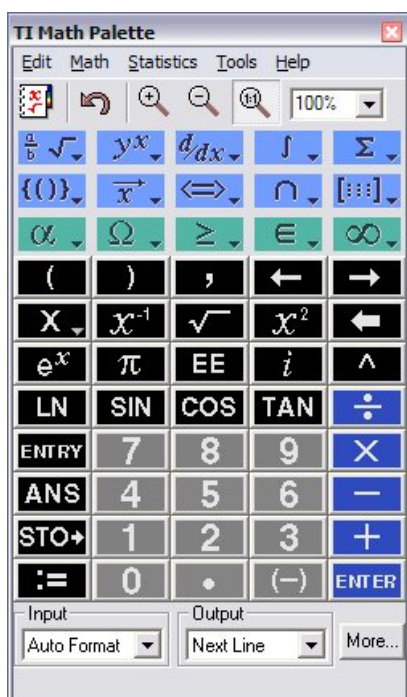


This opens up a blank area of the worksheet and also gives a floating 'Math Palette' which resembles the keyboard of a graphical calculator similar to the TI-83 Plus. Now you can begin to key in a calculation using a mixture of simulated key presses using the mouse and characters typed directly from the PC's keyboard.

Note that when you press the 'ENTER' key, the expression is evaluated according to the format chosen in the 'MODE' setting.

$$45 + \sqrt{32^2 - 5^2}$$

$$3\sqrt{111} + 45$$



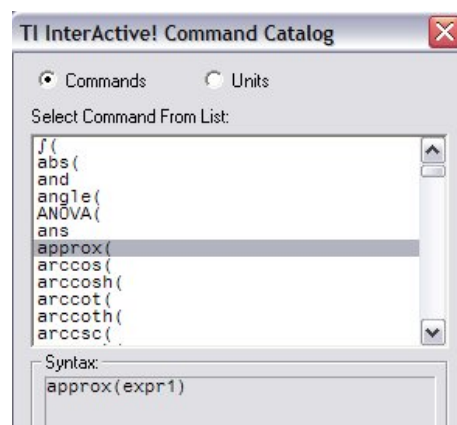
The default mode is with Automatic Computation. But you can force this to be either Exact or Approximate.

However there are also many useful tools in the 'Math Palette' to control the output format. The most recent result can be recalled using the 'ANS' key in the Palette.

You can find the "approx" function either by using the 'Catalog' icon on the Palette, or using the 'Math' pull down menu, and selecting 'Algebra'. Have a look at the range of functions you can find under the 'Math' or 'Stat' menus or from the 'Catalog'.

$$\text{approx}(\text{ans})$$

$$76.607$$



These are not only numerical, but many can be symbolic as well. Two examples of how 'factor' works are shown below.

$$\text{factor}(x^2 - y^2)$$

$$(x + y) \cdot (x - y)$$

$$\text{factor}(100!)$$

$$2^{97} \cdot 3^{48} \cdot 5^{24} \cdot 7^{16} \cdot 11^9 \cdot 13^7 \cdot 17^5 \cdot 19^5 \cdot 23^4 \cdot 29^3 \cdot 31^3 \cdot 37^2 \cdot 41^2 \cdot 43^2 \cdot 47^2 \cdot 53 \cdot 59 \cdot 61 \cdot 67 \cdot 71 \cdot 73 \cdot 79 \cdot 83 \cdot 89 \cdot 97$$

This leads us into the issue of the value of symbolic manipulation within mathematics teaching. For example the following screens show some different approaches for solving a simple linear equation in a single variable, such as $3x + 5 = 17$.

Note that TI InterActive! doesn't 'know' what the variable is – so we have to provide not only the equation, but also the variable to solve for.

$$3x + 5 = 17$$

$$3x + 5 = 17$$

$$\text{ans} - 5$$

$$3x = 12$$

$$3x + 5 = 17$$

$$3x + 5 = 17$$

$$\text{solve}(a \cdot x + b = c, x)$$

$$x = \frac{-(b - c)}{a}$$

In moving from the 3rd to the 4th screen there is no notion of right or wrong – the output is just the manipulated input – but we can now use feedback to tell us whether we have achieved the desired result, that of removing the number 5 from the left hand side.

If you want to try extending the idea, for example, to a set of simultaneous equations, it is convenient to give each output a label to be able to use for reference. The following screens suggest one approach.

Note: The arrow key is obtained by using the 'STO' key: **STO→**.

$$3x + 5y = 11 \rightarrow \text{eq1}$$

$$3x + 5y = 11$$

$$7x - 2y = 12 \rightarrow \text{eq2}$$

$$7x - 2y = 12$$

$$2 \cdot \text{eq1} + 5 \cdot \text{eq2} \rightarrow \text{eq3}$$

$$41x = 82$$

You can also work on lists and find statistics etc. directly from the Math Box. Here we will produce a list of simulated dice throws, compute their mean and median and show how to represent the data as both a histogram and a bar chart.

First use the Math, Probability and Random menus to select Random Integer. 'randInt(1,6,60)' will generate 60 random integers between 1 and 6 inclusive. Store these random integers in the list 'die1' using 'STO'. Use the Math, List, Math menus to find 'mean' and then 'median'.

$$\text{randInt}(1, 6, 25) \rightarrow \text{die1}$$


$$\{1., 3., 1., 5., 6., 4., 5., 1., 6., 4., 3., 3., 4., 5., 4., 3., 2., 1., 6., 5., 3., 4., 4., 4., 3.\}$$

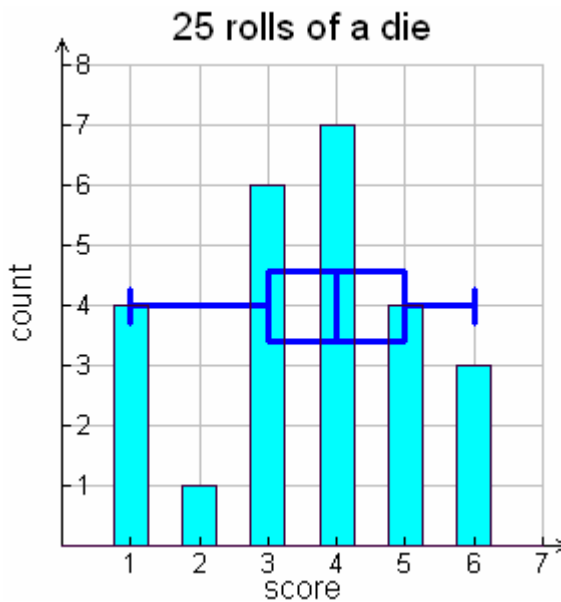
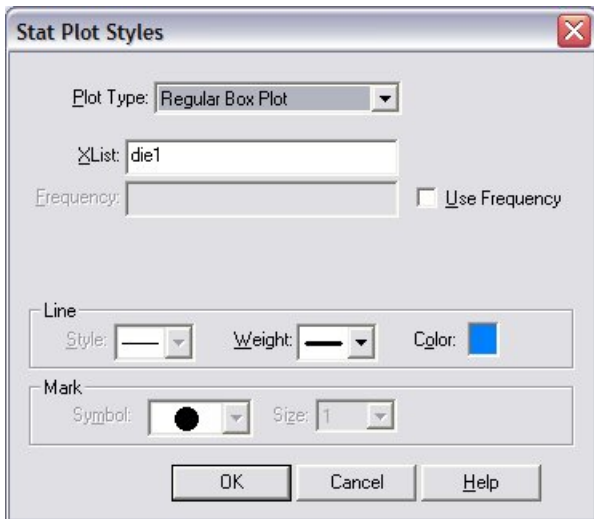
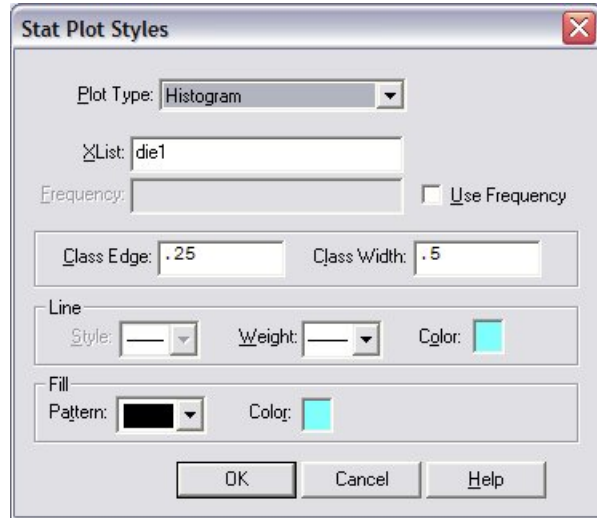
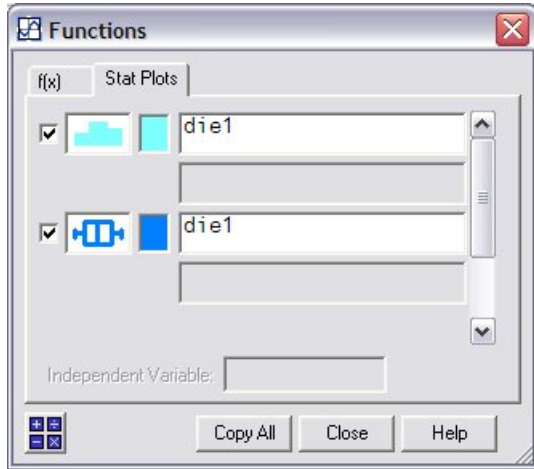
$$\text{mean}(\text{die1})$$

