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# **Applications in the classroom**

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## Preface

The graphing calculator has changed and is still changing the way of teaching mathematics all over the world. For students, the TI-84 Plus is not only a tool to check their results but also a very useful assistant to help them approach a problem from different points of view: a numerical, graphical, statistical and/or geometrical approach. More than ever it makes it possible for students to investigate, to explore and to discover mathematical properties. Different approaches to a problem will improve the students' insight into the material, resulting in an increase of the students' motivation and commitment.

Graphing Calculator Software Applications (APPS) are pieces of software that can be downloaded onto the TI-83 Plus (Silver Edition) and the TI-84 Plus (Silver Edition) <sup>1</sup> as you would add software to a computer to enhance its capabilities. APPS do not only allow you to customize your calculator to meet your class needs, but also to upgrade it from one year to the next. The use of APPS increases the self activity of the students, makes the visualization of problems easier and creates a useful integration of technology during problem solving.

APPS offer new teaching and learning tools, not only for math but also for science, economics, and many other subject areas. The larger memory and the USB port of the TI-84 Plus and the TI-84 Plus Silver Edition turn these graphing calculators into an interdisciplinary IT platform. Combining these calculators with the Vernier Easy products transforms them into a data logger. Just connect the Easy products to the USB port and the data logging starts automatically.

With this book a T<sup>3</sup> Europe interest group, formed by Serge Etienne (FR), Koen Stulens (BE), Hildegard Urban-Woldron (AT) and Martin van Reeuwijk (NL) presents educational examples to show the benefits of integrating APPS in math (and science) education.

Koen Stulens

T<sup>3</sup> Flanders

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<sup>1</sup> Apps are also available for the TI-89 Titanium and the voyage™ 200.



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# 1 Introduction

*Applications in the Classroom* consists of three different parts:

In the first part, *Educational use of applications*, we focus on some educational examples in which we use applications to explore and solve problems. The applications used are shown as tools to discuss subject matter rather than talking about the functionality of the APPS used.

The second part, *Overview of applications*, is more or less a getting started guide of the APPS used in part one and as an extra some other interesting APPS which can be used to solve other problems than the examples of chapter two. The functionality of each application is explained by some educational examples. We classified the APPS into the following four categories:

<b>Tool</b> <i>New functionality</i>	<b>Reference</b>	<b>Mode</b> <i>New graphing mode</i>	<b>Test / Practice</b>
Cabri® Junior	Area Formulas	Inequality Graphing	Area Formulas
CellSheet™	Catalog Help	Transformation Graphing	Guess My Coefficients
EasyData™	Conic Graphing		
Finance	Science Tools		
Probability Simulation			
Science Tools			
StudyCards™			
Polynomial Root Finder and Simultaneous Equation Solver			

In the third part, *Additional information*, we briefly describe some companion software with the APPS of chapter two, where you can download the APPS and how you can install them on your calculator.



## 2 Educational use of applications

### 2.1 Growth processes

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#### TARGET GROUP

Upper secondary students.

#### TOPIC

Financial calculations (growth of amount of money) and exploring growth using various types of functions.

#### PRIOR MATHEMATICAL KNOWLEDGE

Basic knowledge of financial calculations, linear, quadratic, and exponential growth and of statistical calculations.

#### PRIOR CALCULATOR EXPERIENCE

Basic Graphing Calculator experience. Having used APPS before. Being familiar with Finance application, CellSheet™, statistical functions. When familiar with the application Transformation Graphing, this can be used to find the line of best fit as well.

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In this section we provide five problems dealing with growth: financial calculations about growth of money deposited at the bank, financial calculations to find out the ideal age to retire, the growth of body measurements, and the variations in distance cows walk away from the stable.

#### 2.1.1 The joy of credit

In this example we use a spreadsheet and the geometrical progression to investigate the mathematics of credit loans.

##### The problem

To buy a car, one borrows € 15,000 at the bank against an interest rate of 3.9% per year and an additional 0.3% for the insurance, adding up to a total of 4.2% per year. The total time period in which to pay back is set to 4 years, thus 48 months with monthly payments.

##### a. Calculating the monthly interest rate

The monthly interest rate equals  $\sqrt[12]{1.042} = (1.042)^{\frac{1}{12}} = 1.003434\dots$ . This can be calculated directly with the calculator or by using the Equation Solver command.

<pre>1.042^(1/12) 1.003434379</pre>	<pre>NUM CPX PRB 4: J( 5: *J 6: fMin( 7: fMax( 8: nDeriv( 9: fnInt( 0: Solver...</pre>	<pre>EQUATION SOLVER eqn: 0=X^12-1.042</pre>	<pre>X^12-1.042=0 ▪ X=1.00343437929 ▪ bound=(-1e99, 1... ▪ left-rt=0</pre>
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Direct calculation

Menu [MATH] <MATH>

Enter the equation.

[ALPHA] [ENTER] to solve

##### b. Using the geometrical progression

With the geometric progression, you can calculate the value of the monthly payment as follows:  $15,000 = m + m(1 - (1+i)^{-1}) + m(1 - (1+i)^{-2}) + \dots + m(1 - (1+i)^{-47})$ .

$$15,000 = m \left( \frac{1 - (1+i)^{-48}}{1 - (1+i)^{-1}} \right) \text{ or } m = \frac{15000 \times 0.003434}{1 - (1 + 0.003434)^{-48}} = 339.497\dots \text{ results in € 339.50.}$$

### c. Using the spreadsheet, Cellsheet application

One can add all the effective payments in order to find an estimated value for the constant monthly payment that one has to pay.

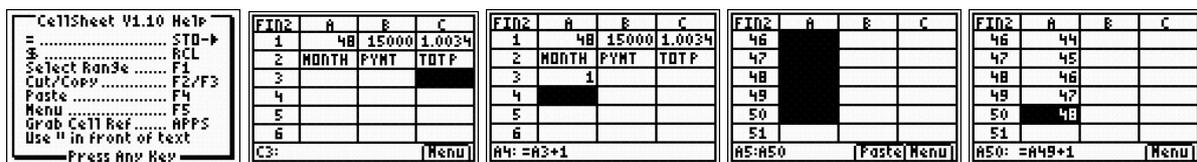
Start the application CellSheet. The start-up screen refreshes the most important features of this application.

Fill in cell A1: 48; and B1: 15000; C1: 1.0034 and leave D1 empty.

A2: MONTH; B2: PYMT; C2: TOT P; and for D2: REST.

Fill in A3: 1 and for A4: =A3+1

When the cursor is in A3 press F3 (COPY) to copy the contents of A3, then press the arrow down key to get to A4. Then press F1 (RANGE) and then the down arrow all the way to A50. Next press F4 (PASTE) to paste the formula from A3 in all these cells.



Do the same for: B3: =C1\*B1/A1; C3: =B3; D3: = (B1-C3) \*C1 and  
 B4: =D3/ (48-A3); C4: =B4+C3; D4: = (D3-B4) \*C\$1.

Copy these formulas all the way to row 50.

FIN2	B	C	D
1	15000	1.0034	
2	PYMT	TOT P	REST
3	313.57	313.57	14737
4			
5			
6			

FIN2	B	C	D
1	15000	1.0034	
2	PYMT	TOT P	REST
3	313.57	313.57	14737
4	313.55	627.12	14447
5	314.63	941.75	
6		941.75	0

FIN2	B	C	D
45	360.87	14454	1810.5
46	362.11	14816	1453.4
47	363.35	15179	1093.8
48	364.6	15544	731.7
49	365.85	15910	367.11
50	367.11	16277	0

D50 shows there is a remaining balance of 0 to pay.

The total amount that is paid (C50) equals €16.277.

The constant monthly payment (the average of all payments) equals  $16,277/48=339.10$

This value is quite close to the € 339.50 that was found previously.

The rounding causes the small differences.

### d. Using the Finance application

With the Finance application, one can perform the calculations quite rapidly.

Fill in the values for N, I%, PV, and P/Y.

Next place the cursor on PMT and start the solver: [ALPHA] [ENTER].

The TVM-solver returns PMT=339.50 for the monthly payment

N=48.00
I%=4.20
PV=15000.00
PMT=-339.50
FV=0.00
P/Y=12.00
C/Y=1.00
PMT: <input type="checkbox"/> END <input checked="" type="checkbox"/> BEGIN

Fill in N, PV, PMT, FV and P/Y.

Next place the cursor on I% and start the solver: [ALPHA] [ENTER]. The TVM-solver returns 0.3434...% for the interest rate per month.

N=12
I%=.343437929
PV=-100
PMT=0
FV=104.2
P/Y=1
C/Y=1
PMT: <input type="checkbox"/> END <input checked="" type="checkbox"/> BEGIN

Please note that when the Finance application is used, it is recommended to set the calculator in the numerical mode with 2 decimals.

**e. A few more calculation**

A loan of € 8000 is paid back in 2 years in monthly payments of € 368.42.

What is the yearly interest rate?

9.80 % per year is pretty high.

24→N:8000→PV:-368.42→PMT:0→FV:12	
→P/Y	12.00
tvm_I%	9.80

When an amount of €15000 is put in the bank against an interest rate of 2.85 % per year.

What will be the value of this amount after 8 years? You will gain less than €3800 in 8 years.

How much money do you need to put in the bank today, against an interest rate of 3.75% per year, to get €5000 in 10 years? The answer is €3460.10.

8→N:2.85→I%:-15000→PV:0→PMT:1→P/Y	
Y	1.00
tvm_FV	18781.30

10→N:3.75→I%:0→PMT:5000→FV:1→P/Y	
MT	1.00
tvm_PV	-3460.10

**f. The following is problem that you may encounter yourself**

A nice motorcycle is advertised for €2005. With only €205 cash at hand, this seems impossible. However the dealer is very helpful and suggests to finance the motorcycle. He has good connections with a reliable bank. The bank will give a permanent credit for an amount of €3000. This means that after registering at the bank, you can spend up to €3000. You only have to pay back a small amount per month as long as needed.

In small print is written that you pay 17.40% interest a year, or 1.45% interest per month. You decide to take a loan of €1800 from the credit account and pay back €15 per month.

The question is: How many months do you need to pay back the €1800?

There are various ways to calculate the number of months.

(i) Simple!

€1800 to be paid back in amounts of €15 per month leads to  $1800/15 = 120$  months, and that equals 10 years.

(ii) Forgot the interest.

The interest is 17.40%, so the total to be paid back equals  $1.174 \times 1800 = €2113.20$ . That leads to  $2113.20 / 15 = 140.88$  months, equals 11.74 years, and that is about 11 years and 9 months.

(iii) But interest is to be paid over the remaining credit amount.

This gets complicated. Each year an amount of 12 times €15 is paid back. So, after the first year left to be paid back is  $1800 - 180 = €1620$ . So for the second year 17.4% interest is to be paid over €1620. That equals €281.88. That is more than what was paid back the first year! How is that possible?

Which of the three calculations is best for this situation?

Before we elaborate on the third situation, let's investigate if 17.40% interest a year is the same as 1.45% interest per month for 12 months.

Proportional method:

N=1.00
I%=17.40
PV=100.00
PMT=0.00
FV=0.00
P/Y=12.00
C/Y=12.00
PMT: <input type="checkbox"/> BEGIN

N=1.00
I%=17.40
PV=100.00
PMT=-101.45
FV=0.00
P/Y=12.00
C/Y=12.00
PMT: <input type="checkbox"/> BEGIN

Equivalent method

N=1.00
I%=17.40
PV=100.00
PMT=0.00
FV=0.00
P/Y=12.00
C/Y=1.00
PMT: <input type="checkbox"/> BEGIN

N=1.00
I%=17.40
PV=100.00
PMT=-101.35
FV=0.00
P/Y=12.00
C/Y=1.00
PMT: <input type="checkbox"/> BEGIN

Let's start with a loan of €100 and a period of paying back of 1 year. The yearly interest is 17.40%. Adjusting the numbers for the payment, and payment/year and the compounding periods per year, you will notice that there is a difference when the compounding periods per year are set to 1 or 12.

Which one is correct 1.45% per month or 1.35% per month?

### An explanation

12 times 1.45 equals 17.40%, but actually you pay 1.45% per month over the remaining credit amount. So, that is over a year  $(1.45)^{12} = 18.86\%$ .

So, actually you should pay per month 1.35% so over a year you pay 17.40% because  $(1.35)^{12} = 17.40\%$ .

Now we go back to the problem of the motorcycle. With CellSheet, we can create a table to see what happens with the amount of €1800 (bank) when we pay back €15.00 each month and use a monthly interest of 1.45%.

In column A we list the amount that is on the bank at the beginning of each month. And column B contains the amounts at the end of each month.

(i)  $A1=1800$  and  $A2=B1$ .

Copy A2 from A3 to A6.

(ii)  $B1=(A1-15) * 1.0145$

Copy B1 from B2 to B6

MOE	A	B	C
1	1800		
2			
3			
4			
5			
6			

A2:=B1

MOE	A	B	C
1	1800		
2		0	
3			
4			
5			
6			

A3:A6 [Paste] [Menu]

MOE	A	B	C
1	1800		
2	0		
3	0		
4	0		
5	0		
6	0		

B1:=(A1-15)\*1.0145

MOE	A	B	C
1	1800	1810.9	
2	1810.9	1821.9	
3	1821.9	1833.1	
4	1833.1	1844.5	
5	1844.5	1856	
6	1856	1867.7	

B6:=(A6-15)\*1.0145 [Menu]

Use Finance or Cellsheet to answer the following questions:

- What is the minimal amount to pay back to the bank each month?
- How much time (number of months) is needed to pay back the complete loan of €1800 when the monthly payment is €75.00?

## 2.1.2 Financial calculations, what age to retire?

We start with the calculation of constant annuities at different expiry terms using the geometrical progression, and a fixed interest per period.

Let  $a_1 = a$  and  $a_2 = aq$  (inflation) where  $q = 1 + i$  ( $i$  being the interest) and  $n$  the number of periods.

Period	0	1	2	3	...	$n-1$	$n$
Value for period	0	$a_1$	$a_2$	$a_3$	...	$a_{n-1}$	$a_n$
Value for period	0	$a$	$aq$	$aq^2$	...	$aq^{n-2}$	$aq^{n-1}$

The total sum (sum of the terms of a geometrical progression, where  $q \neq 1$  ... otherwise the progression would not be geometrical!) is:

$$V_n = a + a.q + a.q^2 + \dots + a.q^{n-1} = a(1 + q + q^2 + \dots + q^{n-1}) = a \frac{1 - q^n}{1 - q}$$

$$V_n = a \frac{1 - (1+i)^n}{1 - (1+i)} = a \frac{1 - (1+i)^n}{-i} = a \frac{(1+i)^n - 1}{i}$$

Since  $V_n = V_0(1+i)^n$  substitution in the previous formula gives:

$$V_0 = a \frac{(1+i)^n - 1}{i \times (1+i)^n} = a \frac{1 - (1+i)^{-n}}{i}$$

Therefore:

- $V_0 = a \times \frac{1 - (1+i)^{-n}}{i}$  where  $V_0$  calculated as a function of a **future value** for which depreciation of money is taken into account (decline in the value of money).
- $V_n = a \times \frac{(1+i)^n - 1}{i}$  where  $V_n$  is the **present value** of the capital (or the total final value).

This relationship is translated on the calculator as follows: FV for  $V_n$ , PV for  $V_0$ , PMT for an annuity (or  $m$  monthly payment), I% for  $100 \times i$  and N for  $n$  the number of payment periods.

### a. The problem situation

In a given country, the (theoretical!) age of retirement is 60 years old. There is a certain disincentive against actually taking retirement at 60, because the sum paid to the retiree is only complete (100%) if (s)he enters retirement at 65. If a person retires from work at 64 years old, the amount paid is cut by 4% (also for the payments after 65 years). Similarly, for retirement at 63 the reduction is 8%, and 12% for retirement at 62, 16% at 61 and 20% for retirement at the legal age of 60...

We assume inflation at 2.5% per year (decline in the value of money) and life expectancy of 20 years after the age of 60.

We also assume that the payments made annually by the pension fund at the due terms are appreciated (i.e. increased in value, indexed to increases in wages) by 1.5% (i.e. less than inflation).

### b. Understanding the problem

- We are comparing the total amount paid by the pension fund to the same person retiring at 60, 61, 62, 63, 64 or 65 years old.
- We do not take into account the amount received by the person continuing to work beyond the age of 60 up to the date of retirement.
- We calculate the "final" value, i.e. at age 80, of the total sum received. It is therefore the calculation of an annuity as a function of a future value.

The first annual payment is therefore made at the end of the 60<sup>th</sup> year for a person retiring on reaching 60 years old, and the last payment is made just before the person's 80<sup>th</sup> birthday (life expectancy...). This person may also of course continue to live and receive his/her pension after 80, remaining blithely ignorant of statistical averages ...

### c. Calculations by hand

*EXCLUDING APPRECIATION* the following formula would be directly applicable (calculation of  $V_0$  because future value):

$$V_{60} = V_{60} = (1 - 0.20) \times a \times \frac{1 - (1.025)^{-20}}{0.025} = 0.80 \times a \times \frac{1 - (1.025)^{-20}}{0.025} \approx 12.47a.$$

Similarly, for the other years, (taking into account annual inflationary distortion):

$$V_{61} = 0.84 \times a \times \frac{1 - (1.025)^{-19}}{0.025} \times (1.025)^{-1} \approx 12.28a \quad V_{62} = 0.88 \times a \times \frac{1 - (1.025)^{-18}}{0.025} \times (1.025)^{-2} \approx 12.02a$$

$$V_{63} = .92 \times a \times \frac{1 - (1.025)^{-17}}{0.025} \times (1.025)^{-3} \approx 11.71a \quad V_{64} = 0.96 \times a \times \frac{1 - (1.025)^{-16}}{0.025} \times (1.025)^{-4} \approx 11.35a$$

$$V_{65} = a \times \frac{1 - (1.025)^{-15}}{0.025} \times (1.025)^{-5} \approx 10.94a$$

Since the values are decreasing, it would be better to retire at 60.

Use of the Finance application:

<pre> APPENDIX D 1: Finance... 2: ALG1CH5 3: ALG1PRT1 4: CSheetFr 5: CabriJr 6: CelSheet 7: Conics </pre>	<pre> NAME VARS 1: TVM Solver... 2: tvn_Pmt 3: tvn_IX 4: tvn_PV 5: tvn_N 6: tvn_FV 7: invPv( </pre>	<pre> tvm_PV(20,2.5,-1 ,0,1,1)*.8 12.47132983 tvm_PV(19,2.5,-1 ,0,1,1)*.84*1.02 5^-1 12.27538412 </pre>	<pre> ,0,1,1)*.96*1.02 5^-4 11.35410055 tvm_PV(15,2.5,-1 ,0,1,1)*1.025^-5 10.94333379 </pre>
---	---	---	--

The same values are obtained, subject to rounding. The conclusion is identical.

*INCLUDING APPRECIATION* (for information purposes, to understand financial calculations), we get the following situation:

Let  $a_1 = a$  and  $a_2 = aqp$  (inflation) where  $q = 1 + i$  ( $i$  being the interest),  $n$  the number of periods and  $p = 1 + k$  (appreciation).

Period	0	1	2	3	...	$n-1$	$n$
Value for period	0	$a_1$	$a_2$	$a_3$	...	$a_{n-1}$	$a_n$
Value for period	0	$ap^{n-1}$	$aqp^{n-2}$	$aq^2 p^{n-3}$	...	$aq^{n-2} p$	$aq^{n-1}$

$V_n$  is a geometrical progression with a ratio of  $\frac{q}{p}$ , and scale factor  $p^{n-1}$ . Therefore:

$$V_n = a \frac{1 - \frac{q^n}{p^n}}{1 - \frac{q}{p}} = a \frac{p^n - q^n}{p^n} \times \frac{p}{p - q} = ap^{n-1} \times \frac{p^n - q^n}{p - q} = ap^{n-1} \times \frac{q^n - p^n}{q - p}$$

Since  $V_n = V_0(1+i)^n$  we have  $V_0 = a \times (1+i)^{-n} \times \frac{(1+i)^n - (1+k)^n}{i - k}$ .