$$3.6$$

$$\text{median}(\text{die1})$$

$$4.$$


Use the Graph icon, but first select the statistical plotting mode .



The box plot which summarizes the data concludes this demonstration of the some relevant features of TI InterActive!.

Appendix A: TI InterActive! manual for student units 7 and 8

intro

If you need help concerning the use of TI InterActive!, you can use the built-in help function by clicking the  icon in the menu bar above.

manual

Collecting and plotting Quick Data lists

By connecting a TI-GRAPH LINK™ cable between your computer and a CBL 2™ you can collect and plot data automatically. With a CBL 2, a variety of probes such as temperature, voltage, light, and motion are supported.

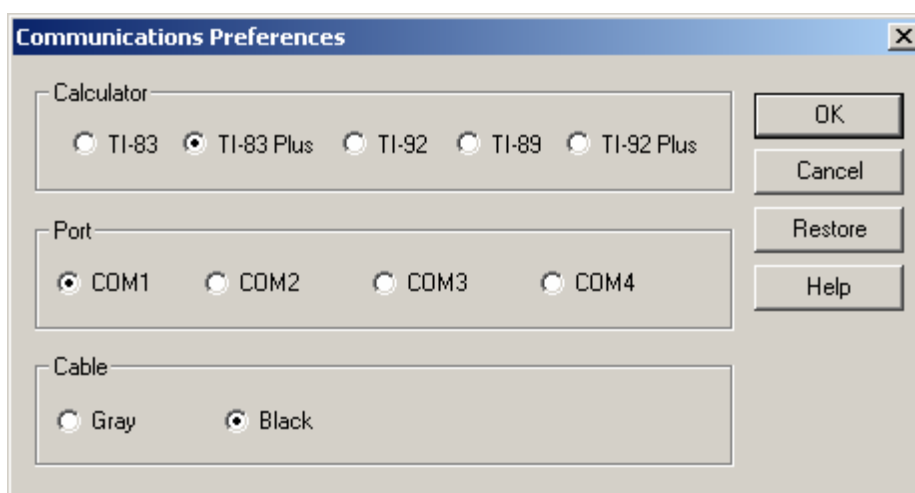
To configure the TI-GRAPH LINK connection (TI InterActive! 1.1):

Connect a probe (e.g. the Pressure sensor) on the CH1 channel of CBL 2.

Insert the other end of the TI-GRAPH LINK into the link port on the calculator. Push the plug in firmly. Click **Edit** → **Preferences** → **Communications** on the document's menu bar.


In the dialog box, click the:

- Calculator type,
- Com port (serial port number) where the TI-GRAPH LINK cable is connected,
- Type of TI-GRAPH LINK cable you are using (gray or black).



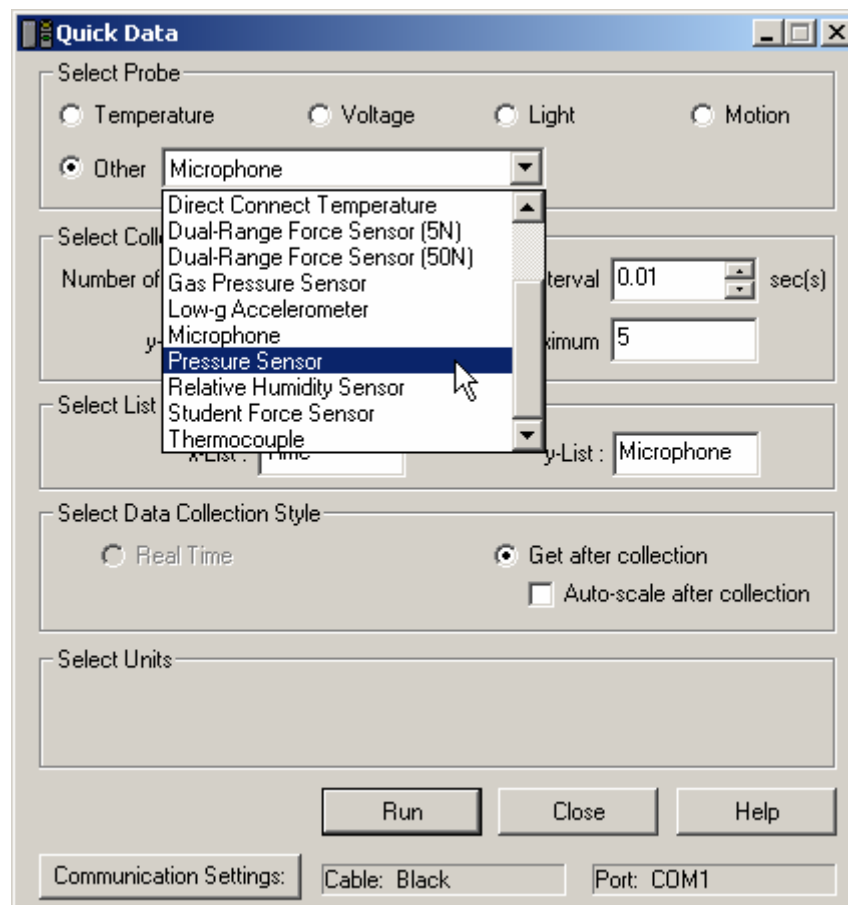
To collect and plot Quick Data lists:

a. Make sure the probe is connected properly and that the unit is turned on.

b. Click Quick Data, , on the document's TI InterActive! toolbar (or click Tools → Quick Data Tool).

A new List Editor is inserted automatically to store the collected data.

c. In the dialog box, select the connected probe and specify the appropriate settings. To select Pressure sensor click Other and then the requested probe. Specify the other appropriate settings.



Quick Data

Select Probe

Temperature
 Voltage
 Light
 Motion

Other Pressure Sensor

Select Collection Parameters

Number of Samples:
Collection Interval: sec(s)

y-Minimum:
y-Maximum:

Select List Names

x-List:
y-List:

Select Data Collection Style

Real Time
 Get after collection

Auto-scale after collection

Select Units

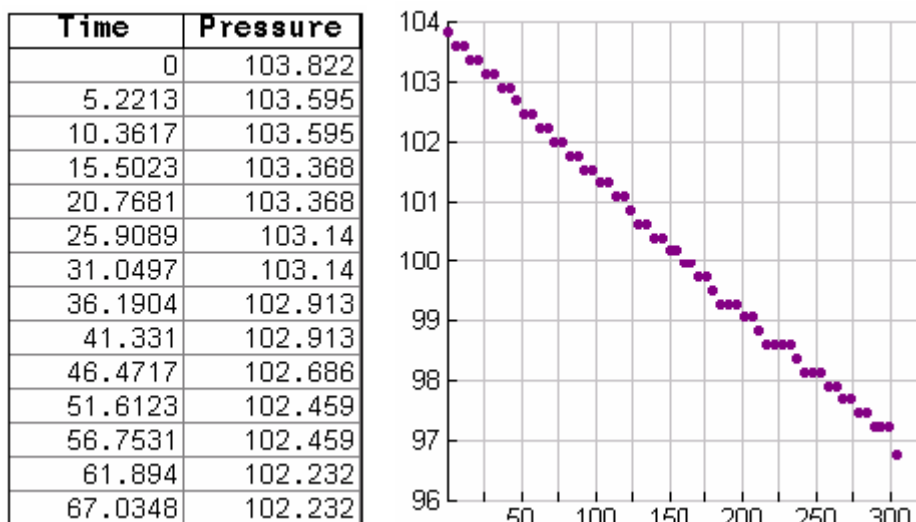
kPa
 mmHg

atm

Communication Settings:
Cable:
Port:

d. Click **Run**.