We now simply have to apply this formula, taking into account changes in the number of years:

$$V_{60} = 0.80 \times a \times 1.025^{-20} \times \frac{(1.025)^{20} - (1.015)^{20}}{1.025 - 1.015} \approx 14.24a$$

$$V_{61} = 0.84 \times a \times 1.025^{-20} \times \frac{(1.025)^{19} - (1.015)^{19}}{0.01} \approx 13.26a$$

⋮

$$V_{65} = a \times 1.025^{-20} \times \frac{(1.025)^{15} - (1.015)^{15}}{0.01} \approx 9.67a.$$

The values start to decrease, so the best choice is to stop working and retire at the age of 60 years.

It is not possible to use the Finance application in this situation. Here, two interest rates are mixed and going in opposite directions. The Finance application uses only one rate, which is in general sufficient.

### 2.1.3 Body measurements

Aurelia's father – a mathematics teacher – measured various body measurements since his daughter's birth. On her tenth birthday, Aurelia asked her father if it would be possible to predict the size of her waist for when she will be 11 years old and on her twelfth birthday.

The measurements of previous years are in the following table:

years	1	2	3	4	5	6	7	8	9	10
waist (cm)	76	87	96	104	110	117	123	129	134	140

With the TI-84 Plus you can plot scatter plots and find the line that best fits the series of points. Deciding on the visual representation which line is best is quite subjective. Therefore, we will use the correlation coefficient as a measurement of fit.

We will enter the data as follows in the lists L1 and L2:

The image shows three screenshots from a TI-84 Plus calculator. The first screenshot shows the 'MATH' menu with options 1:SortA(), 2:SortD(), 3:dim(), 4:Fill(), 5:seq(), 6:cumSum(), and 7:List(). The second screenshot shows the command 'seq(I,I,1,10)→L1' resulting in the list {1 2 3 4 5 6 7 ...}. The third screenshot shows the 'CALC TESTS' menu with options 1>Edit..., 2:SortA(), 3:SortD(), 4:ClrList, and 5:SetUpEditor. To the right, a table shows data entry into lists L1 and L2:

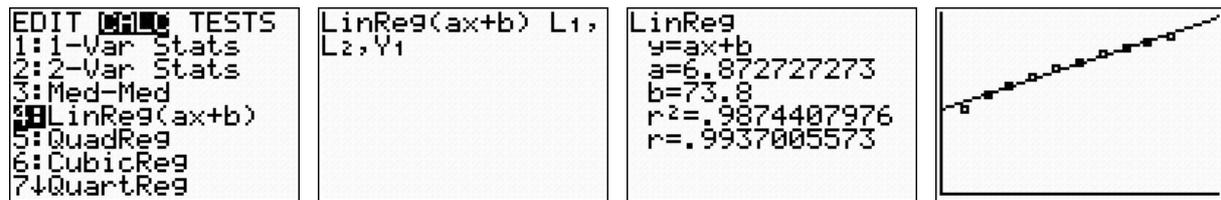
	L1	L2	L3	2
5		110		
6		117		
7		123		
8		129		
9		134		
10		140		
-----				
L2(11) =				

And we become the following scatter plot:

The image shows three screenshots from a TI-84 Plus calculator. The first screenshot shows the 'STAT PLOTS' menu with options 1:Plot1...Off, 2:Plot2...Off, 3:Plot3...Off, and 4:PlotsOff. The second screenshot shows the 'Plot2' settings: On Off, Type: [Scatter], Xlist:L1, Ylist:L2, and Mark: [Square]. The third screenshot shows the 'WINDOW' settings: Xmin=0, Xmax=12, Xscl=0, Ymin=0, Ymax=160, Yscl=0, and Xres=3. To the right, a scatter plot is shown with data points forming an upward curve.

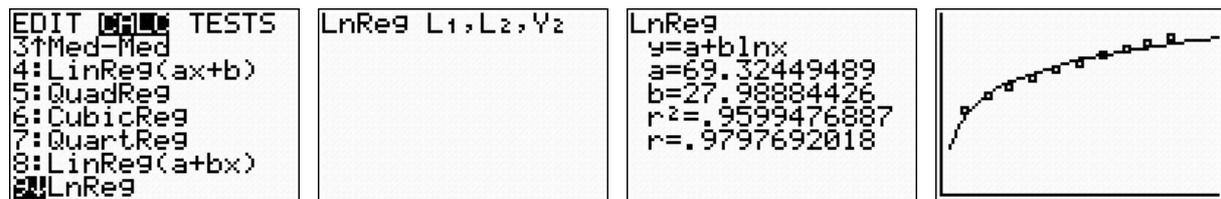
Now various kinds of formulas (functions) can be explored to see which one fits best the data. We will use the following kinds of regression: linear, logarithmic function, exponential and finally cubic regression.

## Linear regression

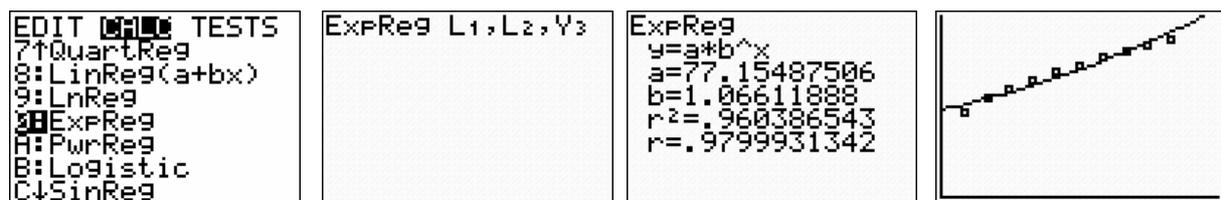


When the correlation coefficient is not displayed, you need run the DiagnosticOn command from the CATALOG.

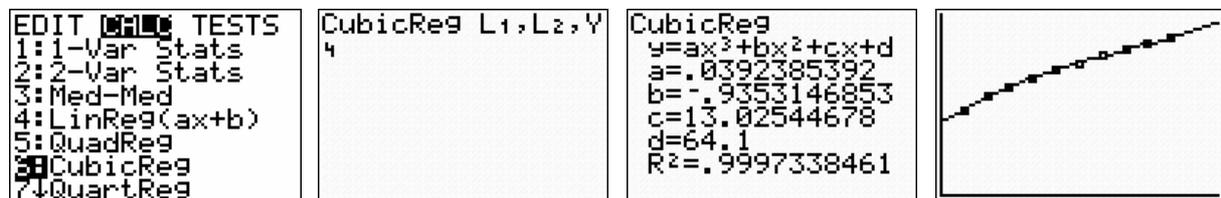
## Logarithmic regression



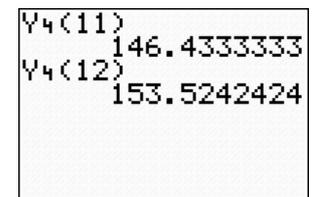
## Exponential regression



## Cubic Regression



This cubic model is quite good. Therefore we use this model to answer our question and to predict Aurelia's size of her waist real when she will be 11 and 12 years old.



### 2.1.4 Moving cattle and logistics

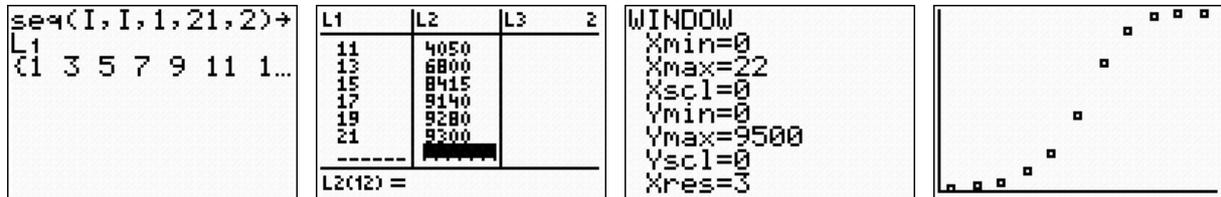
In the winter, cattle lives on the plains in the valleys and in the winter, cattle moves up in the mountains. Every morning the cows leave the stable and each night they return to be milked and to get some concentrates (extra food). Four cows from the herd, the leaders of the pack, have gotten a little clock and a GPS to determine their position. The other cows are used to follow (one of) the four leaders.

The average maximal distance between the stable and the four groups of cows is calculated each day. The table below shows these distances for every other day. From the table, you can state that after a certain number of days the maximum distance stays the same. Probably this is the distance that the cows can walk to be back in time for the milking and treats (the concentrates).

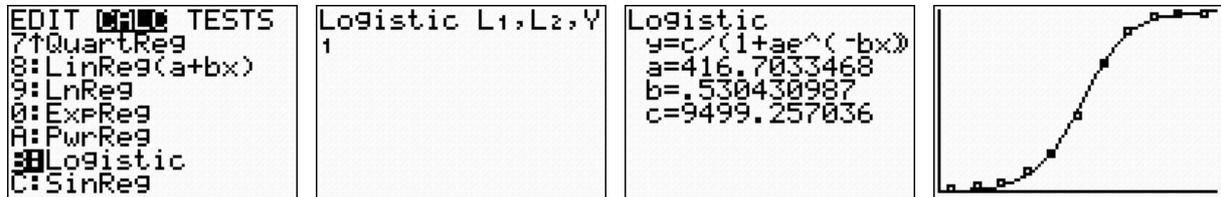
The task is to use the statistical application to investigate these data and find a curve that best fits the scatter plot that graphically represents the data.

Days	1	3	5	7	9	11	13	15	17	19	21
Distance (m)	140	270	520	1120	2015	4050	6800	8415	9140	9280	9300

We will enter the data in the lists L1 and L2 and make a scatter plot.



The scatter plot of the data consist of two parts: the first half shows an increasing growth and the second part a process of decreasing growth that almost stops growing. For such a process, the logistic function can be used to describe the growth.



Phenomena of increasing growth, followed by decreasing growth (like chemical processes, in business, animals of plants, ...) can often be described with a logistic function.



## 2.2 Functions from real experiments

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### TARGET GROUP

Students at upper secondary education, high school students of about 15 - 17 years old.

### TOPIC

Physics, limited growth, mechanics, energy conservation, different real functions

### PRIOR MATHEMATICAL KNOWLEDGE

Linear, quadratic, broken, power and exponential functions, growth and decay processes, use of tables and graphs to organize and display information, some experience with geometric transformations, sine functions

### PRIOR CALCULATOR EXPERIENCE

Basic Graphing Calculator experience, know how to start an APP and know how to use the function keys

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The Vernier EasyData™ application is a data collection application for the TI-83 Plus and TI-84 Plus (Silver Edition). EasyData supports a series of sensors and data collection systems (like CBR 2™, CBL 2™, Vernier's EasyTemp™, ...) and is easy to use.

If you use EasyData with calculators from the TI-83 Plus family for data collection, use the I/O port of the calculator. You can connect the CBR or CBR 2 directly; for other sensors you need a CBL 2.

The USB-port of calculators from the TI-84 plus family provides an additional option for data collection: You can connect EasyTemp temperature sensor directly to the USB port of the calculator which simplifies data collection substantially. For other sensor you need the EasyLink™ as an adaptor. When the EasyData application is installed, the application starts automatically and you collect data and analyze and process the data with your calculator.

The following five examples were used in the classroom and illustrate the didactical potential of the combination of simple data collection and advanced data processing with the help of the calculator. We will show how:

- concepts of physics are tested by visualization and interpretation of data and how
- mathematical models are developed to describe physical experiments.

The user learns to use EasyData from setting up and doing the experiment, through modeling, analyzing and visualizing the data. The example illustrates how the classroom situation can be modified with these new technologies. This technology can also be used for measurements outside the classroom; it is light weight, easy to take with you and can be used everywhere.

Students have a stronger relationship with data measured by themselves than with data presented in a textbook. This may lead to the situation, where all students want to participate in data collection and thus to improved students' participation. This might be challenging for the teacher who has typically only one set of instruments at his disposal. As data can be transferred between calculators, it is feasible to do parallel analysis or as another option the teacher can analyze the data together with the class by using the view-screen.

Compared to the traditional instruments used in the classroom, e.g. thermometer or stop watches, more data can be more precisely collected and the shape of the corresponding curves is obtained easier and faster. Thus, students need less time for data collection and have more time for analysis, investigation and interpretation of data.

Students can investigate variation and the effect of repeated measurements in the so-called what-would-be-if scenarios, which is an additional benefit. Students can analyze the data both algebraically and graphically and associate these relationships with mathematical functions. Finally they can use the data to find the best fit functions and discover the physical meaning of different coefficients and parameters.

## 2.2.1 The bouncing ball

### a. Introduction

The height of a bouncing ball is continuously measured with a distance measuring device (CBR 2) connected to the calculator (TI-84 Plus) and the data collected will be analyzed. The measured movement of the ball is described as a function of time and the gravity law is derived. With energy calculations, insights can be gained where energy is lost during bouncing.

In the classroom the following questions can be asked:

- What is the highest speed of the ball and when does it occur?
- What is the acceleration during falling?
- Which function describes the distance (height) of the ball?
- Is there a model to describe the height of the ball as a function of time?
- How can the total distance of the ball be determined?
- What processes determine the “bouncing back” of the ball from the floor?
- How does the rebound height decrease from one bounce to the next?
- Can you determine how high a ball will rebound on each bounce and make predictions about its motion?

### b. Didactic concepts and methodological hints

The ball is a freely falling and bouncing object where air friction is neglected. Therefore only gravity affects the ball's movements which show that acceleration is approximately constant. The time-distance graphs are parabolic functions, which can be described by the quadratic equation  $y = a(x-b)^2 + c$  where the highest point is described by the coordinates  $(b, c)$  with  $c$  as the maximum height and  $b$  as the corresponding time. The parameter  $a$  represents mathematically the shape of the parabola and depends physically on the degree of acceleration caused by gravity, which is constant during the experiment.

The curves obtained for the time-distance graphs of the individual bounces are first adjusted manually – by determining the parameters  $b$  and  $c$  and by varying parameter  $a$ .

After selecting an individual bounce with `2nd[LIST]<OPS> 8:Select (` and quadratic regression, the function describing the ball's movement is obtained with the help of regression analysis. With the help of the application Transformation Graphing you can do the curve fitting process too.

The maximum height decreases exponentially from bounce to bounce for each ball and its initial height. For  $y = hp^x$ ,  $y$  is the current height,  $h$  is the initial height,  $p$  is a constant depending on the properties of the ball and the floor and  $x$  is the number of the bounce.

For  $x = 0$ ,  $y = h$  (the initial height of the ball, from which it has been dropped). The coefficients of the equation describing the exponential function are determined from the data collected. The experiment can be repeated with different balls, heights and floor types.

In the time-velocity diagram the total distance for a certain time interval is represented by the area under the graph.

### c. Performing the experiment

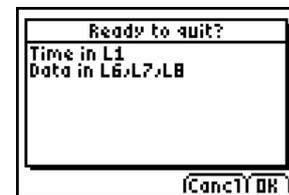
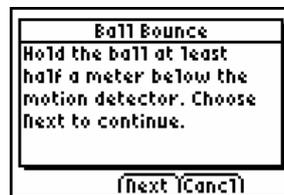
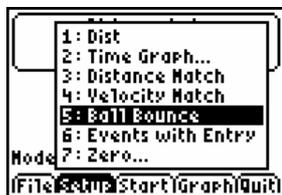
For the experiment, an inflatable ball with about a 25 cm diameter and about an 80 g mass is used.

The CBR 2 is held at about 160 cm above the floor and the ball is positioned roughly 50 cm below the CBR 2 and then dropped.

Students create height-time-plots for a bouncing ball and explain how the ball's height changes mathematically from one bounce to the next.



Data collection starts through the main screen of the application EasyData with the CBR 2 using the option Ball Bounce. The option Ball Bounce from the EasyData APP transforms the data into floor-bounce distances. The data are stored in lists L1 and L6 to L8, which are shown on the screen after quitting EasyData.



The following are data from a real experiment. We use these data to illustrate how Transforming Graphing can be used. In the example the ball was dropped at 0.473 s and we only used data up to four seconds. We have placed the distance (L6) in list L2 and the velocity (L7) in L3.

L1	L2	L3	3
.430	.978	0.000	
.473	.978	-.340	
.516	.948	-1.119	
.559	.881	-1.732	
.602	.799	-2.083	
.645	.702	-2.468	
.688	.587	-2.814	

L3(17) = -2.8108

L1	L2	L3	3
.731	.460	-3.101	
.774	.320	-3.425	
.817	.166	-3.721	
.860	0.000	-3.939	
.903	.085	-2.659	
.946	.229	-3.118	
.989	.353	-2.692	

L3(24) = 2.6943

L1	L2	L3	3
1.032	.460	2.303	
1.075	.551	1.922	
1.118	.626	1.532	
1.161	.683	1.146	
1.204	.724	.768	
1.247	.749	.391	
1.290	.758	.014	

L3(31) = .0143694

L1	L2	L3	3
1.333	.750	-.359	
1.376	.727	-.696	
1.419	.691	-1.097	
1.462	.633	-1.507	
1.505	.561	-1.856	
1.548	.473	-2.220	
1.591	.366	-2.610	

L3(38) = -2.6896

L1	L2	L3	3
1.634	.242	-3.044	
1.677	.104	-2.539	
1.720	.023	-.637	
1.763	.159	2.946	
1.806	.277	2.512	
1.849	.375	2.072	
1.892	.455	1.633	

L3(45) = 1.63295

L1	L2	L3	3
1.935	.515	1.216	
1.978	.560	.812	
2.021	.585	.426	
2.064	.596	.101	
2.107	.594	-.246	
2.150	.575	-.602	
2.193	.542	-.932	

L3(52) = -.93694

L1	L2	L3	3
2.236	.495	-1.259	
2.279	.434	-1.582	
2.322	.358	-1.949	
2.365	.266	-2.310	
2.408	.160	-2.509	
2.451	.020	-.085	
2.494	.167	-.602	

L3(59) = 2.6017

L1	L2	L3	3
2.537	.274	2.241	
2.580	.360	1.778	
2.623	.427	1.349	
2.666	.476	.940	
2.709	.508	.549	
2.752	.523	.168	
2.795	.522	-.213	

L3(66) = -.193138

L1	L2	L3	3
2.838	.506	-1.536	
2.881	.476	-.867	
2.924	.432	-1.221	
2.967	.371	-1.569	
3.010	.297	-1.918	
3.053	.206	-2.300	
3.096	.020	-2.614	

L3(73) = -1.14292

L1	L2	L3	3
3.139	.108	1.237	
3.182	.206	2.065	
3.225	.285	1.657	
3.268	.348	1.275	
3.311	.395	.915	
3.354	.427	.557	
3.397	.443	.198	

L3(80) = .193126

L1	L2	L3	3
3.440	.443	-.172	
3.483	.428	-.530	
3.526	.398	-.892	
3.569	.351	-1.250	
3.612	.290	-1.612	
3.655	.213	-2.011	
3.698	.117	-2.414	

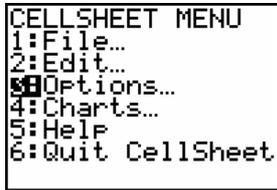
L3(87) = -1.11723

L1	L2	L3	3
3.741	0.000	1.055	
3.784	.208	1.891	
3.827	.279	1.430	
3.870	.331	1.031	
3.913	.368	.675	
3.956	.389	.305	
3.999	.394	-.074	

L3(94) = .060686

With CellSheet™ we can also manipulate the data. In the following example we imported the Lists L1 and L2 in CellSheet columns A and B.

The CBR 2 was at a height of 1.60 m and in column C we computed the distance of the ball to the CBR 2 sensor.



S01	A	B	C
1	.43	.978	
2	.473	.978	
3	.516	.948	
4	.559	.881	
5	.602	.799	
6	.645	.702	

S01	A	B	C
1	.43	.978	.622
2	.473	.978	.622
3	.516	.948	.652
4	.559	.881	.719
5	.602	.799	.801
6	.645	.702	.898

The formula  $C1 := 1.6 - B1$  is copied to the whole range C2:C84 (see 3.4).

In column D we computed the velocity of the ball using the formula

$$D2 := (B2 - B1) / (A2 - A1)$$

S01	B	C	D
1	.978	.622	0
2	.978	.622	0
3	.948	.652	-.6977
4	.881	.719	-1.558
5	.799	.801	-1.907
6	.702	.898	-2.256

S01	C	D	E
1	.622	0	0
2	.622	0	-.34
3	.652	-.6977	-1.119
4	.719	-1.558	-1.732
5	.801	-1.907	-2.083
6	.898	-2.256	-2.469

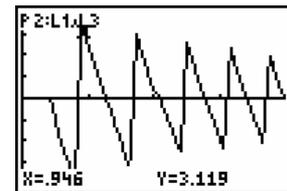
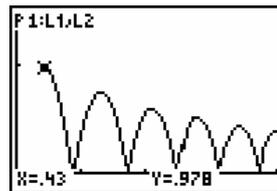
In column E you can see the velocity data computed by the CBR 2. Comparing columns D and E students realize that CBR 2 uses a different algorithm for computing velocity.

#### d. Finding the mathematical model

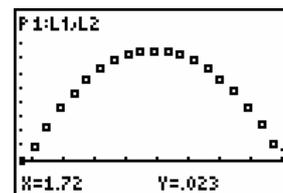
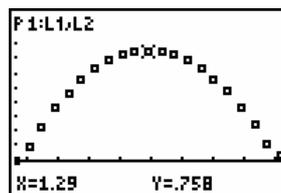
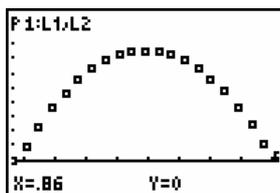
Looking at the time-distance diagram of the ball, you see that the ball first falls and then is reflected by the floor. Next it moves up, slowed down by gravity until it falls down again. This movement corresponds to repeated vertical throws. Therefore both phases of movement, i.e. up and down, can be described by quadratic functions. For this, the data for a complete bounce have to be selected from the total set of data. From this section of the graph (one bounce) the parameters for the ball's movement can be obtained.

The values for height and velocity as a function of time are explored from several points of view.

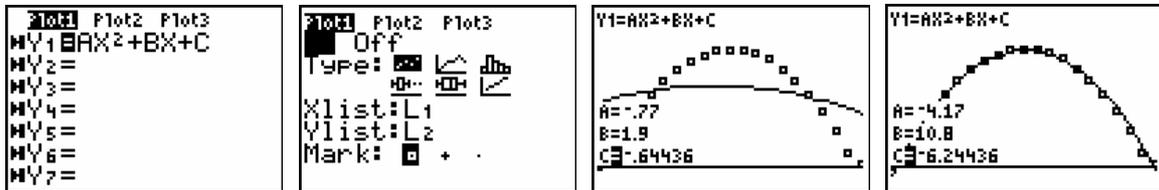
The two figures beside show distance and velocity as functions of time.



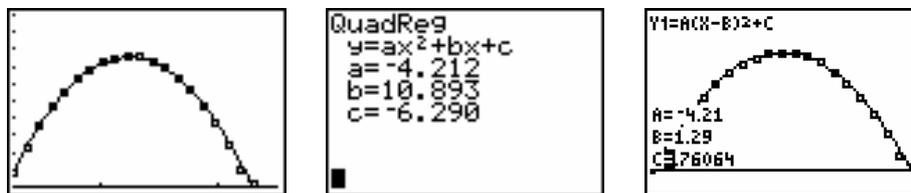
Optionally, a time-distance diagram or a time-velocity can be shown on the screen. When looking at the diagram – using trace – each point can be marked with the cursor and its coordinates can be seen on the bottom of the screen. This way the movement of the ball can be discussed with the class. The figures on the right show the time-distance diagram for the first complete bounce at different time values.



This looks like a parabola. With Transformation Graphing we can look for values of the parameters  $a$ ,  $b$  and  $c$  in the formula  $y = a(x-b)^2 + c$ . In this formula you can easily see the maximum value and the corresponding time. Therefore we already know  $b$  and  $c$  (we can read them from the graph) and we store them to the calculator. For investigation the  $Y=$  screen is used.  $L_1$  and  $L_2$  are graphed as a scatter plot. With the cursor keys the value of  $a$  can be adjusted.



Another way to find the parameters is to do regression on the data in  $L_1$  and  $L_2$ .



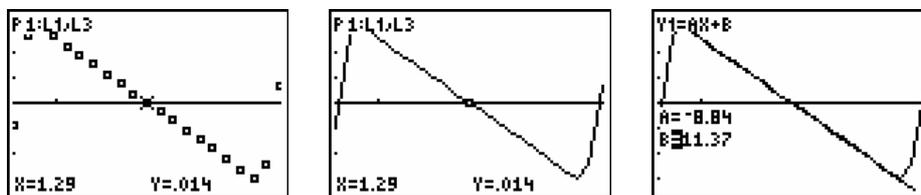
We can use Transformation Graphing again to find the values  $a$ ,  $b$  and  $c$  of the parameters in  $y = ax^2 + bx + c$  it is easier to  $y = a(x-b)^2 + c$ , because then we can read  $b$  and  $c$  from the graph.

### e. Exploring velocity

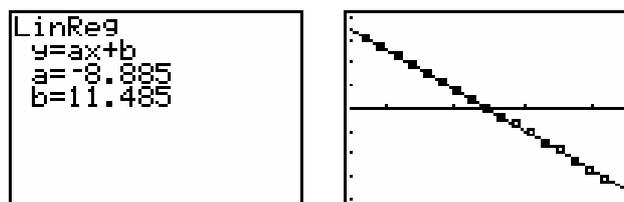
We stored the velocity (speed) in list  $L_3$  as computed by the CBR 2. Graphing the velocity as a function of time ( $L_1$ ), gives a straight line. The diagrams below show the velocity-time graph in the interval of  $x = 0.86$  to  $x = 1.72$ .

With Transformation Graphing we can look for values of the parameters  $a$  and  $b$  in the formula  $y = ax + b$ . For investigation the  $Y=$  screen is used.

$L_1$  and  $L_3$  are graphed as a scatter plot. With the cursor keys the values of  $a$  and  $b$  can be adjusted.



With regression for the time interval of  $x = 0.86$  to  $x = 1.72$  we get the linear function shown in the window below



In this way we get a constant acceleration of  $8.89 \text{ m/s}^2$ . This is significantly below the terrestrial gravity constant of  $9.81 \text{ m/s}^2$ .

It can be seen that the velocity,  $v$ , is  $0 \text{ m/s}$  at the start, decreasing to  $-3.7 \text{ m/s}$  just before touching the floor. Then  $v$  rapidly increases to  $0$  and further up to  $3.1$  which is, comparing absolute values, lower than the  $v$  just before the rebound. With each next bounce energy is lost and finally the ball stops.

#### f. A few further questions

- Why is the experimentally obtained acceleration (calculated by the CBR 2) significantly smaller than  $g$ ?
- Why can air friction not be the cause for a reduction of acceleration?
- Which additional force works against gravity when the ball is falling?
- What is the time course of potential and kinetic energy?
- What can be said about total energy?

When interpreting the time-distance graph, students will recognize the physical concept of movement caused by constant acceleration, the unavoidable transformation of kinetic energy in friction energy and also the mathematical representatives of individual graphs and their subsections. Each individual bounce is described by a convex parabola; its parameters are determined by the experiment and are interpreted physically. Students build mathematical models.

## 2.2.2 Boyle's Law for Gas Pressure

### a. Introduction

When a gas inside a closed container is compressed, its pressure and volume usually change. As the force exerted on the gas increases, the pressure increases while its volume decreases. Two quantities that change in this sort of way are said to vary inversely (inversely proportional). Even so both quantities may change, their product always stays the same.

If we suppose that  $x$  and  $y$  represent the quantities that are inversely related, then  $xy = c$ , where  $c$  represents a positive constant.

In this experiment students will explore the theory that pressure and volume vary inversely and will conclude with a formula that describes the special experiment and will investigate some questions like these:

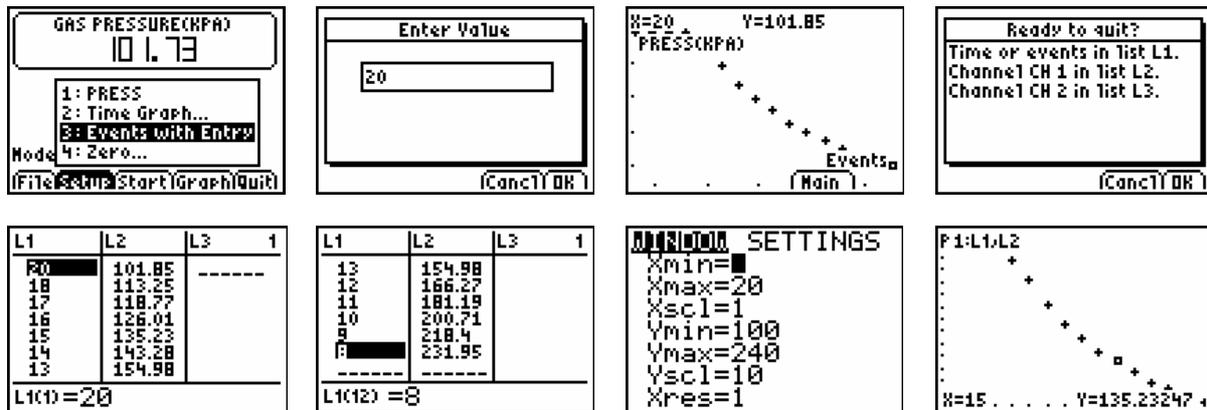
- Could the volume ever be zero cc?
- Why or why not?
- What would be the corresponding pressure?

With the EasyLink adaptor it is very easy to connect sensors directly to the TI-84 Plus and perform real experiments. For example with the Gas Pressure Sensor you can investigate the relationship between volume and pressure of an amount of air in a syringe. The range for the Vernier pressure sensor is  $0$  to  $210 \text{ kPa}$ .



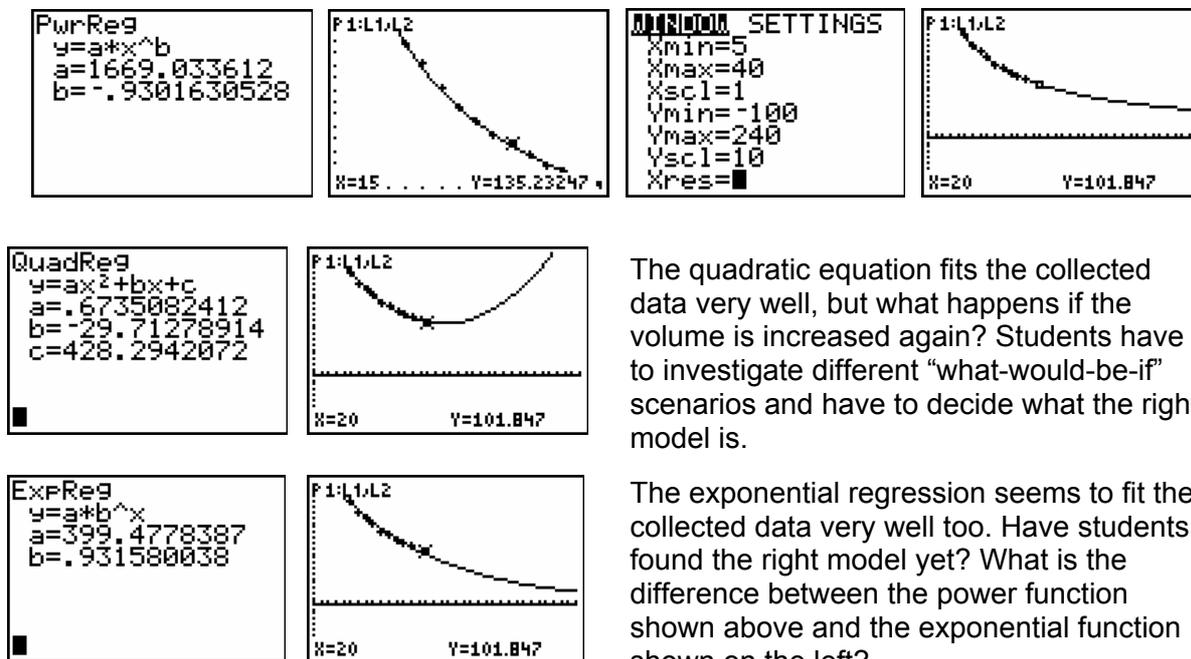
## b. Performing and analyzing the experiments

A volume of 20 ml is connected to the sensor. There is no extra pressure put on the air yet.



The starting values are volume 20 cc and pressure 101.73 kPa. Then the syringe is pressed, and for each of the following volumes, the pressure is measured and stored as events: 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8.

Now we are going to investigate what kind of relationship exists between volume and pressure. From the graph there are several possibilities: a quadratic function (a parabolic graph), an exponential function, a power function or a hyperbolic function (that is a special power function with exponent -1). With regression the best fit function can be found. Below are the results for power, quadratic and exponential regressions with their corresponding graphs.

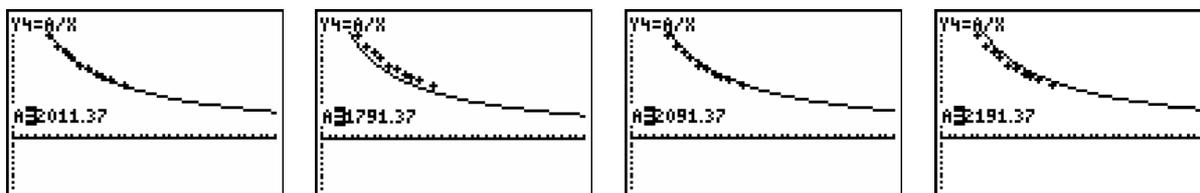


The quadratic equation fits the collected data very well, but what happens if the volume is increased again? Students have to investigate different “what-would-be-if” scenarios and have to decide what the right model is.

The exponential regression seems to fit the collected data very well too. Have students found the right model yet? What is the difference between the power function shown above and the exponential function shown on the left?

In the 17<sup>th</sup> century Boyle discovered that the relation between gas pressure,  $p$ , and volume,  $V$ , is  $pV = \text{Constant}$ . Based on our measurements we conclude that the constant in our example is about 2000 (for 20 ml the pressure was almost 102 kPa).

With Transformation Graphing we can easily explore the effects of different values of the constant on the graph and we conclude that 2011 fits best. See the screen shots below where the formula  $y = A/X$  is explored and the value for  $A$  is adjusted.



## 2.2.3 Newton's law of cooling

### a. Introduction

Students know from everyday life that hot tea cools to room temperature after some time. What determines the cooling process? Does the temperature decrease with a constant rate as shown in Figure 1 or is the decrease faster at the beginning and slower towards the end of cooling to room temperature as in Figure 2? Or is it slow in the beginning and fast towards the end as sketched in Figure 3?

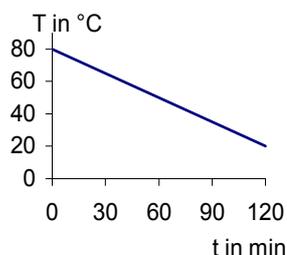


Figure 1

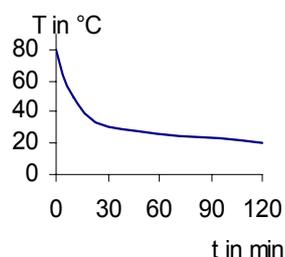


Figure 2

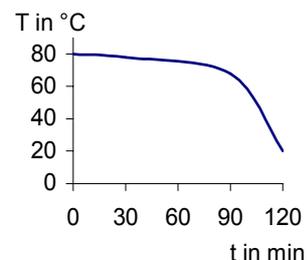
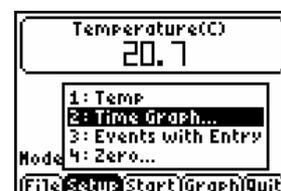


Figure 3

In this experiment students examine the cooling of hot water with the goal to create a model that describes the process.

They can also predict the time it takes for the hot water to cool to room temperature of 20.7°C (see the figure on the right).



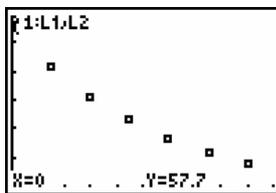
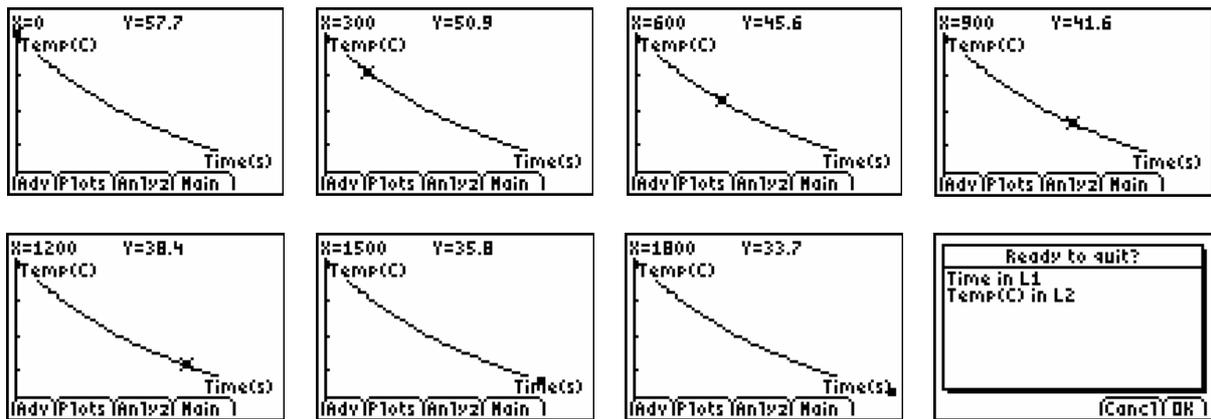
Isaac Newton modeled the cooling process assuming that the rate at which thermal energy moved from one body to another is proportional (by a constant  $k$ ) to the difference in temperature between the two bodies,  $T_{\text{diff}}$ . From this simple assumption he showed that the temperature change is exponential in time and can be predicted by  $T_{\text{diff}} = T_0 e^{-kt}$ , where  $T_0$  is the initial temperature difference.

### b. Performing the experiment

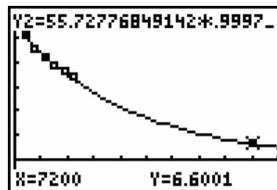
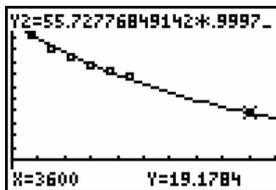
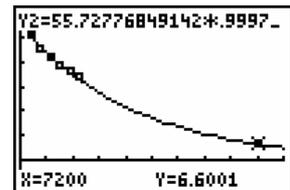
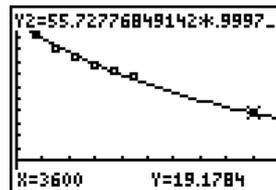
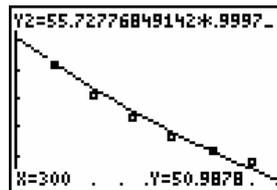
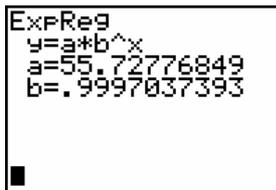
For this experiment we use a small quantity of hot water at a temperature of about 40°C above room temperature. EasyTemp can be connected directly to the USB port of the TI84 Plus. When EasyData is installed, it starts automatically and students can start collecting and analyzing data with their TI-84 Plus.



The graphical representation of the data collection in EasyData:



Now we are going to investigate what kind of relationship exists between time and temperature. The graph allows for various options: a quadratic function (a parabolic graph), an exponential function, a power function or a hyperbolic function (a power function with exponent -1). With regression the best fit function can be found. Below are the results for exponential regression with its corresponding graphs.