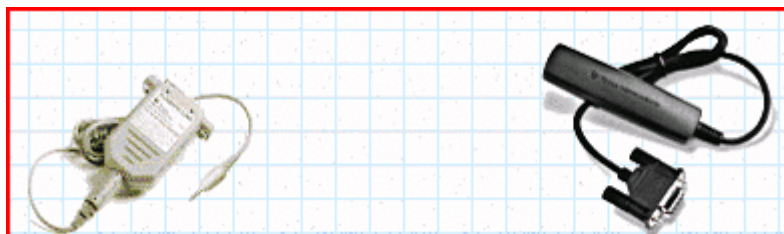
The collected data lists are placed in the associated List Editor and a new **Graph** window shows a stat plot of the data.



If necessary, click **Zoom Statistics**, , on the **Graph** window to see all collected data points.

Transferring data between a calculator and your computer

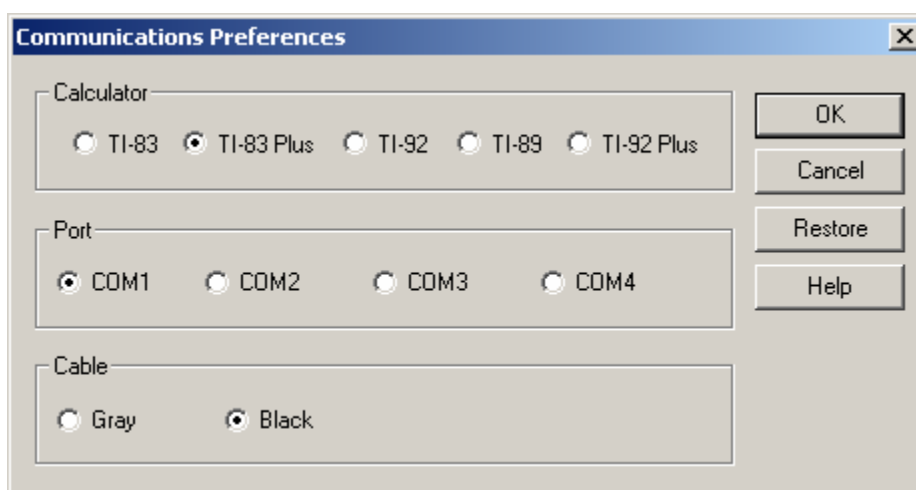
Using a TI-GRAPH LINK™ cable you can connect your TI calculator to your PC. With TI InterActive! 1.1 you can use two types of TI-GRAPH LINK cables: a gray cable and a black cable.



Insert one end of the TI-GRAPH LINK into the link port on the calculator and one end into the serial port of your PC. Push the plug in firmly.

Click **Edit** → **Preferences** → **Communications** (TI InterActive! 1.1) on the document's menu bar. On the dialog box, click the:

- Calculator type,
- Com port (serial port number) where the TI-GRAPH LINK cable is connected,
- Type of TI-GRAPH LINK cable that you are using (gray or black).

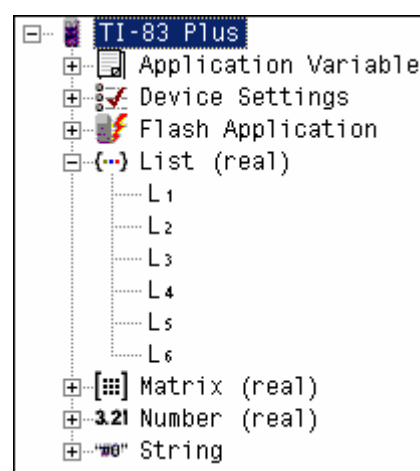


In an open List Editor:

Click **Import Calculator Data**, , on the editor's Standard toolbar (or click **File** → **Import** → **from TI Calculator**).





For a TI-83 Plus the dialog box shows the list names on the calculator.

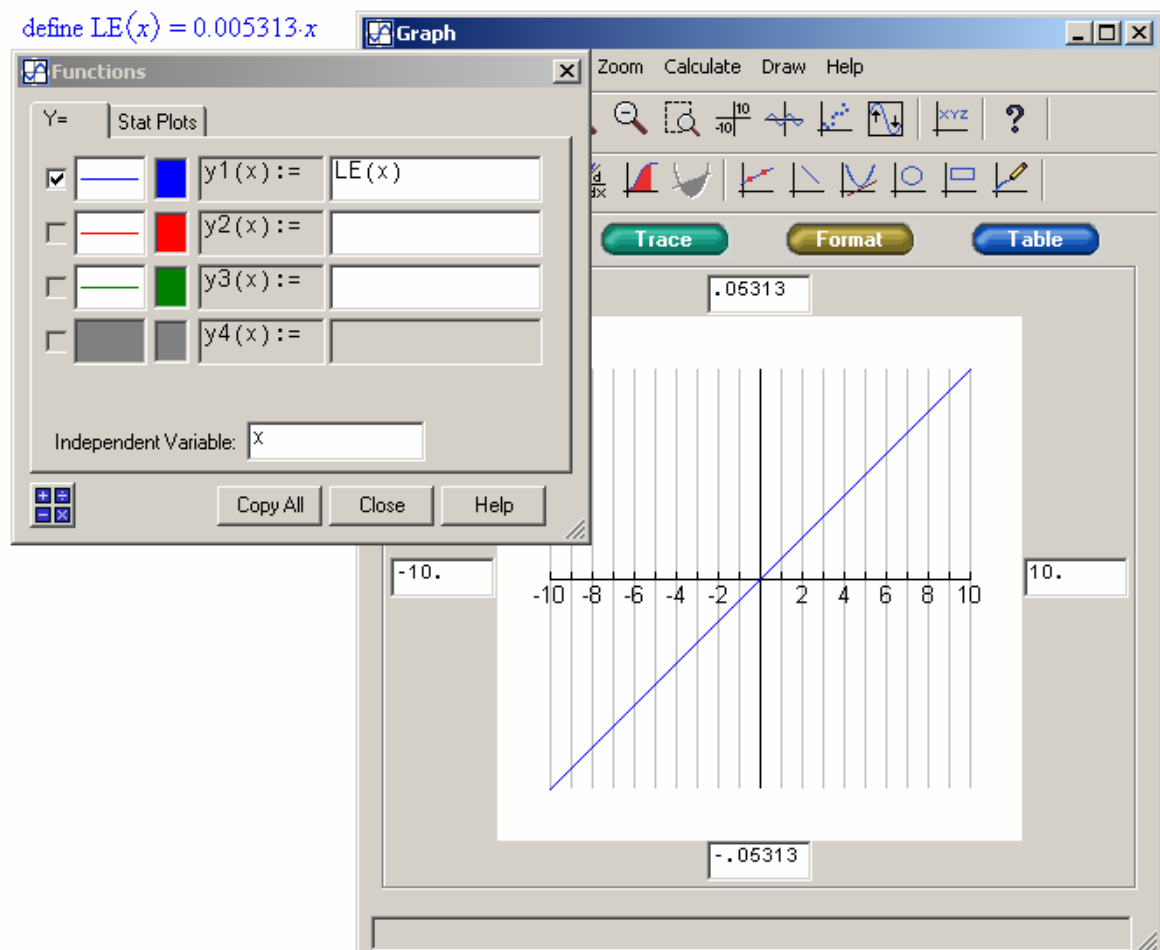
Similar to Windows® Explorer, you can click a (-) sign to collapse the hierarchical listing below it. The (-) changes to (+), which you can click to expand the listing.



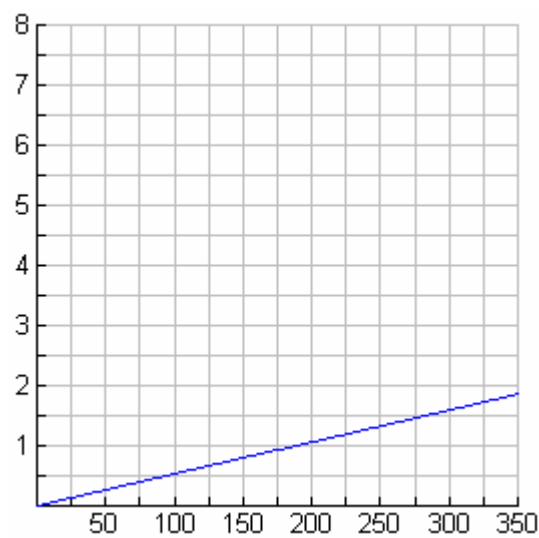
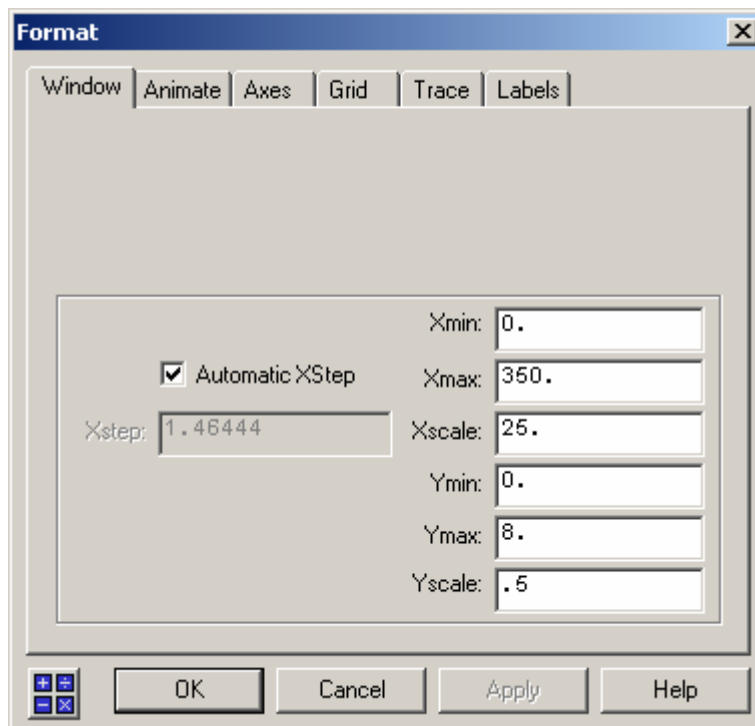
Graphing: Graphing a function defined in your document

To graph the function, for example $LE(x)$, position the entry cursor where you want to insert the graph. Then:


- Use the menu bar or the **Graph** toolbar button, , to insert a graph.
- To use the menu bar, click **Insert** → **Graph** and then click **Y=**, , or **X-Y Function**, .
- To use **Graph**, , on the TI InterActive! toolbar, be sure that the button's image shows the correct graph type.
- In the **Functions** window enter the function to graph and in the **Graph** window click **Zoom** → **Fit**.

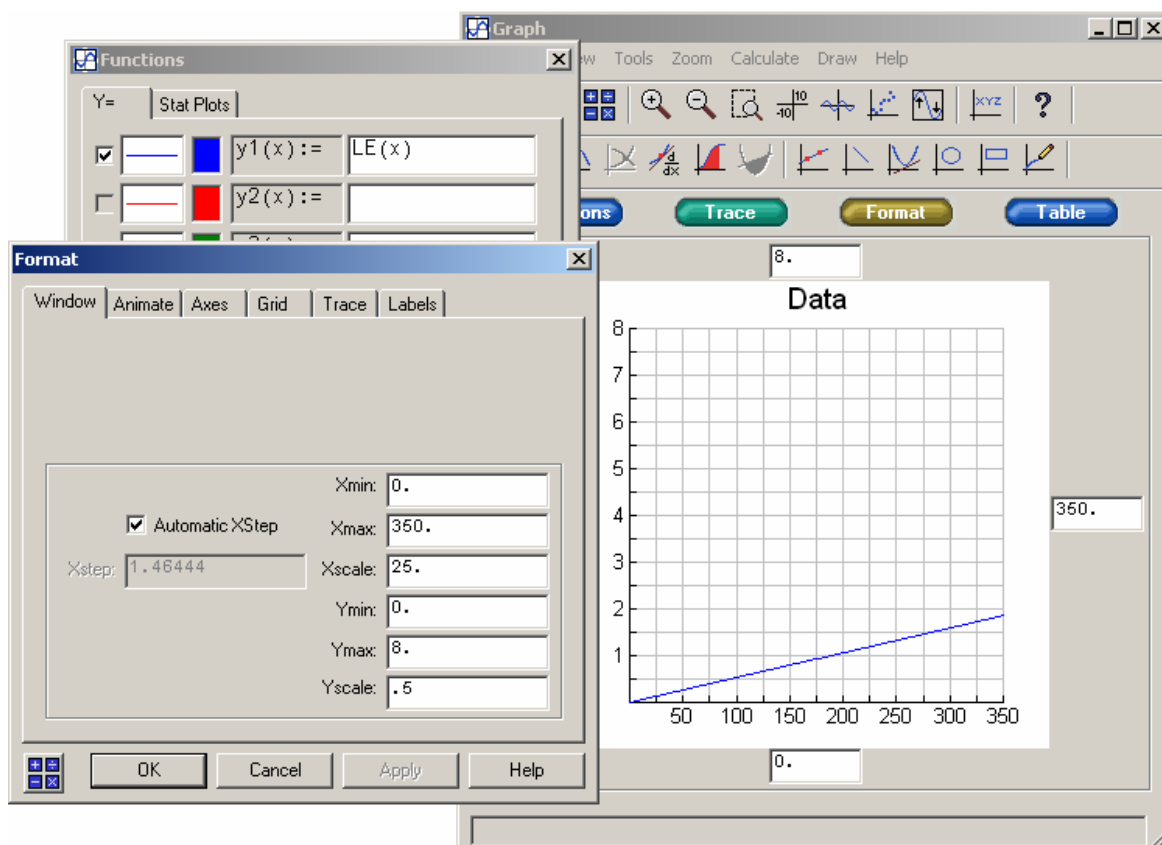


e. Use the  button to set the window:



Graphing: Adding a title and axes labels

- Open the **Graph** window for the graph.
- Click  to show the **Format** dialog box.
- Click the **Labels** tab.




On the dialog box, you can:

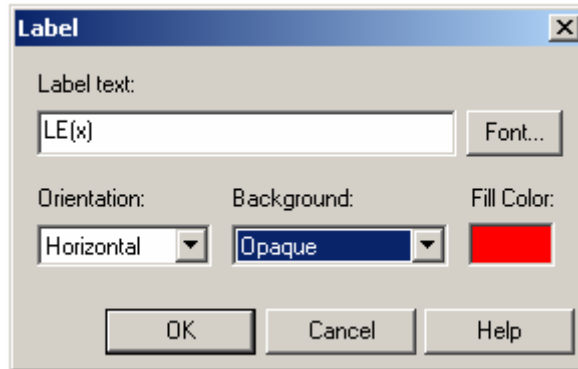
- Show or hide a title for the graph and select its default position. You can also set the title font and color.
- Show or hide a label for the x and/or y axes and select the default position. You can also set the font and color for the label.

To display the **Labels** tab directly, double-click an existing title or axis label on the **Graph** window.

Graphing: Adding text labels

a. Open the **Graph** window.