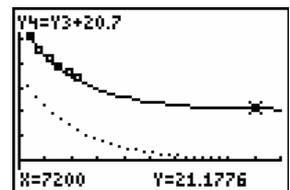
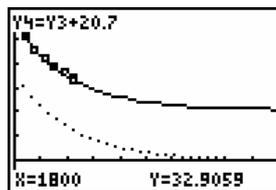
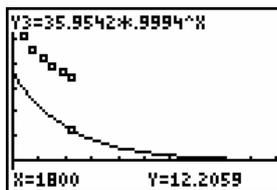
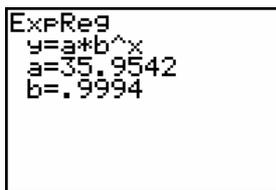


L1	L2	L3	3
0.0000	57.700	-----	
300.00	50.900		
600.00	45.600		
900.00	41.600		
1200.0	38.400		
1500.0	35.800		
1800.0	33.700		

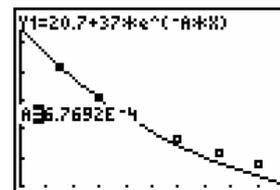
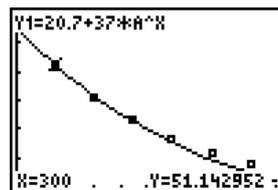
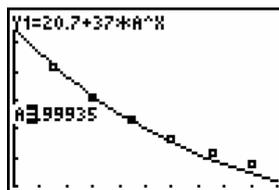
L3=L2-20.7

L1	L2	L3	3
0.0000	57.700	89.0000	
300.00	50.900	30.200	
600.00	45.600	24.900	
900.00	41.600	20.900	
1200.0	38.400	17.700	
1500.0	35.800	15.100	
1800.0	33.700	13.000	

L3(0)=37



And with Transformation Graphing.



### c. Questions in the classroom

In the classroom the following questions can be asked.

- The curve describes the decrease of temperature over time. Analyze the curve and describe in your own words how the temperature decreases. Which of the curves best fits your results?
- The data collected during the experiment are stored in lists  $L_1$  and  $L_2$ .  $L_1$  lists the time in seconds and  $L_2$  the corresponding water temperatures. Plot the data points in a scatter diagram!
- Can you find a regression function that fits your temperature curve?
- Use the regression functions on the calculator to find the best fit, and plot the regression curves together with the recorded data.
- The room temperature  $T_R$  was  $20.7^\circ\text{C}$ . What could be wrong with the regression function you selected? What kind of “growth” do we have here? What are the properties the “right” regression function should have?
- The cooling down of a solution to the temperature  $T$  is described by Newton’s law of cooling:  $T = T_R + T_0 e^{-kt}$  or  $T = T_R + T_0 \cdot a^t$  (time  $t$  in minutes).

$T$  temperature at time  $t$

$T_R$  basic temperature (room temperature)

$T_0$  difference in temperature between the liquid and the room temperature at  $t = 0$

$a$  constant, depending on the fluid’s properties

- Try to fit the temperature function above to the actually measured curve by adjusting the parameters  $k$  and  $a$ . What is the relationship between  $a$  and  $k$ ?
- Make a list  $L_3$  with the temperature differences of measured temperature with room temperature. Now you can do exponential regression analysis. Use the function obtained and determine the values for  $T_0$  and  $a$ ! Then use the room temperature  $T_R$  to find an exponential function describing the decreasing temperature curve.
- Do the values for  $T_R$ ,  $T_0$  and  $a$  make sense to you? Why must  $a$  be smaller than 1? Explain in detail your explanation to these questions.
- Draw the graph of this function obtained together with the curve of the measured data (use coordinates). What do you think? Is this new exponential function a good approximation?
- Given the mathematical approximation for the measured temperature curve, how much time would it take for the water to cool down to room temperature?
- Has the starting temperature of the hot water any impact on the value obtained for  $a$ ? Repeat the experiment with different starting temperatures to answer this question.
- What could you do to your experimental setting to decrease the value of  $k$  in another run? What quantity does  $k$  measure?
- If your starting temperature difference is cut in half, does it take half as long to get  $1^\circ\text{C}$  above room temperature? Why or why not does it take half as long?

- A small research project

A coffee drinker is faced with the following dilemma. She is not going to drink her coffee with cream for ten minutes, but wants it still to be as hot as possible. Is it better to add immediately the room-temperature cream, stir the coffee, and let it sit for ten minutes, or is better to let the coffee sit for ten minutes and then add and stir in the cream? Use an EasyTemp and a calculator to examine this dilemma. Explain your results in terms of the assumptions Newton made about cooling. What changes, if the coffee drinker also wants to add sugar?

## 2.2.4 The pendulum

### a. Introduction

If a pendulum (an object on a string) is pulled back and released, it will swing back and forth over time with a regular pattern occurring. It will eventually stop, but over a short time period the pendulum exhibits simple harmonic motion. This motion can be modeled with a periodic function.

In this activity students collect data from the motion of a pendulum. Then a periodic function is found that models the motion. Its parameters will be related to the time for one period, the distance it was pulled back and how far it is from the motion detector.

There are at least three things students could change in the pendulum that might effect the time for one complete cycle (called the period):

- the amplitude of the pendulum swing,
- the length of the pendulum, measured from the center of the pendulum to the point of support and
- the mass of the pendulum.

To investigate the pendulum students have to do a controlled experiment. They need to make measurements, changing only one variable at the time as a basic principle of scientific investigation. By conducting a series of controlled experiments with the pendulum, students can determine how each of these quantities effects the period, when they measure the period of a pendulum as a function of amplitude, as a function of length and as a function of mass.

The force driving the pendulum back to the equilibrium position is given by  $F = mg \sin \varphi$ .

For small angles we can use  $F = mg \frac{y}{l}$ , where  $y$  is the distance from the starting point to equilibrium position and  $l$  is the length of the string.

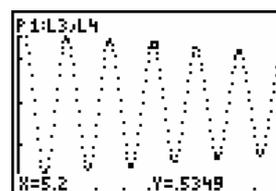
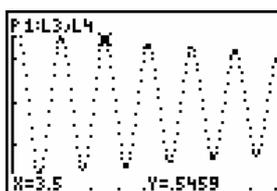
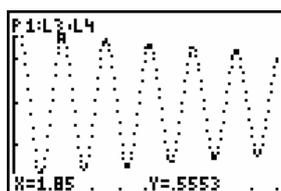
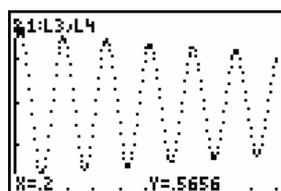
### b. Performing the experiment and collecting the data

In this experiment we will use a motion detector (CBR 2) to plot the position vs. time graph for a simple pendulum. Students will use their data to find a formula that describes the position vs. time graph.

A string is tied to a mass (in the experiment we use a ball – see figure on the right). By trying different masses on the string, students will explore if the period of the pendulum depends on the mass and/or on the length of the string or on the amplitude, too. The mass is held from about  $10^\circ$  from vertical and then released.



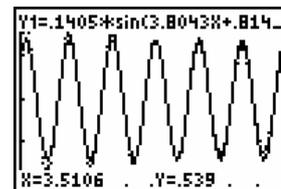
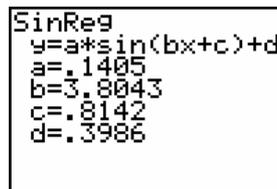
The CBR 2 is aimed at the pendulum mass. Then the mass is pulled back about 10 centimeters and released, so that it swings toward and away from the sensor. The graph appears to be a sine or a cosine graph.



### c. Looking at the results

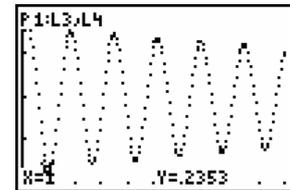
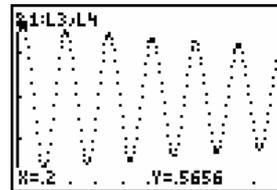
A pendulum completes a cycle as it moves from one extreme position to the other and back again. The time required for the pendulum to complete a cycle is called the period. To find the pendulum's period students have to trace to the first peak and record the time ( $x$ -value). Then they have to trace to the second peak and record the time value again. The period  $T$  is the difference between two time peaks (in the example  $T = 1.65$  s).

With regression the best fit sine function can be found. The screens below show the result for sine regression with its corresponding graph.



The position of the pendulum can be modeled with  $y = a \sin(bx + c) + d$ . In this formula  $y$  is the horizontal distance from the equilibrium position,  $a$  is the amplitude of the motion,  $b$  depends on the frequency of the oscillation ( $b = \frac{2\pi}{T} \approx 3.81$ ),  $x$  is the time and  $c$  is a phase constant. The average between the maximum and the minimum values is the vertical shift  $d$ . It can be found by adding the maximum and the minimum together and dividing by two. You get the distance from the CBR 2 to the pendulum's rest position.

The distance from the maximum to the minimum is twice the amplitude. To find the amplitude, we subtract the minimum from the maximum and then divide by two. You get the value to the distance from the pendulum's rest position to the point it was pulled back.

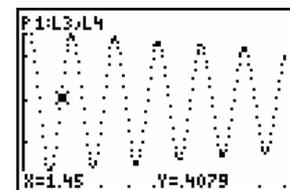


The formula  $y = a \sin(bx + c) + d$  used by regression on the calculator can be more difficult to students than the form  $y = a \sin(b(x - c)) + d$  or  $y = a \cos(b(x - c)) + d$ .

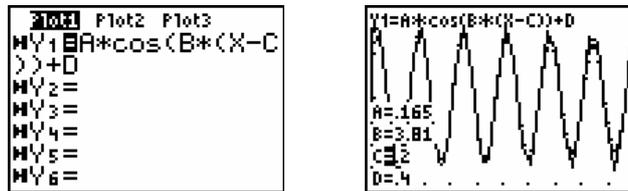
If the graph is modeled with the cosine function the horizontal shift is easier to identify than the horizontal shift of the sine curve. The phase shift for a cosine curve would be the time at which the first maximum occurs.

In this activity we use the sine curve  $y = a \sin(bx + c) + d$  created by regression. Sine and cosine graphs only differ by a horizontal shift.

The phase shift will be the  $x$ -value of the point halfway between the minimum and the next maximum. The  $y$ -value will correspond to the vertical shift. Therefore students can trace to the point, where the  $y$ -value is most nearly the value of  $d$  and record the  $x$ -value as  $e$ . They get  $c$  by evaluating  $c = -(be - 2\pi)$ .



With Transformation Graphing we can look for values of the parameters A, B, C and D in the formula  $y = A\cos(B(x - C)) + D$ . For investigation the Y= screen is used. With the cursor keys the values of A, B, C and D can be adjusted to find a graph that best fits the scatter plot.



A is the amplitude. That distance from the maximum to the minimum is twice the amplitude.

$$A = (0.5656 - 0.2353)/2 \approx 0.165$$

The vertical shift D is the average between the maximum and the minimum values.

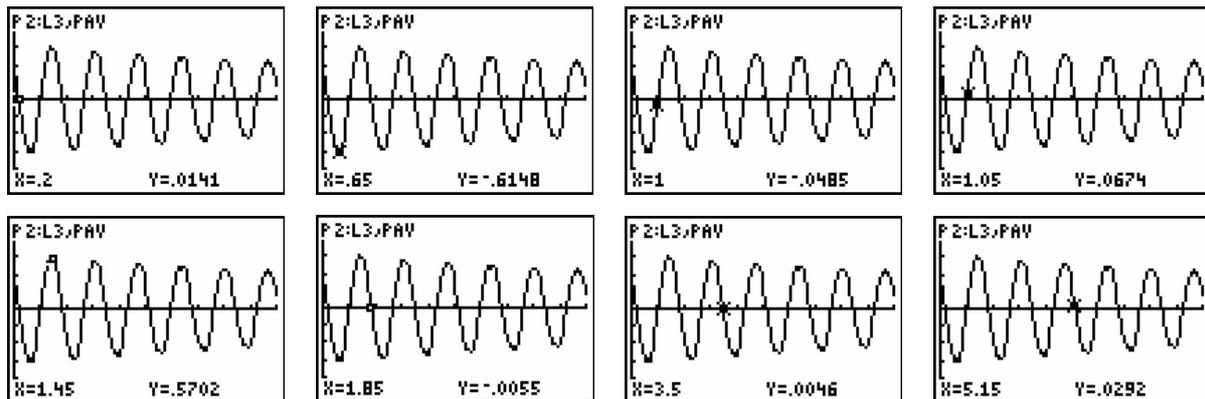
$$D = (0.5656 + 0.2353)/2 \approx 0.400$$

The difference in time between the first two maximum values is the period T.

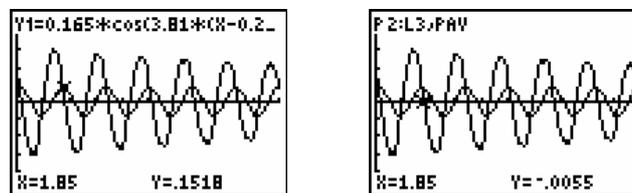
We get B by evaluating  $B = \frac{2 \cdot \pi}{T} \approx 3.81$ .

The phase shift C for a cosine curve is the time at which the first maximum occurs.

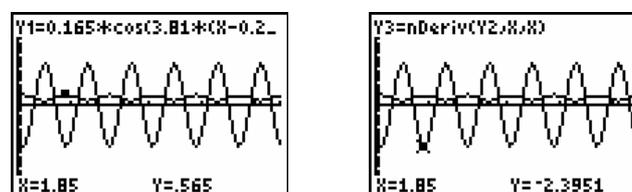
**d. Modeling velocity of the pendulum**



The period of the displacement and velocity are equal. When the distance is at a maximum, the velocity is zero. This makes sense because the mass stops as it turns around to change direction. The mass moves the fastest as it passes through the equilibrium position.



The period of the acceleration is the same as the period for the distance plot. When the distance is maximum, the acceleration is minimum. The acceleration is zero when the mass passes through the equilibrium position.



### e. Extensions and Questions in the classroom

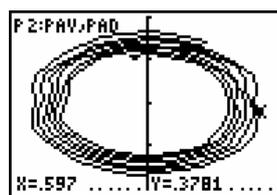
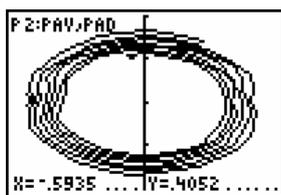
Once students have a formula for the position vs. time graph of the pendulum motion, they can take the derivative of the formula. This represents the velocity of the pendulum at any time  $t$ . The derivative of velocity is acceleration.

- How does the velocity vs. time graph compare with the position vs. time graph?
- When during the pendulum motion is the velocity zero?
- When is the velocity maximum?
- Describe the position and velocity when the acceleration is maximum. Do the same when the acceleration is zero.
- Give a general description of the pendulum's position, velocity and acceleration when the pendulum mass is passing through the at-rest position and when it is farthest from the detector.
- Determine how the period depends on amplitude. Measure the period for five different amplitudes.
- Investigate the effect of changing pendulum length on the period.
- Determine if the period is effected by changing the mass. Does the period appear to depend on the length of the pendulum (the string attached to the ball)? Do you have enough data to answer this for sure?
- To examine more carefully how the period  $T$  depends on the pendulum length  $l$ , you can create two additional graphs of the same data:  $T^2$  vs. length and  $T$  vs. length<sup>2</sup>.

Using Newton's law, you could show that for a simple pendulum the period  $T$  is related to the length  $l$  and free fall acceleration  $g$  by  $T = 2\pi\sqrt{\frac{l}{g}}$  or  $T^2 = \left(\frac{4\pi^2}{g}\right)l$ .

Does one of the graphs support this relationship?

- Determine a value for  $g$  from your graph  $T^2$  vs. length.
- Try a larger range of amplitudes. What can you find for large amplitudes?
- The two figures below show distance vs. velocity graphs of the pendulum motion. Explain what you can see there and how the figures would look like if the motion wasn't damped.



## 2.2.5 A mass on a spring – a further simple harmonic motion

### a. Introduction

A mass hanging on a spring is a simple system that can be put to vibration. The force applied on an ideal spring is proportional to how much it is stretched or compressed. Given this proportional force, the up and down motion of the mass is called simple harmonic and the position can be modeled with  $y = A\cos(2\pi ft + \phi)$ . In this formula,  $y$  is the vertical distance from the equilibrium position,  $A$  is the amplitude of the motion,  $f$  is the frequency of the oscillation,  $t$  is the time and  $\phi$  is the phase constant.

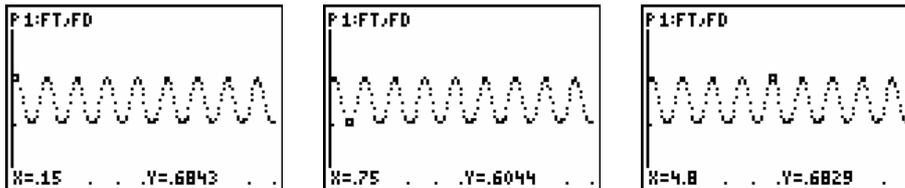
The following experiment will help to clarify each of these terms and describe harmonic motion with the help of mathematical functions. Students measure the position and velocity as a function of time for a vibrating mass and spring system. After determining the amplitude, period and phase constant of the observed motion of a mass and spring system they compare their data to a mathematical model of simple harmonic motion.

**b. Performing the experiment and collecting the data**

A spring is attached to a horizontal rod connected to a ring stand and a mass hangs on the spring. The motion detector is placed at least 75 cm below the mass.

Then the mass is lifted upward five to ten centimeters and then released. It should vibrate along a vertical line only and should never come closer than 40 cm to the motion detector (in this experiment CBR 2 is used, directly connected to the calculator).

The distance graph should show a clean sine curve. For these data the period  $T$  of the motion can be calculated by  $(4.8 - 0.15)/4 = 1.2$  s.



The frequency  $f$  is the reciprocal of the period,  $f = \frac{1}{T}$ .

Based on the experiment the frequency is calculated as 0.83 Hz.

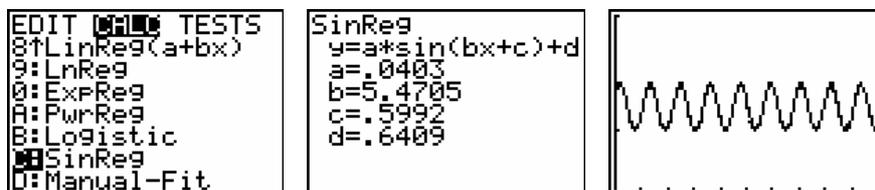
The amplitude,  $A$ , of simple harmonic motion is the maximum distance from the equilibrium position.  $A$  can be calculated as  $(0.6843-0.6044)/2 = 0.04$  m.

**c. Looking at the results**

With regression the best fit sine function can be found. The position of the mass can be modeled with  $y = a \sin(bx + c) + d$ .

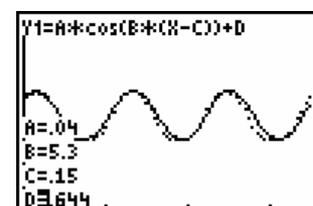
In this formula  $y$  is the vertical distance from the equilibrium position,  $a$  is the amplitude of the motion,  $b$  depends on the frequency of the oscillation ( $b = \frac{2\pi}{T} \approx 3.81$ ),  $x$  is the time and  $c$  is a phase constant.

The average between the maximum and the minimum values is the vertical shift  $d$ . It can be found by adding the maximum and the minimum together and dividing by two. You get the distance from the CBR 2 to the rest position of the mass on the spring.



Experimental data can be compared to the sine function model using the formula entered in your calculator. The model formula in the introduction, which is similar to the one in many textbooks, gives the distance from the equilibrium position of the mass. The CBR 2 reports the distance from the detector. To compare the model to the experimental data the equilibrium distance to the model has to be added.

With Transformation Graphing we can look for values of the parameters  $A$ ,  $B$ ,  $C$  and  $D$  in the formula  $y = A \cos(B(x - C)) + D$ , which is much easier. The  $Y=$  screen is used for investigation and with the cursor keys the values of  $A$ ,  $B$ ,  $C$  and  $D$  can be adjusted to get the graph that best fits the scatter plot.