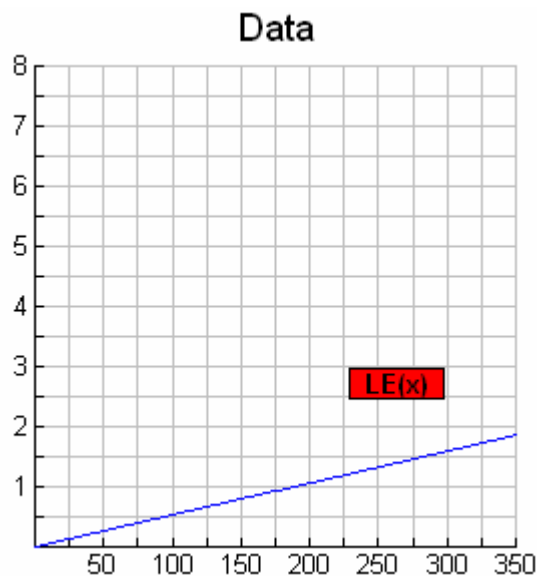
b. Click **Label**, , (or click **Draw** → **Label**).



c. On the dialog box:

- Type the text for the label. Click **Font** to set attributes such as font, size, underlining and color.
- Select an orientation of horizontal (left to right) or vertical (rotated 90 degrees). Select the background for the text as transparent (the graph's background and grid shows through the label's background) or opaque (a solid color).
- For an opaque background, set the color. This **Fill Color** is the background color, not the text color.


d. Click **OK**.

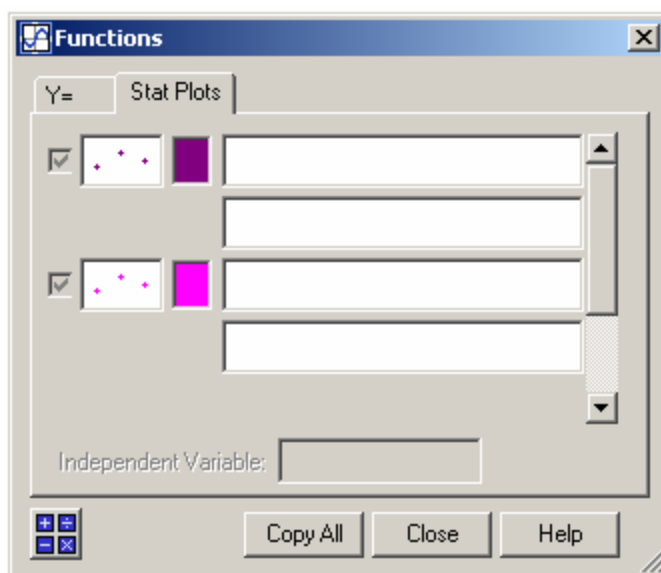



e. The dialog box closes and the label appears at the center of the graph.

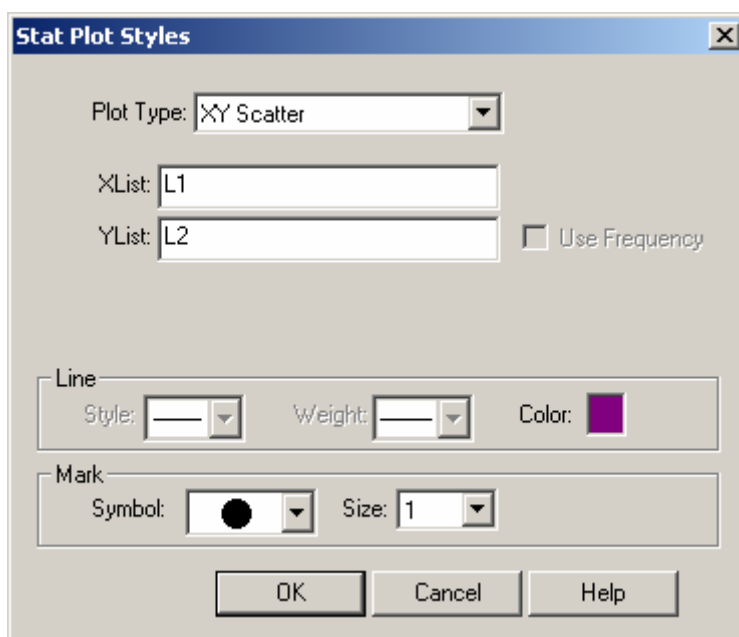
f. Click the label, hold down the left mouse button and drag the label to the appropriate location.

Graphing: Inserting a statistical plot

- Position the entry cursor where you want to insert the plot.
- Click **Graph**, , on the TI InterActive! toolbar (or click **Insert** → **Graph** → **Statistical Plot**).
- The **Graph** window and **Functions** window appear.
- In the **Functions** window be sure that the **Stat Plot** tab is selected.

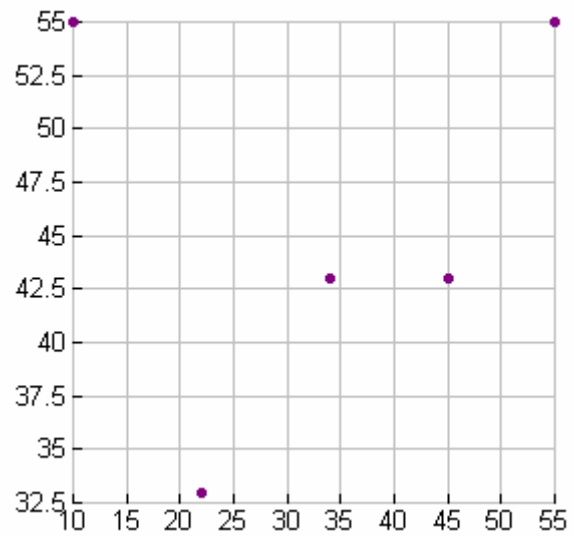
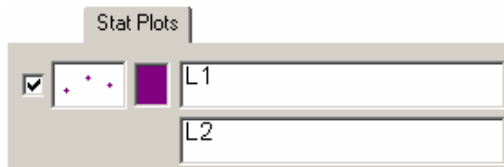


- Enter the data lists to plot. Whether you need to enter one- or two-variable data depends on the plot type you select later.
- Click the  style box to the left of the data to show the **Stat Plot Styles** dialog box.

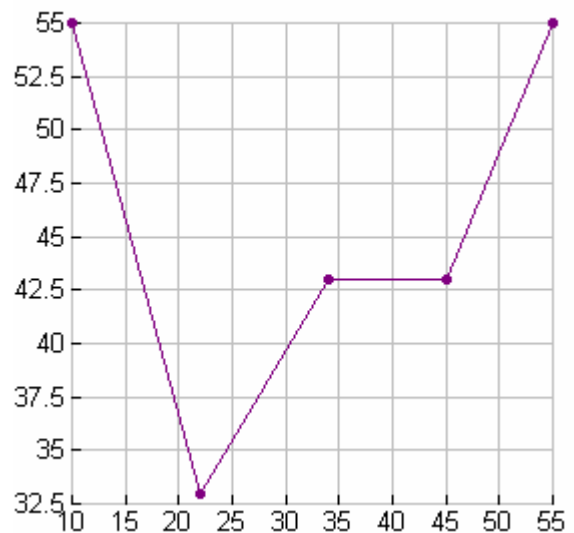
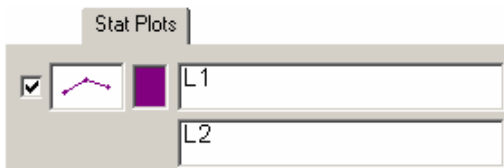


- For **Plot Type**, click  to select a plot type.


h. Select **XY Scatter** to plot single points



i. Select **XY line** to plot data as points connected by line:




j. Click **OK**.

k. The **Graph** window shows the plotted data. Click **Zoom Statistics**, , on the **Graph** window to change the window boundaries to show all data points.