$A$  is the amplitude – it is the maximum distance from the equilibrium position. That distance from the maximum to the minimum is twice the amplitude:  $A = (0.6843 - 0.6044) / 2 \approx 0.04$  m.

The vertical shift  $D$  is the average between the maximum and the minimum values:

$$D = (0.6843 + 0.6044) / 2 \approx 0.644.$$

The difference in time between the first two maximum values is the period  $T$ . We get  $B$  by

this formula:  $B = \frac{2\pi}{T} \approx 5.236$ .

The phase shift  $C$  for a cosine curve is the time at which the first maximum occurs.

#### d. Further Questions and extensions

- Compare the position-time graph and the velocity-time graph. How are they the same? How are they different?
- Trace the velocity graph to view the data values. Record a time when the velocity is maximized and another time when the velocity is zero. Then record the position of the mass at these times. Where is the mass, when the velocity is zero, relative to the equilibrium position? Where is the mass, when the velocity is maximized?
- Predict what would happen to the plot of the model if you doubled the parameter  $A$  (the amplitude).
- Similarly, predict how the model plot would change if you doubled  $f$  and then check by modifying the model definition.
- Does the frequency,  $f$ , depend on the amplitude of the motion? Try to get enough data to draw a firm conclusion.
- Does the frequency,  $f$ , depend on the mass used? Try to get enough data to draw a firm conclusion.
- Investigate how changing the spring amplitude changes the period of motion.
- How will damping change the data? Tape a card to the bottom of the mass and collect additional data for more than 10 seconds. Does the model still fit well in this new situation?
- Do additional experiments to discover the relationship between the mass and the period of motion.

## 2.3 Probability experiments

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### TARGET GROUP

Students at secondary education; high school. Students of about 15-17 years old.

### TOPIC

Probability and Simulation

### PRIOR MATHEMATICAL KNOWLEDGE

Rational Numbers; percentages and decimals, ratio & proportion. Simple statistics; using tables and graphs to organize and display information.

### PRIOR CALCULATOR EXPERIENCE

Basic Graphing Calculator experience, having used an APP before (know how to start an APP and knowing how to use the function keys)

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Reasoning about probability is difficult. Especially because certain intuitions seem to be very evident, but they can be very wrong. For example, when asking for the probability of the outcomes of tossing a coin (head or tail) – assuming the coin is fair and not tricked – most people will quickly and confidently report back with the correct response  $\frac{1}{2}$ . But the outcome of a single random toss of the coin is unpredictable. A famous French philosopher d'Alembert said "When a coin is tossed, it has forgotten what face came up the previous time it was tossed."

This can get complicated when you do an experiment. A first toss results in tail. What will be the result for the next toss? Now the reasoning can go into two directions. Some may follow d'Alembert, but others may think that on the average the number of tails and heads will be equal, so it is more likely that the other face – head – will show up.

The same difficulty in reasoning can appear when dealing with "a large number of tosses". When a large number of tosses are done the frequency of heads and tails will be equal. But again, what is a large number?

What we can say is that in the long run the relative frequency of an event approximates its probability as close as we want but nobody can tell us after how many trials.

To reflect on this problem we will analyze later the following two situations in detail:

- CHEATING AND NOT WINNING

The idea is to toss a coin that is modified so many times that it becomes clear that the number of times heads show up is really more than the number of tails.

- I LIKE GRAPEFRUIT!

1000 persons have given their opinion about liking or not liking grapefruit. Of these thousand persons, 55% said they like grapefruit. When from this population of 1000, a random sample of 100 persons is selected, how many will like grapefruit? 55? Are you sure?

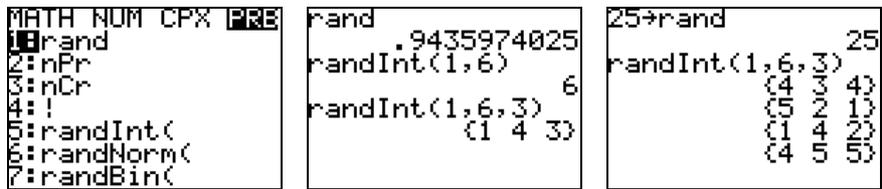
But first we will start with some fair probability experiments.

### 2.3.1 At random?

On the TI-84 Plus it's possible to generate at random a series of numbers with the following commands ([MATH] <PRB>) :

- rand generates a random number >0 and <1
- randInt generates a random integer in a specific range

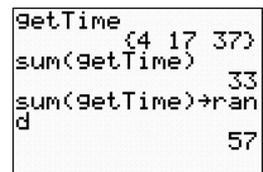
Therefore the TI-84 Plus uses a procedure that starts from a seed that is stored in rand.



When you use the calculator for the first time or when it is reset the value of rand is 0. This can be annoying when the students do a probability experiment and all get the same results while they expect random numbers.

To avoid this let each student choose a different seed to start with, e.g. the sum of the figures of their date of birth and the date of today: 01/01/1992 gives 23 and 10/03/2006 gives 12, so together 35.

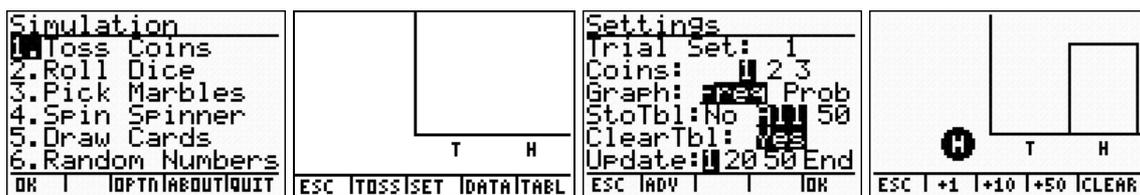
On the TI-84 Plus it's also possible to use the value of the clock as a seed to start a random procedure. Therefore you need the getTime command from the CATALOG.



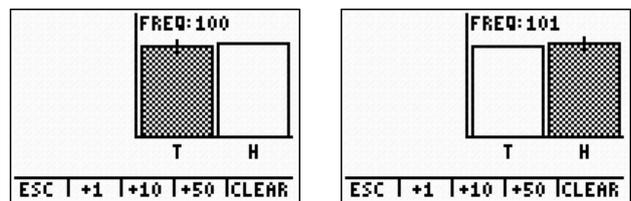
### 2.3.2 Coin tosses

It's very easy to simulate the toss of a coin with the application Probability Simulation and to use the results to visualize that the relative frequency of the event heads is in the long run  $\frac{1}{2}$ , which is the theoretical probability of the event heads.

The experiment below starts with rand equals to 8 (8 STO ▶ rand)<sup>2</sup>. Once 1. Toss Coins is started press [F3], SET, to define the Settings, leave the Settings window, OK, and press [F2], TOSS, for the first toss.



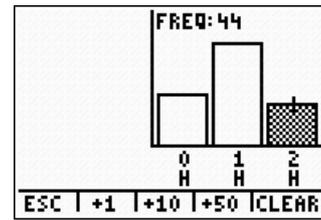
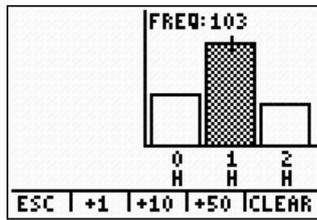
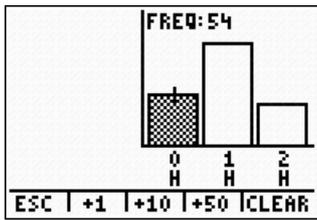
After 200 extra tosses we get the following results:



<sup>2</sup> It is also possible to define the seed by pressing [F3], OPTN, at the menu window of Probability Simulations.



After 201 tosses we note a very clear difference between the appearance of Heads-Heads, Heads-Tails and Tails-Tails.



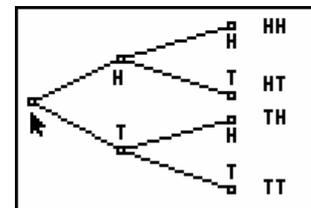
$$p(HH) \approx \frac{54}{201} \approx 0.27$$

$$p(HT \text{ or } TH) \approx \frac{103}{201} \approx 0.51$$

$$p(TT) \approx \frac{44}{201} \approx 0.22$$

Remember Cardano's error during a dice game with three dice. He considered {1,3,5} and {1,5,3} as the same events to reach a sum of 9 points.

A handy tool to miss no events is a tree diagram. The tree diagram for tossing two coins tells us immediately the exact probability distribution.

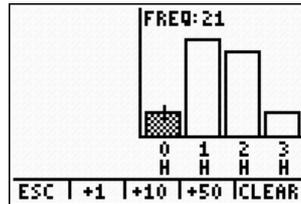
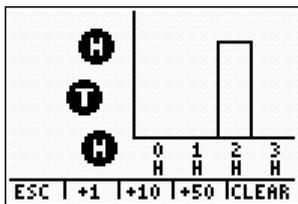


Made with Cabri Junior

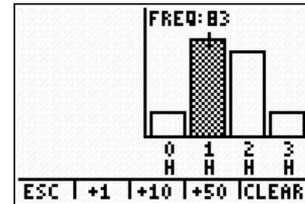
$$p(HH) = p(HT) = p(TH) = p(TT) = 0.25$$

$$\Rightarrow p\{1 \text{ times Tails}\} = 0.5$$

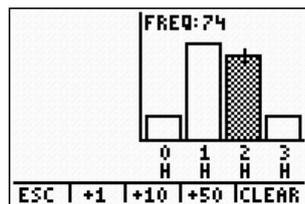
And what about three coins?



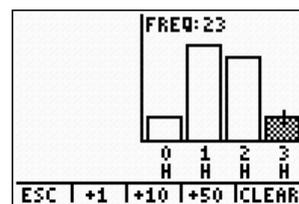
$$p(HHH) \approx 0.10$$



$$p(TTH \text{ or } THT \text{ or } HTT) \approx 0.37$$



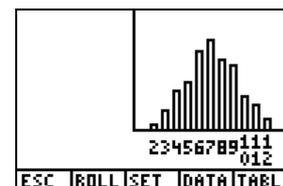
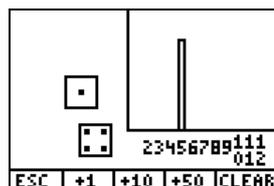
$$p(THH \text{ or } HTH \text{ or } HHT) \approx 0.41$$



$$p(TTT) \approx 0.11$$

### 2.3.3 Throwing dice

2. Roll Dice you can simulate the throwing of dice, up to 3 dice. Let's start with two dice.



After saving the data in lists we can do several calculations.

Dice 1	Dice 2										
1	1	2	1	3	1	4	1	5	1	6	1
1	2	2	2	3	2	4	2	5	2	6	2
1	3	2	3	3	3	4	3	5	3	6	3
1	4	2	4	3	4	4	4	5	4	6	4
1	5	2	5	3	5	4	5	5	5	6	5
1	6	2	6	3	6	4	6	5	6	6	6

```
Save data to:
Roll num. - 'LROLL'
D1 Data - 'LD1'
D2 Data - 'LD2'
Sum Dice - 'LSUM'
ESC | | | OK
```

```
sum(LSUM<7)/300
.33
sum(LSUM=7)/300
.15
sum(LSUM>7)/300
.3533333333
```

UPNOVER 7

The Grand Duke of Tuscany noted during gambling that the sum of 10 points appeared more than the sum of 9 when he threw three dices. He could not explain what was happening because the amount of possibilities to get 9 is the same as to get 10.

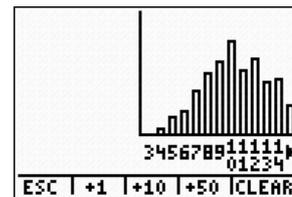
$$9=1+2+6=1+3+5=1+4+4=2+2+5=2+3+4=3+3+3$$

$$10=1+3+6=1+4+5=2+2+6=2+3+5=2+4+4=3+3+4$$

```
Settings
Trial Set: 1
Dice: 1 2
Sides: 8 10 12 20
Graph: Prob
StoTbl: No H11
ClearTbl: Yes
ESC ADV | | | OK
```

```

3 4 5 6 7 8 9 10 11 12
0 1 2 3 4
ESC | +1 | +10 | +50 | CLEAR
```



### 2.3.4 Cheating and not winning

We start with initializing the rand variable. In this experiment the coins are really modified! We are going to toss a coin with the probability for tails 0.55 and for heads 0.45.

```
Set Random Seed...
Seed=6
ESC | | | OK
```

To modify the coin press [F2], ADV, in the Settings window. Don't forget to save the settings with OK, [F5].

```
Settings
Trial Set: 1
Coins: 1 2 3
Graph: Prob
StoTbl: No H11
ClearTbl: Yes
Update: 20 50 End
ESC ADV | | | OK
```

```
Side Wght Prob
Tails 1 .55
Heads 1 .5
ESC | | | OK
```

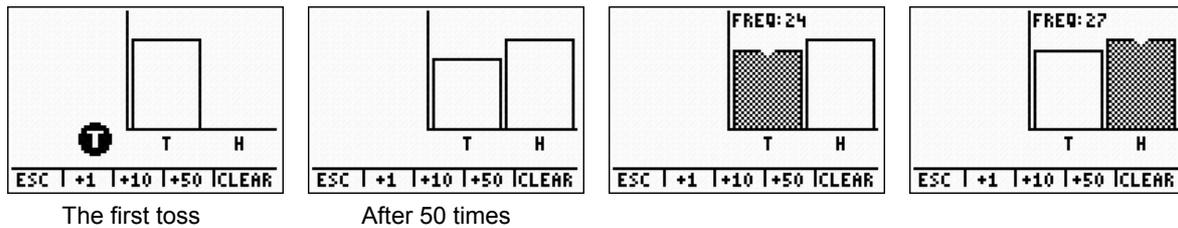
```
Side Wght Prob
Tails 11 .55
Heads 9 .45
ESC | | | OK
```

We will now gamble about the result after fifty tosses of the coin.

Will you choose more tails (55% chance) or more heads (45 % chance)?

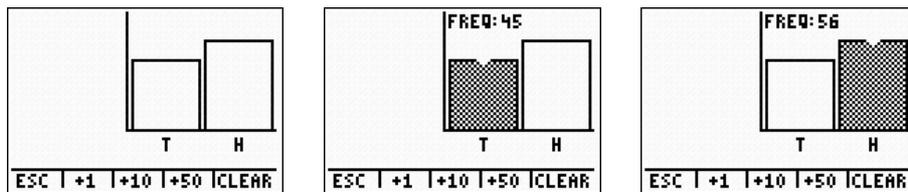
This seems easy, tails of course!

First we will toss the coin once and then we will start gambling.

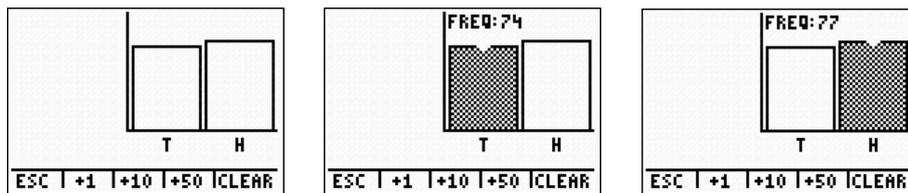


The result of the first toss seems to be OK but after 50 tosses more you lose.

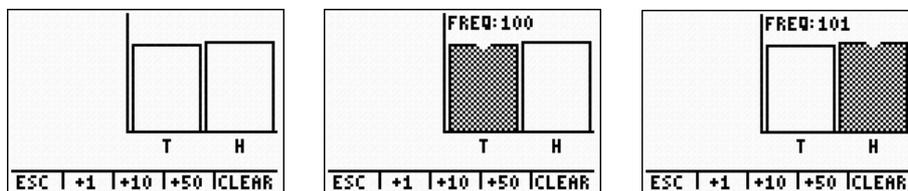
And after again 50 times more? You lose again!



Keep going on and toss the coin another 50 times. Still no win yet and for sure you cheated the coin.



Keep going on... Still lost!?



Here we stop, because the next tosses will give you the result you predicted.

The conclusion is that the results of the tossing are unpredictable, even with a cheated coin. In other words, the law of the large numbers is only true with really large numbers.

### 2.3.5 I like grapefruit!

For each person in a large group of 1000 persons, we know their preference: liking or not liking grapefruit. We assume the persons are honest and that when they are asked a second time will give the same answer.

A sample of 100 persons is selected from the large group. The problem we have to face is that this sample is randomly selected from the large group, so each time we select another random sample of 100 persons, the results may be different.

The question now is how to simulate this sample.

With the graphing calculator, we can design a small program that allows us to simulate the random selection of 100 persons from the large group of which 55% of the people do like grapefruit.

Below we have listed the program and the results of a series of samples. When you do the experiment yourself you will certainly get different results.

```

Q->H
Input "SAMPLE ?", S
For(I,1,S)
int(rand+.55)->A
A+N->N
End
Disp N/S*100
    
```

<pre> EDIT NEW FLUCTUA     </pre>	<pre> Pr9mFLUCTUA SAMPLE ?100            52            Done SAMPLE ?100            57            Done     </pre>	<pre> SAMPLE ?100 Done            52            Done SAMPLE ?100 Done            48            Done     </pre>	
<pre> SAMPLE ?100 Done            53            Done SAMPLE ?100 Done            59            Done     </pre>	<pre> SAMPLE ?100 Done            60            Done SAMPLE ?100 Done            58            Done     </pre>	<pre> SAMPLE ?100 Done            49            Done SAMPLE ?100 Done            53            Done     </pre>	<pre> SAMPLE ?100 Done            59            Done SAMPLE ?100 Done            62            Done     </pre>

What can you say about the differences between the results and your expectation? Using experiences and insight gained from this experiment, you can reflect on surveys in general (political surveys, preferences of people, ...) and the effect of repeating a survey over time.

A conclusion is that you need to take into account the fluctuation, the variation.

### 2.3.6 A historical note

The probability theory has its roots in the world of gambling. Perhaps we can say that the first steps in the development of the probability theory were taken in search for all possible solutions of several dice games.

One of the first dissertations about probability, *Liber de Ludo Aleae*, dates from 1525 and is written by Girolamo Cardano. It is a book about throwing dice.



Cardano  
1501-1576

Mathematical discussions about probability arise in the seventeenth century. The correspondence between Pascal and Fermat, which consists of five letters ( $\pm 1624$ ), forms the fundamentals of probability. Based on this Cristiaan Huygens wrote the first printed work on probability, *De Ratiociniis in Ludo Aleae* in 1657.



Blaise Pascal  
1623-1662

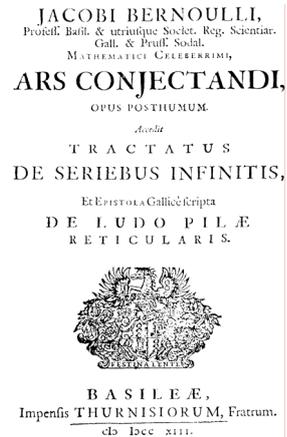


Christiaan Huygens  
1629-1695

The work *Ars Conjectandi* of Jacob (James) Bernoulli is a masterpiece in the history of probability. It was published posthumously by his nephew Nikolaus Bernoulli in 1713.



Jacob Bernoulli  
1654-1705



It was in this masterpiece the law of the large numbers appears for the first time. James formulates and proves that the relative frequency of an event approximates its probability in the long run.

It is this idea that will be used in this article to get information from the simulations.

Another important moment in the development of probability as a modern science is the axiomatic approach of probability by Kolmogorov (1903-1987) around 1933.



Andrey Kolmogorov  
1903-1987

## 2.4 Linear programming

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### TARGET GROUP

Students upper secondary education – Age: 16-18

### TOPIC

Applied mathematics – Matrix Algebra

### PRIOR MATHEMATICAL KNOWLEDGE

Linear functions, Solving of systems of linear equations, linear inequalities, matrix calculations and elementary row operations with matrices.

### PRIOR CALCULATOR EXPERIENCE

Graphing, Stat editor, Statistical plots, Lists, Matrices

---

Linear programming is the branch of applied mathematics that deals with problems like the following example.

### 2.4.1 Apples and pears

Suppose you have € 3.6 for which you want to buy apples and pears. The price of one apple is € 0.2 and € 0.3 for a pear. How many apples and pears can I buy if you know that there are only 12 apples and 10 pears in the store?

#### Solution

Let's  $x$  represent the number of apples and  $y$  the number of pears.

Obviously the following conditions count:  $x \geq 0$  and  $y \geq 0$ .

And there are the following constraints for  $x$  and  $y$ :  $20x + 30y \leq 360$ ,  $x \leq 12$  and  $y \leq 10$ .

To solve the problem we need to find all the points  $(x, y)$  that satisfy:

$$\begin{cases} 20x + 30y \leq 360 \\ x \leq 12 \\ y \leq 10 \\ x \geq 0, y \geq 0 \end{cases}$$

We try to solve the problem by a graphical approach by plotting the linear relations

$20x + 30y = 360$ ,  $x = 0$ ,  $x = 12$ ,  $y = 10$  and  $y = 0$ .

Therefore we define the functions:  $Y_1 = 12 - \frac{2}{3}x$ ,

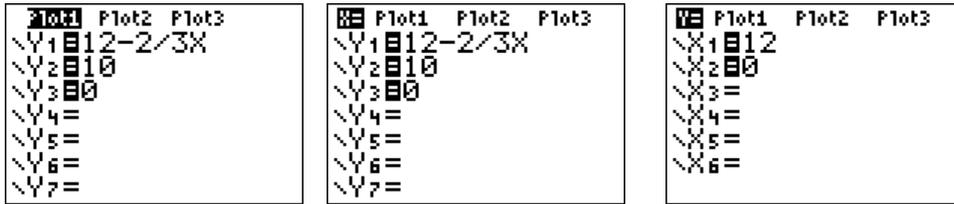
$$Y_2 = 10,$$

$$Y_3 = 0,$$

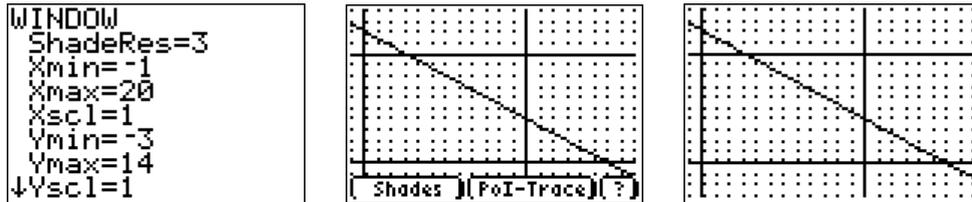
$$X_1 = 12,$$

$$X_2 = 0.$$

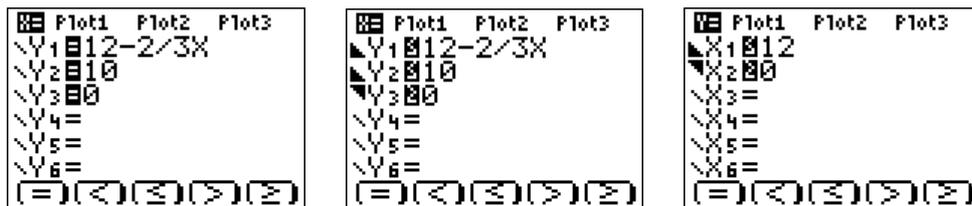
To define  $X_1=12$  and  $X_2=0$  you first need to activate the application *Inequality Graphing*. And then select  $X=$ .



These definitions result into the following graph (press TRACE CLEAR to remove the menu at the bottom of the screen):



All the points in the enclosed area are solutions for our problem. It's possible to shade this area and to calculate its vertices. To shade we need to change the equal signs with F1 through F5 as follow:

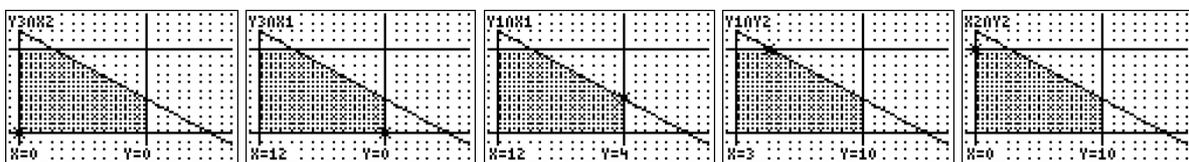


Press GRAPH, then select Shades and 1: Ineq Intersection.

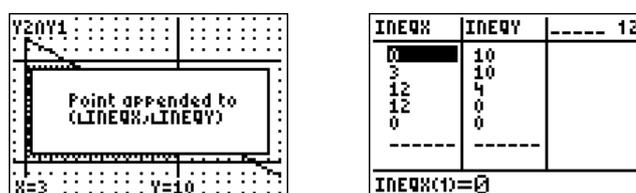


We will now calculate the vertices of shades with PoI-Trace:

◀ ▶ = change the second function      ▲ ▼ = change the first function



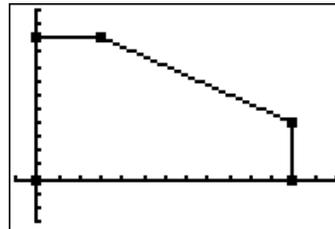
You can store a selected vertex by pressing STO ▶. The coordinates of the vertex will automatically be stored in the lists INEQX and INEQY.



With these lists it's still possible to plot the area even after quitting *Inequality Graphing* and/or deleting the functions. On the graph below the grid is turned off.

```

P1ot2 P1ot3
Off Off
Type: L1 L2 L3 L4 L5 L6 L7 L8 L9 L0
Xlist:INEQX
Ylist:INEQY
Mark: [ ] + .
    
```

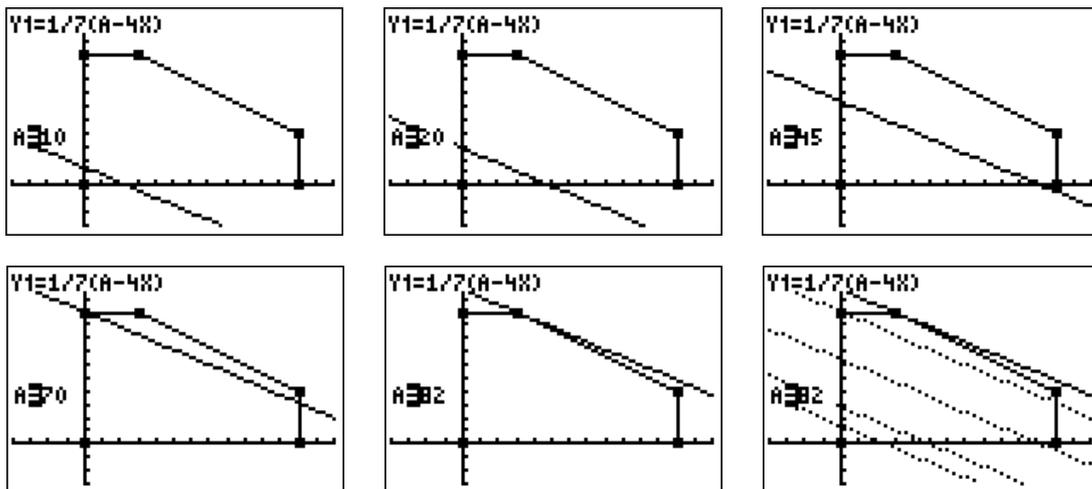


Let's make our example a little bit more complicated. Try to maximize the amount of vitamin C in our purchase. Suppose an apple contains 4 grams of vitamin C and a pear 7 grams.

To solve this problem we need to find the maximum value of  $4x + 7y$  over the area determined above.

To investigate this problem graphically we define the parameter  $A$ ,  $A = 4x + 7y$ , and the function  $Y_1 = \frac{1}{7}(A - 4x)$ .

Activate the application *Transformation Graphing* (deactivate first *Inequality Graphing*) and investigate the value of  $A$  for several points in the enclosed area.



TrailOn - 2nd[FORMAT]

When we study the variation of the parameter  $A$  we see that the maximum value of  $A$  will be found in one of the vertices. With STAT 1:Edit... we can calculate these values for  $A$  as follows:

INEQX	INEQY	A	14
0	10	-----	
3	10		
12	4		
12	0		
0	0		
-----	-----		
A=4 LINEQX+7 LINEQY			

INEQX	INEQY	A	14
0	10	-----	
3	10		
12	4		
12	0		
0	0		
-----	-----		
A=...NEQX+7 LINEQY			

INEQX	INEQY	A	14
0	10	70	
3	10	82	
12	4	76	
12	0	48	
0	0	0	
-----	-----		
A(1) =70			

The maximum amount of vitamin C is 82 grams with a purchase of 3 apples and 10 pears.

## 2.4.2 The simplex method

We can write the previous example as follows:

$$\begin{aligned}
 \text{Maximize} \quad & 4x + 7y \\
 \text{Subject to} \quad & 20x + 30y \leq 360 \\
 & x \leq 12 \\
 & y \leq 10 \\
 & x \geq 0, y \geq 0
 \end{aligned} \tag{2.4.1}$$

The simplex method always starts from a feasible solution. For our use we will take the origin  $x = 0$  and  $y = 0$ . Of course these  $x$  and  $y$  values aren't the ones that gives us the maximum value for  $4x + 7y$ .

We will rewrite the inequalities into equalities by introducing three new variables  $u, v, w$ ; called slack variables:  $u = 360 - 20x - 30y \leq 360$

$$v = 12 - x$$

$$w = 10 - y$$

We define  $z = 4x + 7y$ . The old variables  $x$  and  $y$  are called the decision variables.

So now we can rewrite our problem as follows:

$$\begin{aligned}
 \text{Maximize} \quad & z = 4x + 7y \\
 \text{Subject to} \quad & u = 360 - 20x - 30y \\
 & v = 12 - x \\
 & w = 10 - y \\
 & x \geq 0, y \geq 0, u \geq 0, v \geq 0, w \geq 0
 \end{aligned} \tag{2.4.2}$$

### Note

- Each feasible solution of (2.4.1) can be extended to a feasible solution of (2.4.2).
- Each feasible solution of (2.4.2) can be restricted to a feasible solution of (2.4.1).
- Each optimal solution of (2.4.1) corresponds with an optimal solution of (2.4.2).

$$\text{Our feasible solution to start from is } x = 0, y = 0, u = 360, v = 12, w = 10. \tag{2.4.3}$$

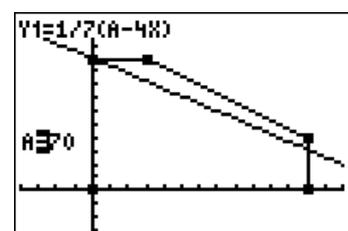
This solution gives  $z = 0$ .

We will try to find successive improvements out of this feasible solution  $x, y, u, v, w$  to end with a maximized solution. This means that out of  $x, y, u, v, w$  we try to deduce a feasible solution  $\tilde{x}, \tilde{y}, \tilde{u}, \tilde{v}, \tilde{w}$  with  $4\tilde{x} + 7\tilde{y} \geq 4x + 7y$ .

If we look at  $z = 4x + 7y$  we see that if we increase  $y$ ,  $z$  will increase faster as we increase  $x$ . So we will increase  $y$  and keep  $x = 0$ . How much can we increase  $y$ ?

For  $x = 0, u \geq 0, v \geq 0, w \geq 0$  the following constraints count:

$$\begin{cases} 360 - 30y \geq 0 \\ 12 \geq 0 \\ 10 - y \geq 0 \end{cases} \Leftrightarrow \begin{cases} y \leq 12 \\ 12 \geq 0 \\ y \leq 10 \end{cases} \Rightarrow y \leq 10.$$



In other words  $y$  can increase up to 10.

So we become our next solution:  $x = 0, y = 10, u = 60, v = 12, w = 0$  which yields  $z = 70$ .

In our next step we are going for an ever better feasible solution. How can we do this?

We need to manufacture a new system of linear constraint to continue.

If we look at (2.4.2) we see that it expresses the variables  $u, v, w$  that assume positive values in (2.4.3) in terms of those variables  $x, y$  that assume zero. And also  $z$  is expressed in (2.4.2) in terms of  $x, y$ .

Note that  $y$  changed its value from zero to positive and  $w$  from positive to zero. So we need to change their position in the system of equations, from the right-hand side to the left-hand side and vice versa. We call  $y$  the entering variable and  $w$  the leaving variable.

We start with the newcomer  $y$  on the left-hand side. With the third equation of (2.4.2) we can express  $y$  in terms of  $x, w$ :  $w = 10 - y \Leftrightarrow y = 10 - w$ .

Next we express  $u, v$  and  $z$  in terms of  $x, w$

$$u = 360 - 20x - 30y = 360 - 20x - 30(10 - w) = 60 - 20x + 30w$$

$$v = 12 - x$$

$$z = 4x + 7y = 4x + 7(10 - w) = 70 + 4x - 7w$$

So we can rewrite our problem as follows:

$$\begin{aligned} \text{Maximize} \quad & z = 70 + 4x - 7w \\ \text{Subject to} \quad & u = 60 - 20x + 30w \\ & v = 12 - x \\ & y = 10 - w \\ & x \geq 0, y \geq 0, u \geq 0, v \geq 0, w \geq 0 \end{aligned}$$

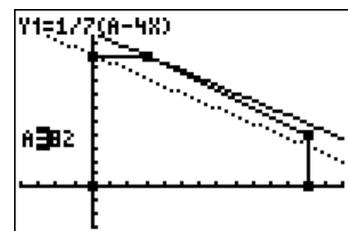
From our second feasible solution  $x = 0, y = 10, u = 60, v = 12, w = 0$  with  $z = 70$  we will again try to find an improvement.

If we look at  $z = 70 + 4x - 7w$  the only way to increase  $z$  is to increase  $x$ .

How much can we increase  $x$ ?

For  $w = 0, y \geq 0, u \geq 0, v \geq 0$  the following constraints count:

$$\begin{cases} 60 - 20x \geq 0 \\ 12 - x \geq 0 \\ 10 \geq 0 \end{cases} \Leftrightarrow \begin{cases} x \leq 3 \\ x \leq 12 \\ 10 \geq 0 \end{cases} \Rightarrow x \leq 3$$



In other words  $x$  can increase up to 3 and our next feasible solution is:

$x = 3, y = 10, u = 0, v = 9, w = 0$  with  $z = 82$ .

Now we express all variables and  $z$  in terms of  $u, w$ .

Again we will start with the newcomer  $x$ :

$$u = 60 - 20x + 30w \Leftrightarrow 20x = 60 - u + 30w \Leftrightarrow x = 3 - \frac{1}{20}u + \frac{3}{2}w.$$

It follows that:  $v = 12 - x = 9 + \frac{1}{20}u - \frac{3}{2}w$

$$y = 10 - w$$

$$z = 70 + 4x - 7w = 70 + 4\left(3 - \frac{1}{20}u + \frac{3}{2}w\right) - 7w = 82 - \frac{1}{5}u - w$$

When we look at  $z$  it's clear we can not increase  $z$  anymore by increasing  $u$  or  $w$ .

This means we found an optimal solution  $z = 82$  for  $x = 3$  and  $y = 10$ .

The method we just used to find an optimal solution is called the simplex method. In this particular example  $x$  and  $y$  has to be integers but everything stays the same if we consider  $x$  and  $y$  as real variables.

### 2.4.3 The simplex method using matrices

We rewrite our example into the following modified form.

$$\begin{array}{rcl} 20x + 30y + u = 360 & & 20x + 30y + u = 360 \\ x + v = 12 & \text{or} & x + v = 12 \\ \hline y + w = 10 & & y + w = 10 \\ -z + 4x + 7y = 0 & & -z + 4x + 7y = 0 \end{array}$$

Using only the coefficients we can use the following matrix to represent our example.

$$\begin{pmatrix} 20 & 30 & 1 & 0 & 0 & 360 \\ 1 & 0 & 0 & 1 & 0 & 12 \\ 0 & 1 & 0 & 0 & 1 & 10 \\ 4 & 7 & 0 & 0 & 0 & 0 \end{pmatrix}$$

#### Step 1

Examine the elements of the last row, except the one in the last column (which represents the present value of  $-z$ ). If all the elements are negative, the matrix represents an optimal solution. Otherwise select the column associated with the largest positive number. This column is called the pivot column and corresponds with the entering variable.

$$\begin{pmatrix} 20 & \boxed{30} & 1 & 0 & 0 & 360 \\ 1 & 0 & 0 & 1 & 0 & 12 \\ 0 & 1 & 0 & 0 & 1 & 10 \\ 4 & 7 & 0 & 0 & 0 & 0 \end{pmatrix}$$

#### Step 2

We will calculate the ratios  $\frac{p}{q}$  of the elements  $p$  of the rightmost column and the positive elements  $q$  of the pivot column (except for the last row). If they are all negative the problem is unbounded (see point d).

The row with the smallest ratio  $\frac{p}{q}$  is called the pivot row and corresponds with the leaving variable.

$$\begin{pmatrix} 20 & 30 & 1 & 0 & 0 & 360 \\ 1 & 0 & 0 & 1 & 0 & 12 \\ 0 & 1 & 0 & 0 & 1 & 10 \\ 4 & 7 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} \longrightarrow \frac{360}{30} = 12 \\ \longrightarrow \frac{10}{1} = 10 \end{matrix}$$

### Step 3

In this step we divide each element of the pivot row with the pivot (= intersection of the pivot column and the pivot row). In our case (pivot = 1) we don't need to do anything.

It's not a bad idea to add a column with the positive variables of our present solution.

$$\begin{pmatrix} 20 & 30 & 1 & 0 & 0 & 360 \\ 1 & 0 & 0 & 1 & 0 & 12 \\ 0 & 1 & 0 & 0 & 1 & 10 \\ 4 & 7 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} u \\ v \\ w \end{matrix}$$

### Step 4

Use elementary row operations (2nd [MATRIX] <MATH>) to make all the elements of the pivot column, except the pivot, zero.

Input of the matrix

```
NAMES MATH [EDIT]
1:[A]
2:[B]
3:[C]
4:[D]
5:[E]
6:[F]
7:[G]
```

```
MATRIX[A] 4 x6
[ 20 30 1 0 0 -
[ 1 0 0 1 0 12
[ 0 1 0 0 1 10
[ 4 7 0 0 0 0
1, 1=20
```

```
MATRIX[A] 4 x6
-0 0 360 1
-1 0 12 1
-0 1 10 1
-0 0 0 1
1, 6=360
```

-30 R<sub>3</sub> + R<sub>1</sub>

```
[A]
[[20 30 1 0 0 360
[1 0 0 1 0 12
[0 1 0 0 1 10
[4 7 0 0 0 0
*row+(-30,Ans,3,1)
```

```
[4 7 0 0 0 0...
*row+(-30,Ans,3,1)
[[20 0 1 0 -30 ...
[1 0 0 1 0 ...
[0 1 0 0 1 ...
[4 7 0 0 0 ...
```

```
[4 7 0 0 0 0...
*row+(-30,Ans,3,1)
... 0 1 0 -30 60]
... 0 0 1 0 12]
... 1 0 0 1 10]
... 7 0 0 0 0 0]
```

-7 R<sub>3</sub> + R<sub>4</sub>

```
*row+(-30,Ans,3,1)
[[20 0 1 0 -30 ...
[1 0 0 1 0 ...
[0 1 0 0 1 ...
[4 7 0 0 0 ...
*row+(-7,Ans,3,4)
```

```
[4 7 0 0 0 ...
*row+(-7,Ans,3,4)
[[20 0 1 0 -30 ...
[1 0 0 1 0 ...
[0 1 0 0 1 ...
[4 0 0 0 -7 ...
```

```
[4 7 0 0 0 ...
*row+(-7,Ans,3,4)
...0 1 0 -30 60 1
...0 0 1 0 12 1
...1 0 0 1 10 1
...0 0 0 -7 -70 1]
```

So we become the following new matrix, with  $z = 70$  and  $x = 0, y = 10, u = 60, v = 12$  and  $w = 0$ :

$$\begin{pmatrix} 20 & 0 & 1 & 0 & -30 & 60 \\ 1 & 0 & 0 & 1 & 0 & 12 \\ 0 & 1 & 0 & 0 & 1 & 10 \\ 4 & 0 & 0 & 0 & -7 & -70 \end{pmatrix} \begin{matrix} u \\ v \\ y \\ \end{matrix}$$

We need to redo the previous four steps, starting from this matrix, to find a better feasible solution.