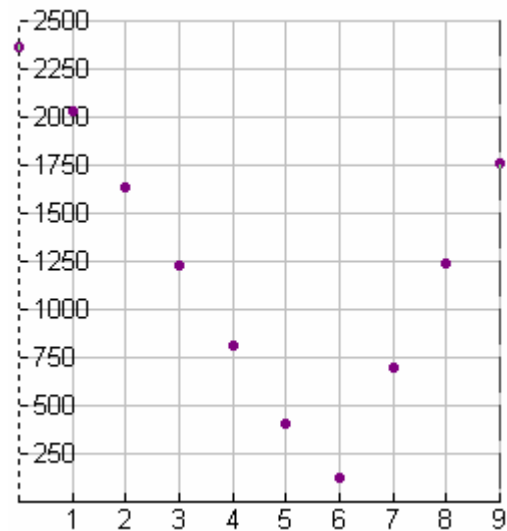
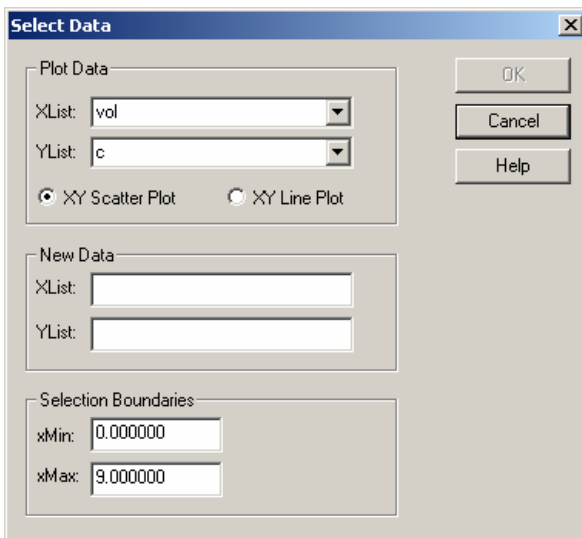
Graphing: Selecting data from a plot

In an open List Editor:

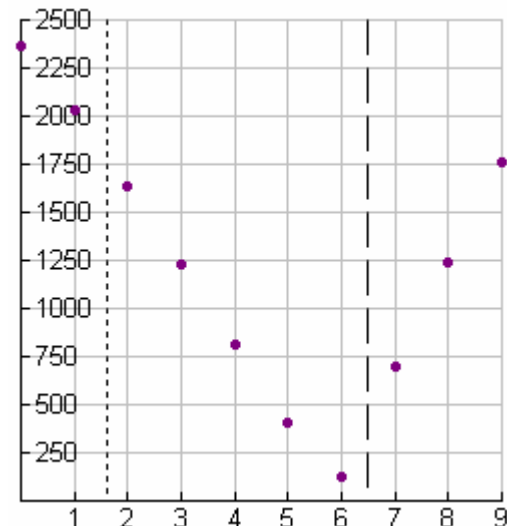
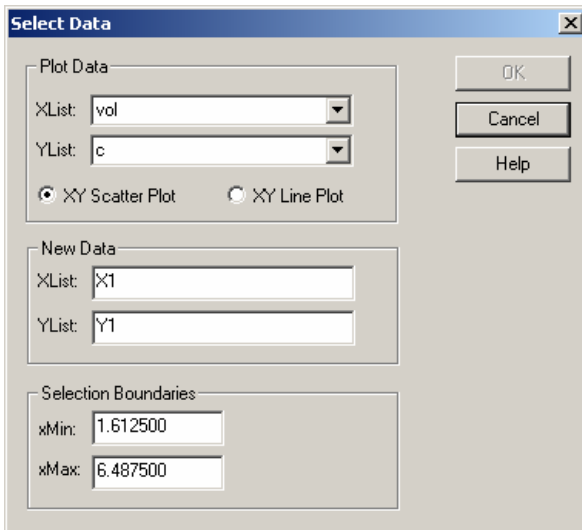
- Select the lists that contain the original x and y data points.
- Click **Select data**, , on the Data Editor's Standard toolbar (or click **Data _Select Data**).

The **Select Data** dialog box appears and a **Graph** window is placed behind the dialog box and editor.

VOL	C
0	2364.38
1	2028.97
2	1631.46
3	1229.81
4	815.73
5	409.935
6	124.223
7	695.648
8	1242.23
9	1763.96

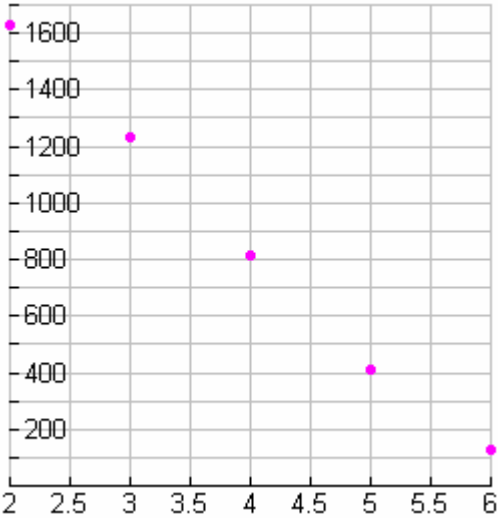


- On the dialog box for **New Data** specify where to store the selected data. You can specify new list names. In the Graph window vertical dashed lines on each side of the graph indicate the selection boundaries.
- Set the left and right boundary lines. Click a line, hold down the left mouse button and drag the line to an appropriate location. The **Selection Boundaries** area on the dialog box shows x values that correspond to the lines.




e. On the **Select Data** dialog box click **OK**. If you can't see the dialog box then click the **Data Editor** button on the taskbar.

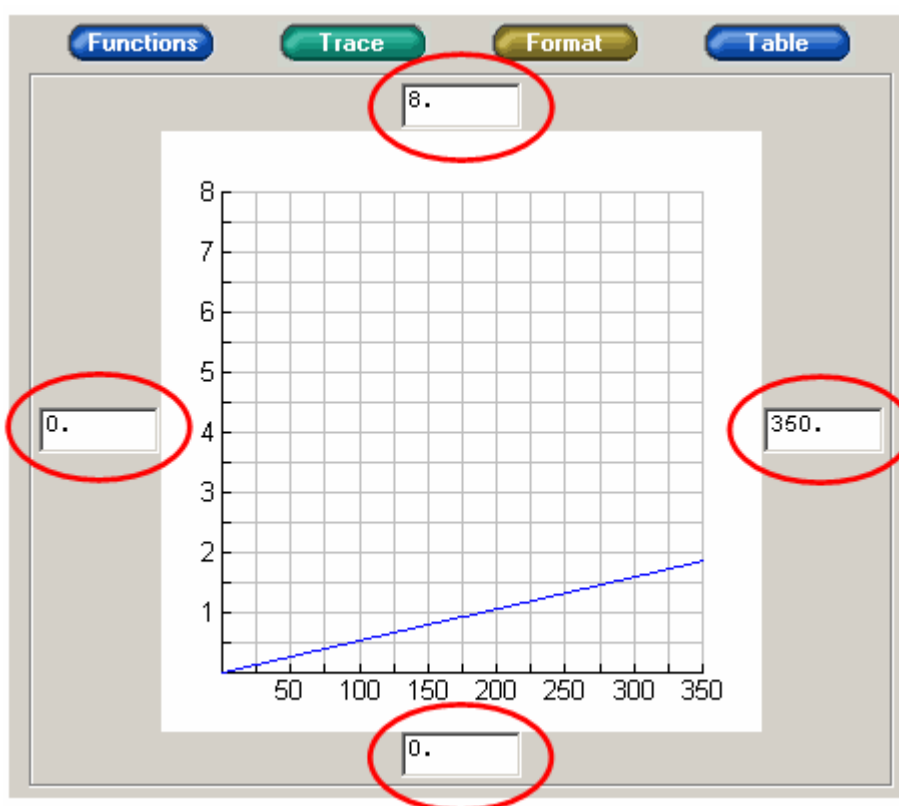
VOL	C	X1	Y1
0	2364.38	2	1631.46
1	2028.97	3	1229.81
2	1631.46	4	815.73
3	1229.81	5	409.935
4	815.73	6	124.223
5	409.935		
6	124.223		
7	695.648		
8	1242.23		
9	1763.96		



Graphing: Resizing a graph by setting windows boundaries


- Open the **Graph** window for the graph.
- Click  to show the **Format** dialog box.
- Click the **Window** tab.
- Enter the appropriate values.
- Click **OK**.

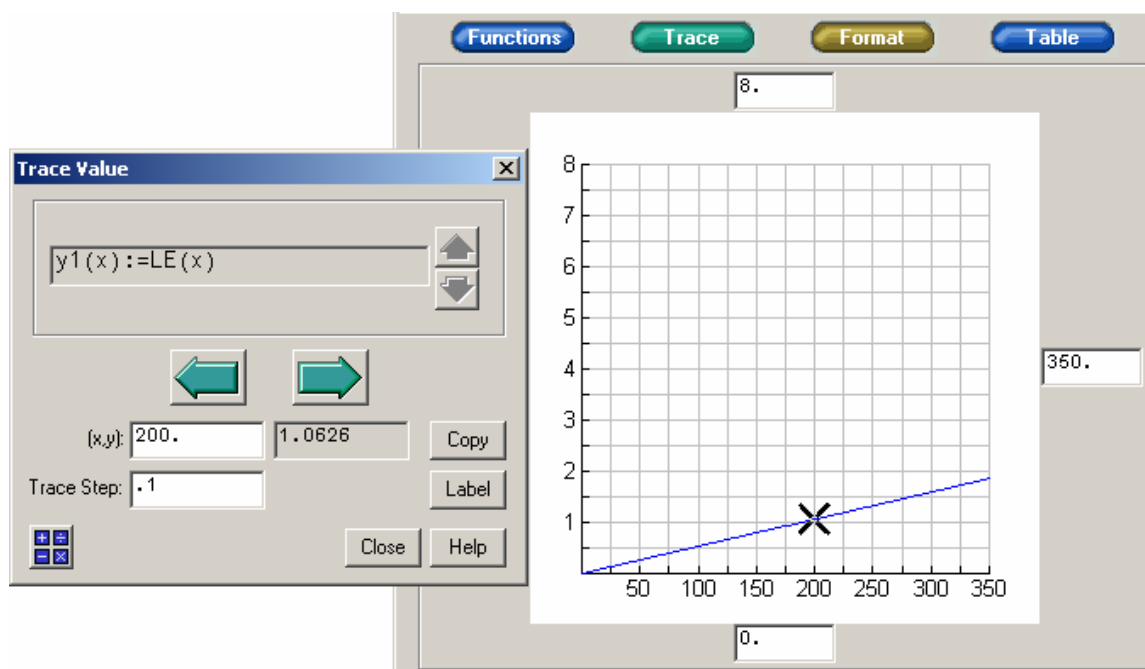
You can set window boundaries directly in the **Graph** window without having to display the **Format** dialog box. Enter appropriate values in the input boxes above, below and on either side of the graph.





Graphing: Tracing or evaluating a graphed function

For each graphed function, you can use the Trace feature to show the exact coordinates of points on the curve. You can also evaluate the function at a specified value of its independent variable.

- Open the **Graph** window for the graph.
- If you have more than one function graphed then you can go to the **Graph** window and select the function curve to use. Otherwise, the first one selected in the **Functions** window is used although you can select a different one later.
- Click  (or click **Tools** → **Trace** or **Calculate** → **Value**).
A dialog box shows the function and the coordinates of the trace cursor.



- On the dialog box:

- To select a different function, click .
- To move the trace cursor, click . The trace cursor moves along the function in increments set by **Trace Step**.
- To evaluate the function at a particular point and move the trace cursor to that point, click the coordinate box for the independent variable. (This is the only coordinate box that accepts input as all others are grayed out.) Type a value, and press **Enter** or **Tab**.
- To change the **Trace Step** click the input box. Type a value and press **Enter** or **Tab**.
- To mark the specified point on the graph and add a label that shows its coordinates, click **Label**. (After you close the dialog box, you can move, edit, or delete the label.)

- When finished tracing, click **Cancel**.

To move the trace cursor using the mouse:

- Drag it to a different location on the function.
- Click any place on the graph.


To trace a different graphed function simply click that curve on the graph.

Inserting or editing a Math Box

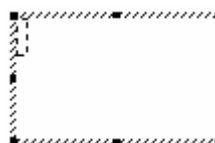
To perform a calculation in your document, you cannot simply type a math expression as text. You must insert a Math Box and then type the expression in the Math Box.

To insert a Math Box:


- a. Position the entry cursor where you want to insert the Math Box.


- b. Click **Math Box**, , on the document's TI InterActive toolbar (or click **Insert** → **Math Box**). An empty Math Box containing an entry cursor is inserted, and a Math Palette appears.

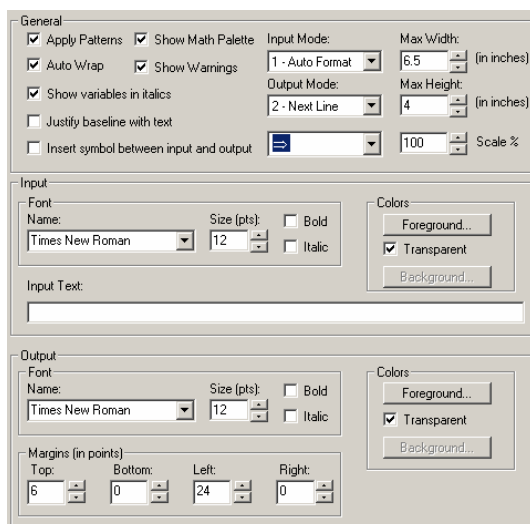
- c. Type a math expression on the keyboard, or use the Math Palette to enter numbers and functions similar to a calculator. You may use a combination of both the keyboard and Math Palette.



- d. To evaluate the expression, press **Enter** on the keyboard or click **Enter** on the Math Palette. You can also evaluate by clicking outside the Math Box.

- e. Click  to set Input and Output format.

- f. Click  to set any other Math Box properties.




To edit a Math Box:


- a. Double-click the Math Box to open it for editing.
- b. An edit cursor is shown in the Math Box and the Math Palette appears. You can change the entered expression as necessary and re-evaluate it.

Performing a regression calculation

For a regression calculation, you must have two lists of corresponding X and Y data points. You can refer to list names such as L1 and L2 in the calculation.

a. Position the entry cursor where you want to insert the results.

b. Click **Stat Calculation Tool**, , on the TI InterActive! toolbar (or click **Insert** → **Stat Calculation Tool**).

- For **Calculation Type**, click  to select the type of regression.
- For **X List** and **Y List**, type the appropriate list name. If you selected lists in a List Editor their names are shown.
- Specify names for the output residual list and regression equation. By default, **RESID**→ and **regEQ(x)** are used respectively.

c. Click **Calculate**.


The **Calculation Results** tab shows the regression equation and the values of associated statistical results. The number of digits in the values, etc., are shown according to the current mode settings. By default, the regression equation is selected to be saved in the document.

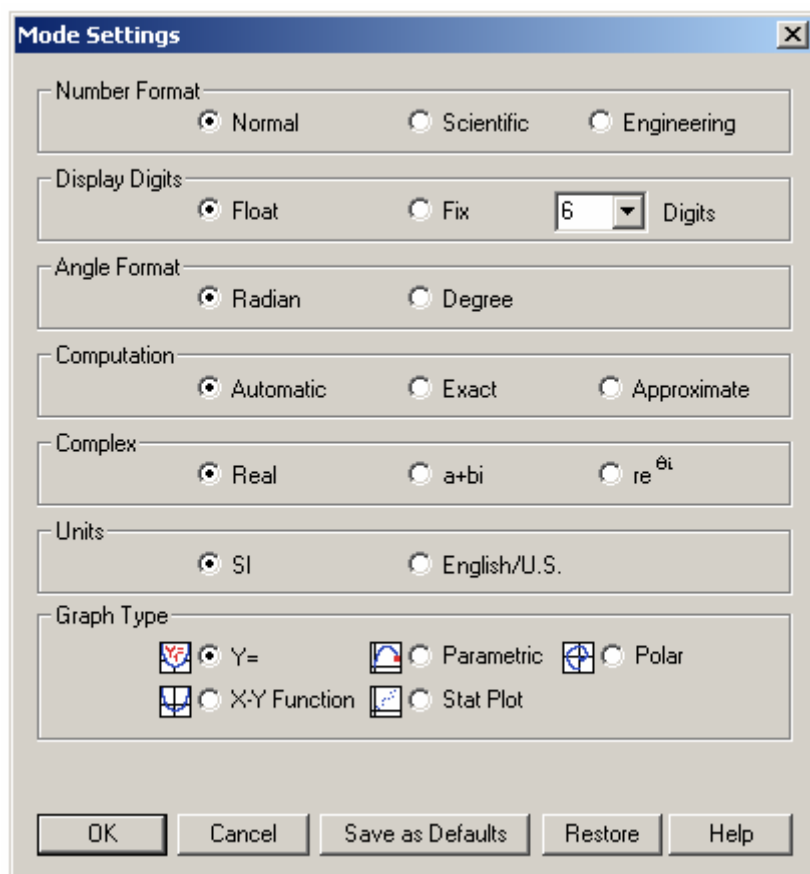
d. On the **Calculation Results** tab:

- (Optional) Type a title to display with the results.
- To select or deselect a result item, click it to check or uncheck its box.
- Click **Save Results** to insert the selected items into your document.