Step 1 & 2

$$\begin{pmatrix} 20 & 0 & 1 & 0 & -30 & 60 \\ 1 & 0 & 0 & 1 & 0 & 12 \\ 0 & 1 & 0 & 0 & 1 & 10 \\ 4 & 0 & 0 & 0 & -7 & -70 \end{pmatrix} \begin{matrix} u \\ v \\ y \\ \end{matrix}$$

```
[B]
[[20 0 1 0 -30 ...
 [1 0 0 1 0 ...
 [0 1 0 0 1 ...
 [4 0 0 0 -7 ...
```

```
[B]
...0 1 0 -30 60 1
...0 0 1 0 12 1
...1 0 0 1 10 1
...0 0 0 -7 -70 1]
```

Step 3 & 4

$-\frac{1}{20}R_1$

```
[B]
[[20 0 1 0 -30 ...
 [1 0 0 1 0 ...
 [0 1 0 0 1 ...
 [4 0 0 0 -7 ...
 *row(1/20, Ans, 1)
```

```
[4 0 0 0 -7 ...
 *row(1/20, Ans, 1)
 [[1 0 .05 0 -1...
 [1 0 0 1 0 ...
 [0 1 0 0 1 ...
 [4 0 0 0 -7 ...
```

```
[4 0 0 0 -7 ...
 *row(1/20, Ans, 1)
 ...05 0 -1.5 3 1
 ... 1 0 12 1
 ... 0 1 10 1
 ... 0 -7 -70 1]
```

$-R_1 + R_2$

```
*row(1/20, Ans, 1)
 [[1 0 .05 0 -1...
 [1 0 0 1 0 ...
 [0 1 0 0 1 ...
 [4 0 0 0 -7 ...
 *row+(-1, Ans, 1, 2)
```

```
[4 0 0 0 -7 ...
 *row+(-1, Ans, 1, 2)
 [[1 0 .05 0 -1...
 [0 0 -.05 1 1...
 [0 1 0 0 1 ...
 [4 0 0 0 -7...
```

```
[4 0 0 0 -7 ...
 *row+(-1, Ans, 1, 2)
 ...5 0 -1.5 3 1
 ...05 1 1.5 9 1
 ... 0 1 10 1
 ... 0 -7 -70 1]
```

$-4R_1 + R_4$

```
*row+(-1, Ans, 1, 2)
 ...5 0 -1.5 3 1
 ...05 1 1.5 9 1
 ... 0 1 10 1
 ... 0 -7 -70 1]
 *row+(-4, Ans, 1, 4)
```

```
... 0 -7 -70 1]
 *row+(-4, Ans, 1, 4)
 [[1 0 .05 0 -1...
 [0 0 -.05 1 1...
 [0 1 0 0 1 ...
 [0 0 -.2 0 -1...
```

```
... 0 -7 -70 1]
 *row+(-4, Ans, 1, 4)
 ...5 0 -1.5 3 1
 ...05 1 1.5 9 1
 ... 0 1 10 1
 ...2 0 -1 -82 1]
```

We can put the last result into fractions with [MATH]<MATH> 1: ▶ Frac.

<pre>...2  0  1  10  1 ...2  0  -1  -82  1] Ans▶Frac [[1  0  1/20  0  - [0  0  -1/20  1  3... [0  1  0  0  1... [0  0  -1/5  0  -...</pre>	<pre>...2  0  1  10  1 ...2  0  -1  -82  1] Ans▶Frac ...0  0  -3/2  3  1 ...20  1  3/2  9  1] ...0  1  0  0  1... ...5  0  -1  -82  1]</pre>
--	--

Our new matrix is:

$$\begin{pmatrix} 1 & 0 & \frac{1}{20} & 0 & -\frac{3}{2} & 3 \\ 0 & 0 & -\frac{1}{20} & 1 & \frac{3}{2} & 9 \\ 0 & 1 & 0 & 0 & 1 & 10 \\ 0 & 0 & -\frac{1}{5} & 0 & -1 & -82 \end{pmatrix} \begin{matrix} x \\ v \\ y \\ \end{matrix}$$

The last row of our matrix contains only negative numbers which means we reached an optimal solution  $x = 3, y = 10, u = 0, v = 9, w = 0$  with  $z = 82$ .

### 2.4.4 Always a unique solution?

Without giving a complete discussion we will end with two examples to show that there is not always a unique solution.

(i) Several solutions – infinite many

<p>Maximize <math>z = 2x + 4y</math>  subject to <math>x - y \leq 2</math>  <math>x + 2y \leq 16</math>  <math>x \geq 0, y \geq 0</math></p>	or	<p>Maximize <math>z = 2x + 4y</math>  subject to <math>u = 2 - x + y</math>  <math>v = 16 - x - 2y</math>  <math>x \geq 0, y \geq 0, u \geq 0, v \geq 0</math></p>
--	----	--

The second constraint gives already an indication that the line which represent  $2x + 4y - z = 0$  is parallel to one side of the area enclosed by the constraints.

In a following step we become:

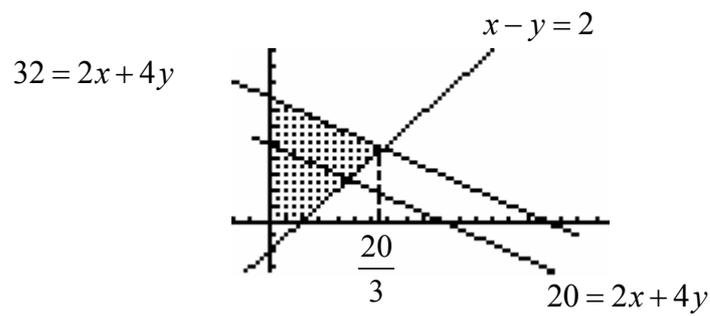
Maximize  $z = 32 - 2v$   
subject to  $y = 8 - 0.5x - 0.5v$   
 $u = 10 - 1.5x - 0.5v$   
 $x \geq 0, y \geq 0, u \geq 0, v \geq 0$

For each optimal solution ( $z = 32$ ) counts  $v = 0$ , but not necessary  $x = 0$ .

The condition for  $x$  is  $10 - 1.5x \geq 0 \Leftrightarrow x \leq \frac{20}{3}$ .

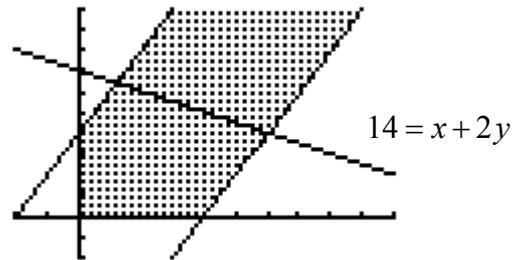
For all  $x$  in  $\left[0, \frac{20}{3}\right]$  we find an optimal solution  $x, y = 8 - 0.5x, u = 10 - 1.5x, v = 0$ .

Note:  $y = 8 - 0.5x \Leftrightarrow x + 2y = 16 \Leftrightarrow 2x + 4y = 32$ .



(ii) No solution – an unbounded problem

Maximize  $z = x + 2y$   
 subject to  $-2x + y \leq 4$   
 $2x - y \leq 8$   
 $x \geq 0, y \geq 0$



## 2.5 Parameters and functions

### TARGET GROUP

Students upper secondary education – Age: 14-18

### TOPIC

Real functions: the effect of changes in a function's parameters to its graph.

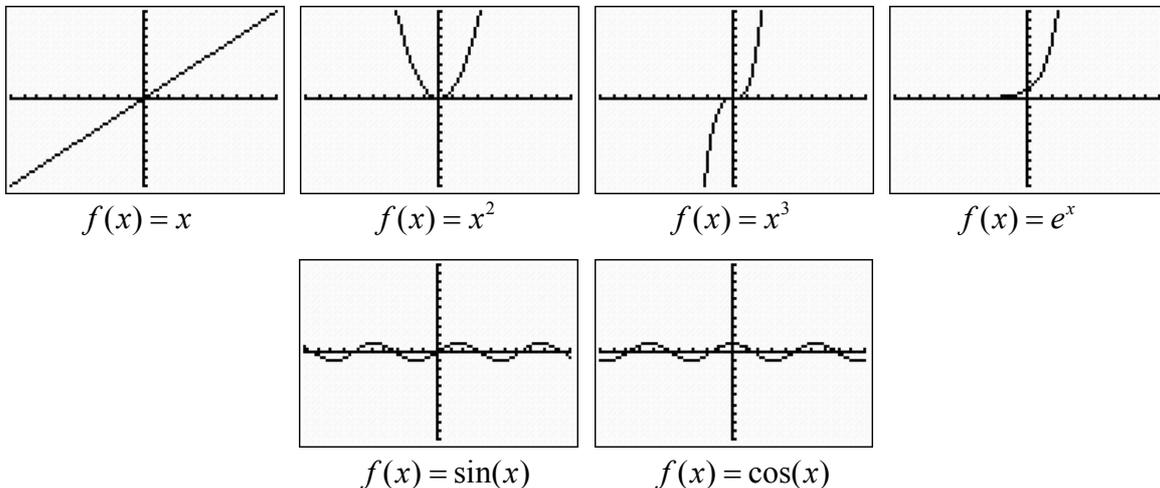
### PRIOR MATHEMATICAL KNOWLEDGE

Basic real functions:  $f(x) = x$ ,  $f(x) = x^2$ ,  $f(x) = x^3$ ,  $f(x) = e^x$  and  $f(x) = \sin(x)$

### PRIOR CALCULATOR EXPERIENCE

Graphing, Stat editor, Statistical plots, Lists, Matrices

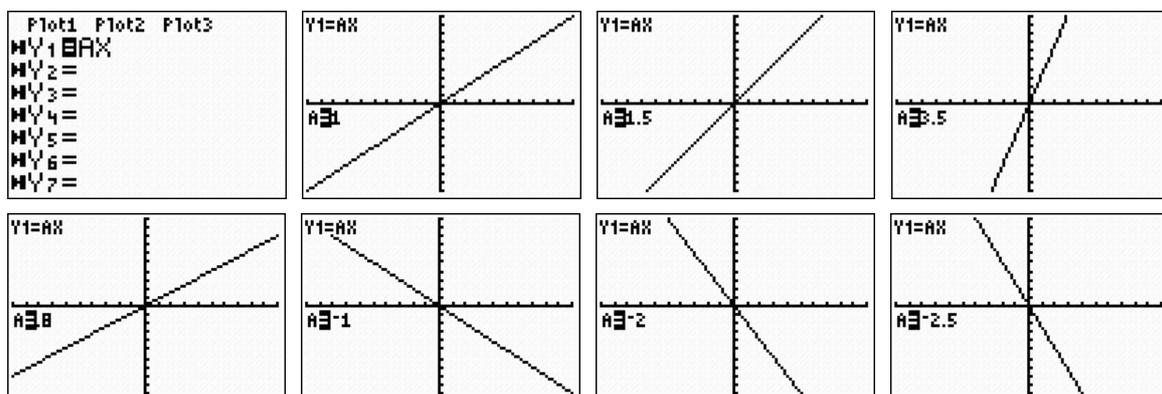
We will start our investigation about the changes in parameters of a function to its graph with the following elementary real functions, all plotted in the standard window (6:Zstandard).



With Transformation Graphing we will graphically analyze linear, quadratic, exponential and trigonometric functions. By exploring the graph, students can discover properties of the functions on their own. A next step is to confirm them algebraically, with or without computer algebra. But this is not the meaning of this chapter.

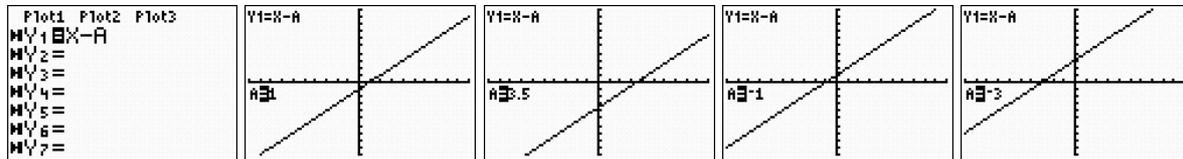
### 2.5.1 Linear functions

#### a. $y = ax$



The students will observe that the value of  $a$  will determine if the function is decreasing ( $a < 0$ ) or increasing ( $a > 0$ ) and that the parameter  $a$  corresponds with the slope of the graph. Note that for  $a=0$  the graph will be equal to the  $x$  axis.

**b.  $y = x - a$**



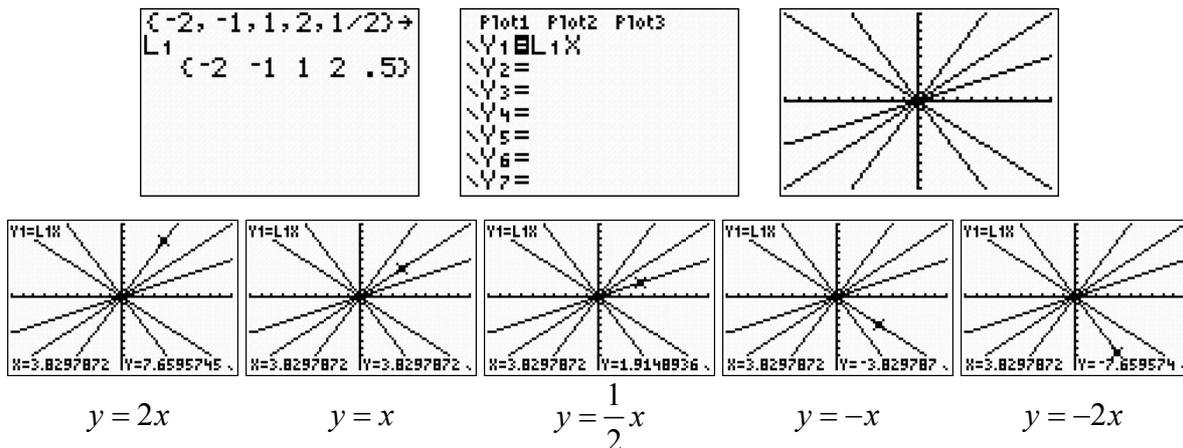
We can conclude that changes in the parameter have no effect on the slope of the line. The transformation  $x \mapsto x - a$  move the intersection point of the graph with the  $y$  axis upwards ( $a < 0$ ) or downwards ( $a > 0$ ).

**Activity 1**

Make a rough sketch of the graph of the function  $f(x) = -2x + 3$  and control your graph with your calculator.

A combination of  $a$  and  $b$  leads to the conclusion that the graph of  $f(x) = ax + b$  is a parallel line to the graph of  $f(x) = ax$ .

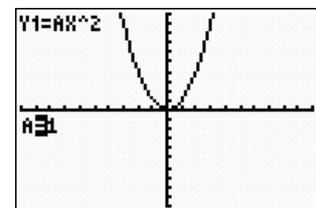
Although Transformation Graphing is a very handy tool to manipulate dynamically the graph of a function it's also interesting to do investigations using lists. An example is shown below.



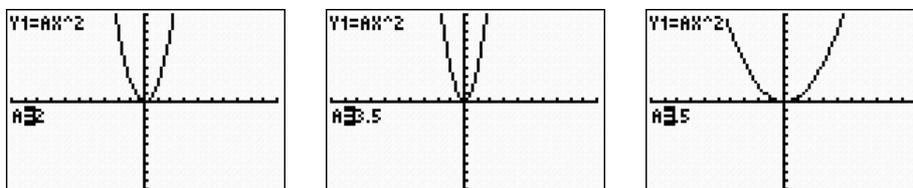
**2.5.2 Quadratic functions**

**a.  $f(x) = ax^2$**

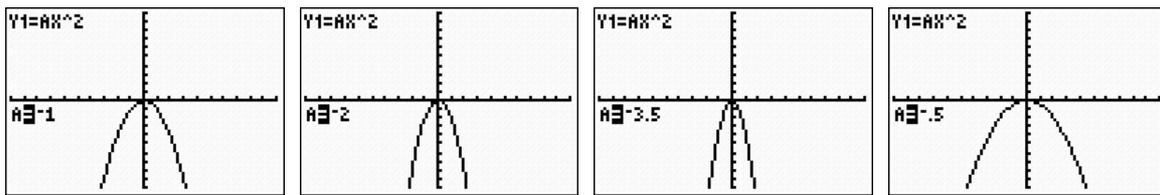
The graph of the function  $f(x) = x^2$  is a concave up parabola with the origin as the minimum and the  $y$  axis,  $x = 0$ , as the axis of symmetry.



If we change the parameter  $a > 0$  we will discover vertical stretches ( $a > 1$ ) and compressions ( $a < 1$ ) of the parabola. The origin stays as the minimum and  $x = 0$  is the axis of symmetry.



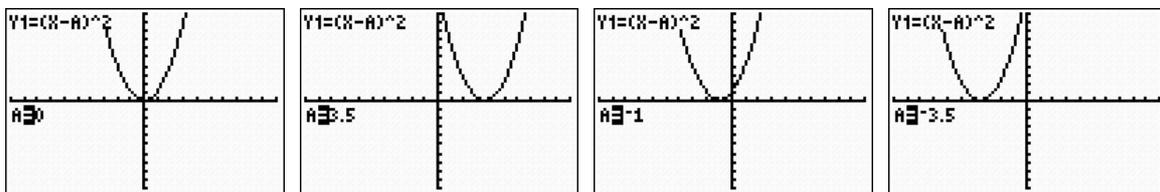
What will happen if  $a$  becomes negative?



The graph changes from concave up to concave down and the minimum becomes a maximum. Again we see vertical stretches ( $|a| > 1$ ) and compressions ( $|a| < 1$ ).

**b.**  $f(x) = (x - a)^2$

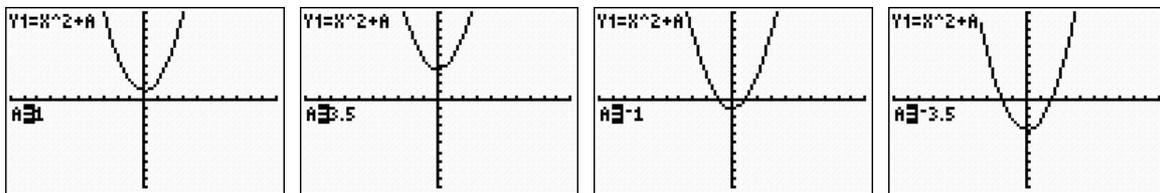
This transformation generates horizontal shifts as shown below.



The minimum will be the vertex  $(0, a)$  and  $x = a$  the axis of symmetry.

**c.**  $f(x) = x^2 + a$

Changing the parameter in this case results into vertical shifts.



$x = 0$  is always the axis of symmetry but the minimum is dependent of the parameter  $a$ . The vertex  $(0, a)$  is the minimum.

**d.**  $f(x) = ax^2 + bx + c$

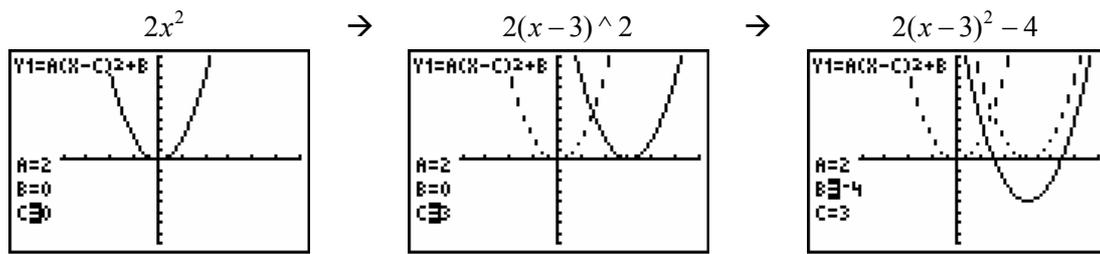
Each quadratic function,  $f(x) = ax^2 + bx + c$ , can be transformed as follows:

$$ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = a\left(x^2 + 2\frac{b}{2a} + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a}\right) = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 - \frac{b^2}{4a} + c.$$

This means for the corresponding parabola that the vertex  $\left(-\frac{b}{2a}, \frac{-b^2 + 4ac}{4a}\right)$  is the minimum

or maximum and  $x = -\frac{b}{2a}$  is the axis of symmetry.