Setting the number format

Mode settings control the format in which numbers are displayed, whether angles are treated as radians or degrees, the Computation mode, etc.

- a. Click **Mode Settings**, , on the document's TI InterActive! toolbar (or click **Edit** → **Mode Settings**).




- b. Select the number format and/or displayed digits.
- c. Click **OK**.

Using the Command Catalog to select functions

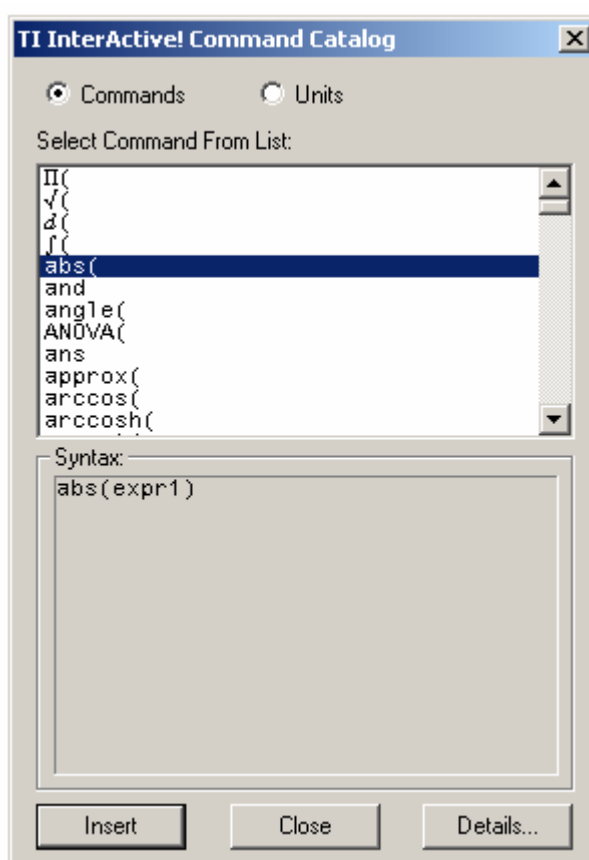
The Command Catalog is an alphabetical list of all TI InterActive! math functions and operators (commands). Although these commands are available in various menus and using buttons on the Math Palette, the Command Catalog lets you access them from one convenient list. It also gives you help information about the commands.

When you select a command its name or symbol is inserted at the entry cursor. You should position the cursor as necessary before selecting the command. Then:

- a. Click  in the upper-left corner of the Math Palette.

The TI InterActive! Command Catalog appears. Commands are listed in alphabetical order.

Symbols (+, %, $\sqrt{\quad}$, \sum etc.) are at the end of the list.



- b. Scroll through the list and select the appropriate command to highlight it.
- c. Click **Insert** to insert the abs() command and then close the Command Catalog. You can also double-click the command to insert it.
- d. The syntax for the highlighted command is shown below the list of commands. For more detailed information about the command and examples of how it is used, click Details.

Appendix B

About the authors

Tor Jan Aarstad

lives in Tau (Norway). He has been contact teacher for Texas Instruments in Norway for about ten years. He has higher education in studies in math, physics, chemistry and computer science, with a higher university degree in theoretical chemistry.

He teaches in Upper Secondary school and he is also the leader of a national group appointed by the Department of Education and Science.

The group's main task is to produce math examination questions for computer science for all levels in the grammar school.

He has written some articles, distributed to schools in Norway, to show how to solve problems in math with TI InterActive!.



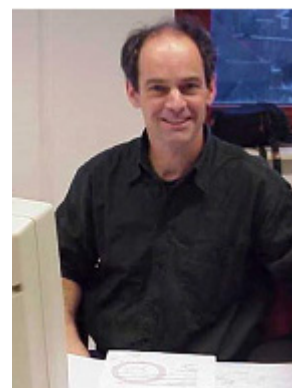
Paul Drijvers

is a researcher in mathematics education at the Freudenthal Institute (Utrecht University, the Netherlands).

He is involved in in-service teacher training; in particular, he is Program Director of the Dutch T3 project ('Teachers teaching with Technology'), hosted by the Algemeen Pedagogisch Studiecentrum APS. His main research interest is the integration of technologies such as graphing calculators and computer algebra systems in mathematics education at upper secondary level.

In 2003, Utrecht University awarded Paul a PhD degree in Mathematics Education for his theses entitled 'Learning algebra in a computer algebra environment' (www.fi.uu.nl/~pauld/dissertation).

Paul is assistant-editor of the International Journal of Computer Algebra in Mathematics Education.



Renée Gossez

lives in Brussels (Belgium). She has a mathematics degree from ULB (Brussels University) and taught mathematics in a secondary school in Brussels (upper level) until she retired in September 2003.

At Brussels University, Renée is still an active member of UREM (Unité de Recherche pour l'Enseignement des Mathématiques), a group involved in pedagogy of mathematics, under the direction of Professors Francis Buekenhout and Jacqueline Sengier.

She is also a T3 instructor for the French speaking part of Belgium.

Since September 2002, Renée has been the editor of a journal called SBPM-Infor, which aims to inform members of the Belgian association of mathematics teachers (SBPM) about ongoing activities such as conferences and in-service courses.



Adrian Oldknow

lives in London. He is emeritus professor of mathematics and computing education at University College Chichester and visiting fellow with the mathematical sciences group at the London University Institute of Education.

Retired from full-time teaching, Adrian now does a variety of free-lance work - which includes working with T-cubed in the UK and in Europe (including attending the 2001 ADT conference).

He chairs the professional development committee of the UK's Mathematical Association and is co-editor of the journal 'Teaching Mathematics and Its Applications' of the Institute of Mathematics and Applications (www.teamat.oupjournals.org).

Adrian has recently worked on revising the book 'Teaching Mathematics with ICT', co-authored with Ron Taylor and published by Continuum (ISBN 0-8264-4806-2) for a second edition, due December 2003.



Salvatore Regalbuto

lives in Syracuse (Italy). He has an Agrarian Science degree in Natural Science (Biology, Chemistry and Earth Science).

Salvatore teaches in Quintiliano Secondary school in Syracuse. He has an interest in Botany and Ecology and has co-operated with Catania University and local Authorities on studies on protected areas.

Concerning the new technologies he has worked in RTL applications to the Natural Science teaching. Some of his writings have been published on web sites and reviews.

As ADT (T³ Italy) instructor he acts as a teacher trainer.



Friedrich Tinhof

lives in Muellendorf (Austria). He has a maths and physics degree from Vienna University.

Since 1981 he has worked as a teacher at the Bundeshandelsakademie Eisenstadt (secondary college for business administration which is a commercial college with a focus on business studies) in Burgenland.

Friedrich teaches Mathematics, with a focus on Financial Maths, Economics and Statistics. Friedrich is co-author of mathematics and physics textbooks and is T³ coordinator for in-service teacher training courses on TI graphing calculators and use of the technology.

In 2003 he wrote a German textbook entitled "Einführung in TI InterActive!", which is published by bk-teachware.