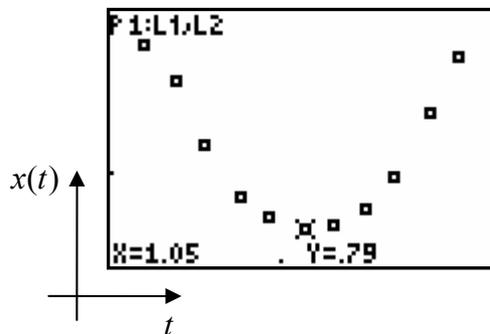
An example:  $f(x) = 2x^2 - 12x + 14 = 2(x^2 - 6x + 7) = 2(x^2 - 2 \cdot 3x + 9 - 2) = 2(x - 3)^2 - 4$ .



### Activity 2

Determine a quadratic model,  $x(t) = a(t - b)^2 + c$ , for the following data, the bounce of a ball. Use Transformation Graphing to find a value for  $a$ .

L1 $t$	L2 $x(t)$
0.67	1.46
0.75	1.33
0.82	1.09
0.9	0.91
0.97	0.83
1.05	0.79
1.12	0.8
1.2	0.86
1.27	0.98
1.35	1.21
1.42	1.42



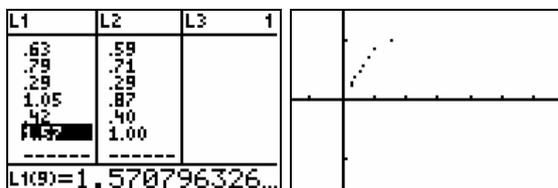
### Activity 3

Sketch the graph of the function  $f(x) = -(x - 2)^3 + 3$  based on the graph of  $f(x) = x^3$ . Control your plot with your calculator.

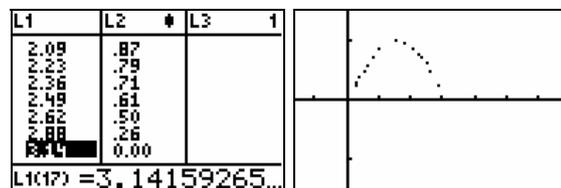
### 2.5.3 Trigonometric functions

Let  $P$  be the point where the terminal side of an angle  $x$  meets the unit circle. The  $y$ -coordinate of this point is equal to the number  $\sin(x)$  (and the  $x$ -coordinate to  $\cos(x)$ ). We will plot some points  $(x, \sin(x))$ .

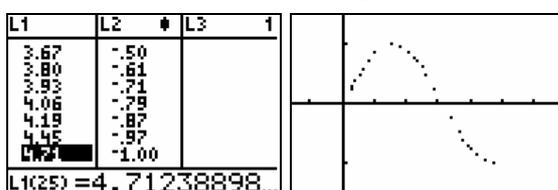
From 0 to  $\frac{\pi}{2}$



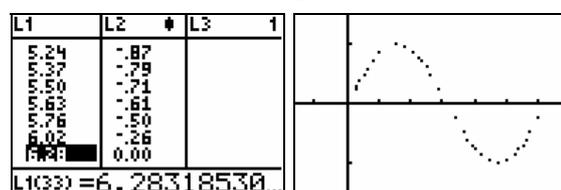
From  $\frac{\pi}{2}$  to  $\pi$



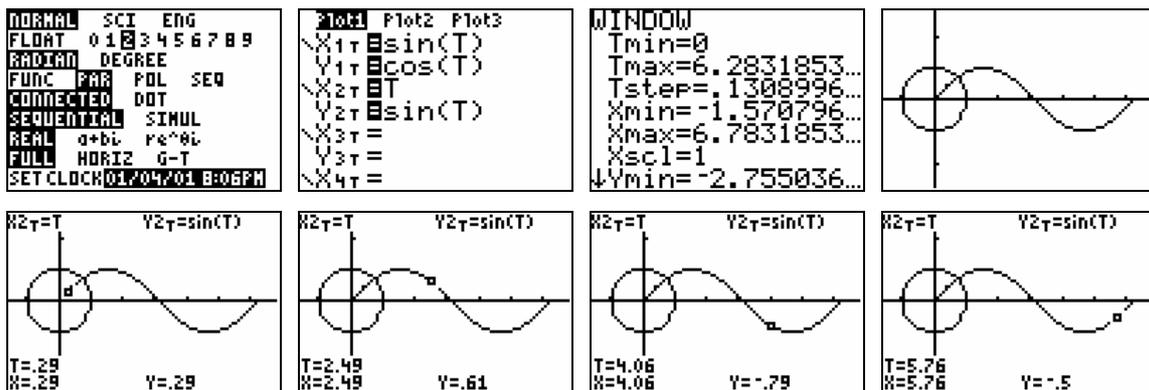
From  $\pi$  to  $\frac{3\pi}{2}$



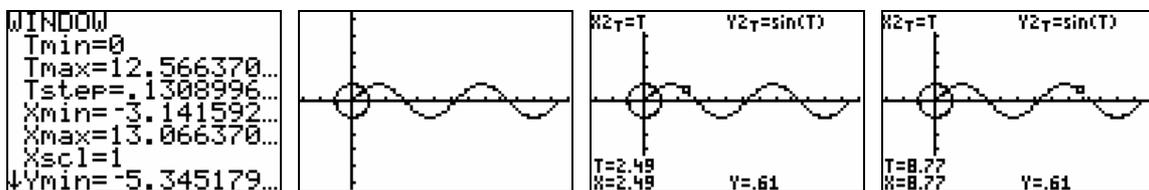
From  $\frac{3\pi}{2}$  to  $2\pi$



For another visualisation of the sine function we will use parametric functions as follows.

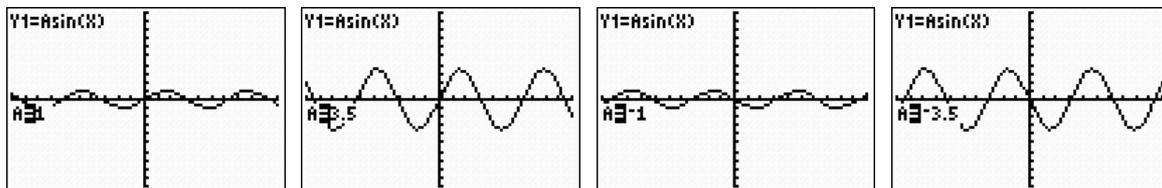


If we extend the range of the parameter from 0 to  $4\pi$  we can see that  $\sin(t) = \sin(t \pm 2\pi)$ . We say that the sine function has a period of  $2\pi$ .



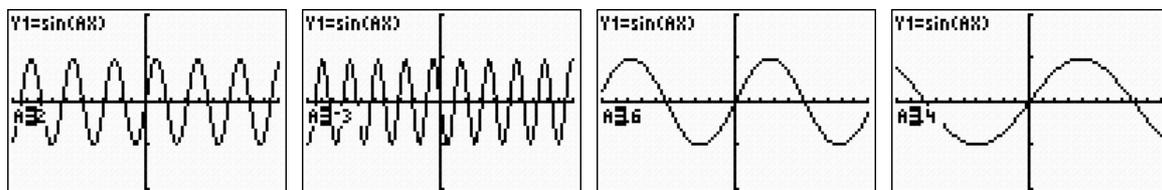
Now we will show some basic transformations of the sine function. The mode has to be equal to RADIAN.

**a. Vertical stretch or compression -  $f(x) = a \sin x$**

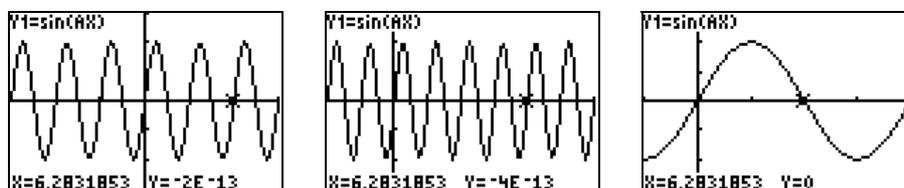


The maximum distance the graph reaches above and below the  $x$ -axis is called the amplitude. The amplitude of  $f(x) = a \sin(x)$  is equal to  $|a|$ .

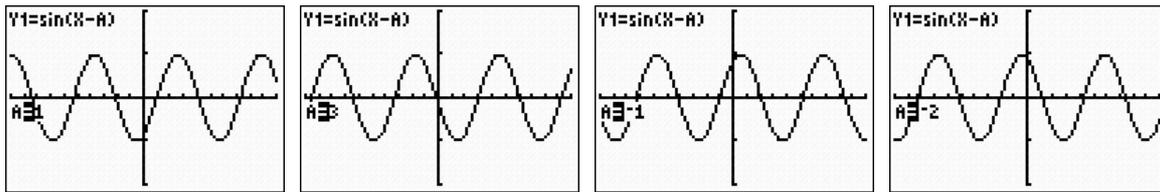
**b. Horizontal stretch or compression -  $f(x) = \sin(ax)$**



All the sine functions we have already seen showed a periodic behaviour and a graph consisting of a series of identical waves. A single wave is called a cycle and its length is called the period. Using TRACE leads you to the fact that between 0 and  $2\pi$  the graph of  $f(x) = \sin(ax)$  has  $|b|$  complete cycles, in other words the period is  $\frac{2\pi}{|b|}$ .



**c. Horizontal shift -  $f(x) = \sin(x - a)$**



In this case the parameter causes a horizontal shift, called the phase shift.

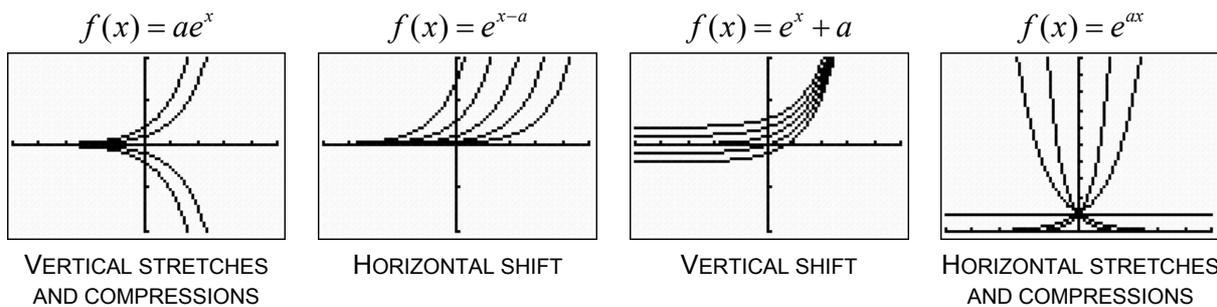
For  $f(x) = \sin(x - a)$  the phase shift is equal to  $a$  and you can work out that the phase shift for  $f(x) = \sin(ax - b)$  is equal to  $\frac{c}{|b|}$ .

**Activity 4**

Use Transformation Graphing to model data created by harmonic motion (2.2.4 and 2.2.5)

**2.5.4 Exponential functions**

The following screens show some transformations of the graph of  $f(x) = e^x$  generated with the list  $L1 = \{-2, -1, 0, 1, 2\}$  to show the effect of the changes of the parameter  $a$ .



**Activity 5 – The Eiffel Tower**



The Eiffel tower is designed according to an exponential model. Stress resistance calculations led Gustave Eiffel (wind, ground support forces etc.) to an exponential frame for the tower, which has become a veritable symbol.

The Eiffel Tower has a height of 300 metres between the ground at the center of the tower and the floor of the upper terrace slightly below the antenna. The floor of the last level is at a height of 280 metres and is approximately 10 metres wide.

The collected data below come from a scaled figure with a height of 15 cm, which is a good scale to work with. We shall define the coordinates of the left foot of the tower as (0,0). Since this value is not actually possible, given that we wish to find a curve that as much as possible resembles an exponential function, we shall only start the measurements from an offset point (5,10).

We shall consider the base of the tower to be a square with sides of approximately 115 m.

x	5	10	15	20	25	30	35	40	45	50	54
y	10	22	29	37	48	62	80	104	135	175	280

When modeling the data, certain coordinates may not fit. We shall adopt two attitudes: either suppress the point that is incompatible with the calculations or use a nearby point (at the chosen scale, a pencil line is at least 1/10 mm, which represents 0.2 m in life size).

### a. Towards an exponential function

We will store the data in the lists L1 and L2. This can be done from the home screen or in the STAT editor.

```
seq(I, I, 5, 55, 5)→
L1
(5 10 15 20 25 ...
(10, 22, 29, 37, 48,
62, 80, 104, 135, 17
5, 280)→L2
(10 22 29 37 48...
```

L1	L2	L3	#
30	62		
35	80		
40	104		
45	135		
50	175		
54	280		

L2(12) =

With these data we obtain the right-hand side of the tower. But we want to have two sides.

Five metres from the top, the width of the tower is 110 metres (while at the base, without counting the foundation bulges, it is about 115 metres). It is sufficient to carry out a reflection or axial symmetry in relation to the line  $x = 60$ .

```
115-L1→L3
(110 105 100 95...
```

L1	L2	#
5	10	-----
10	22	
15	29	
20	37	
25	48	
30	62	
35	80	

L3 = 115 - L1

without dynamic link<sup>3</sup>

L1	L2	#
5	10	-----
10	22	
15	29	
20	37	
25	48	
30	62	
35	80	

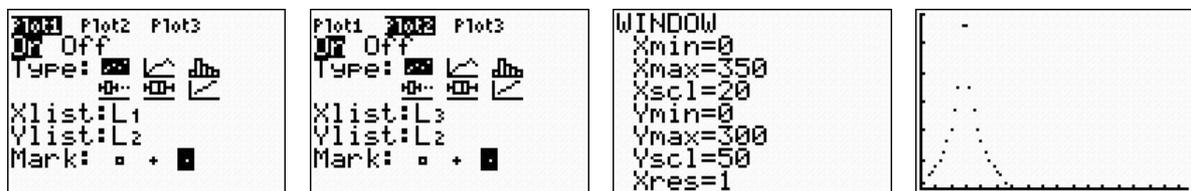
L3 = "115 - L1"

with dynamic link

L1	L2	L3	#
5	10	110	
10	22	105	
15	29	100	
20	37	95	
25	48	90	
30	62	85	
35	80	80	

L3(10) = 110

Scatter plots of these data result in the following plot.



Note that the ratios of consecutive ordinates (discounting 280 for which the abscissa is not constantly increasing) are “almost” equal. Why “almost” equal?

Because this study is experimental! And because the recorded measurements are approximate ... but there is more. Our eyes deceive us to see an exponential frame. In fact, the curve consists of segments of straight lines forming panels between the levels. This is clear from 0 to 50 metres, the floor of the first level.

<pre>L2(10)/L2(9) 1.296296296 L2(9)/L2(8) 1.298076923 L2(8)/L2(7) 1.298076923</pre>	<pre>1.298076923 L2(8)/L2(7) 1.3 L2(7)/L2(6) 1.290322581 L2(6)/L2(5) 1.291666667 L2(5)/L2(4)</pre>	<pre>1.297297297 L2(4)/L2(3) 1.275862069 L2(3)/L2(2) 1.318181818 L2(2)/L2(1) 2.2</pre>
---	--	--

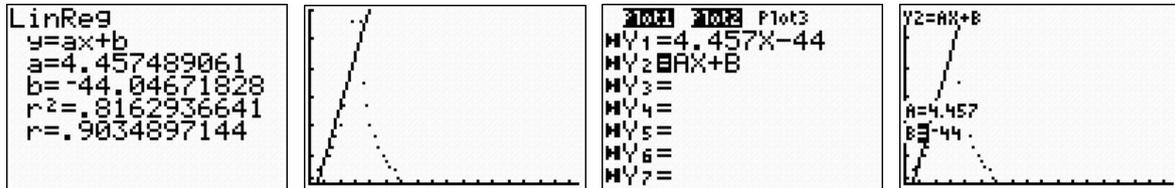
<sup>3</sup> When we write the formula between quotation marks, “115 - L1”, the cells of L3 are linked to those of L1, which means that if we change an element of L1, its image in L3 will be recalculated. If the formula is written without changes in L1 will not cause changes in L3.

We will now look for functions that best fit the dotted tower.

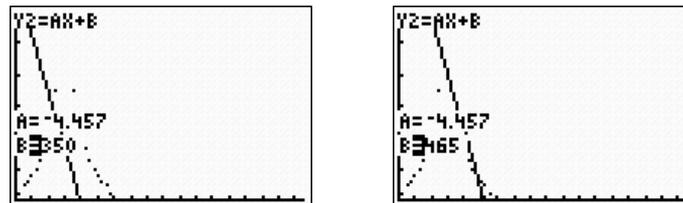
### b. Some model layouts

We will start with a linear model,  $y = ax + b$  for the left-hand side, a long distance view.

Define  $Y_1 = 4.457x - 44$  and use Transformation Graphing to attempt to find a straight line for the right-hand side. Start by assigning the values determined for  $Y_1$  to the parameters A and B of the function  $Y_2 = AX + B$ .



The function we are looking for needs to be decreasing, a line that goes downhill. Therefore the slope, A, of the line has to be negative. B has to be positive and as you see on the graph below more than 350. It is easy to see that A has to be equal to  $-4.457$  and some exploration with Transformation Graphing shows that for B we can accept a value between 450, too small, and 470, too high.



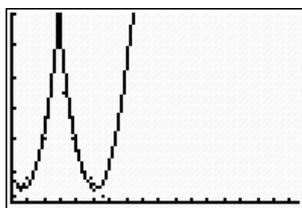
#### A LITTLE BIT OF MATH

The line  $Y_2 = a_2x + b_2$  is symmetrical to the line  $Y_1 = a_1x + b_1$  in relation to the line  $x = 5.75$ . Therefore the two slopes needs to be opposite:  $a_2 = -a_1$ .

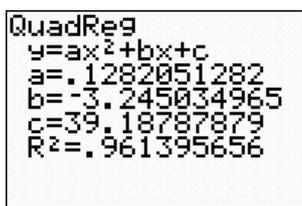


Because  $Y_2(57.5) = Y_1(57.5)$  ( $\Leftrightarrow 4.457 \times 57.5 - 44 = -4.457 \times 57.5 + b_2$ ) we get that  $b_2 \approx 468$ .

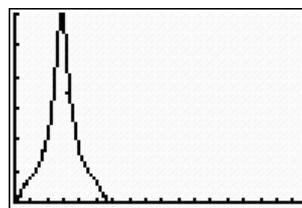
Three other attempts to model the Eiffel Tower. Only the regression results for the left-hand side are shown.



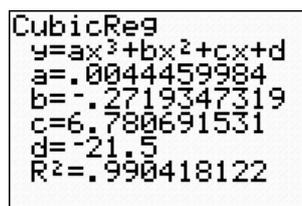
Quadratic model  
 $ax^2 + bx + c$



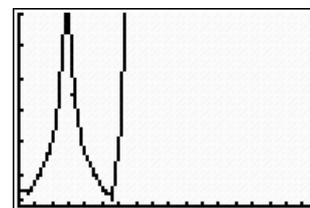
Good correlation coefficient



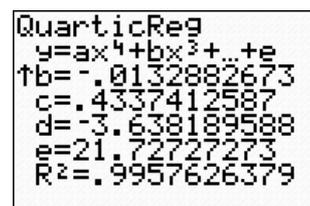
Cubic model  
 $ax^3 + bx^2 + cx + d$



Very good correlation coefficient



4<sup>th</sup> degree model  
 $ax^4 + bx^3 + cx^2 + dx + e$



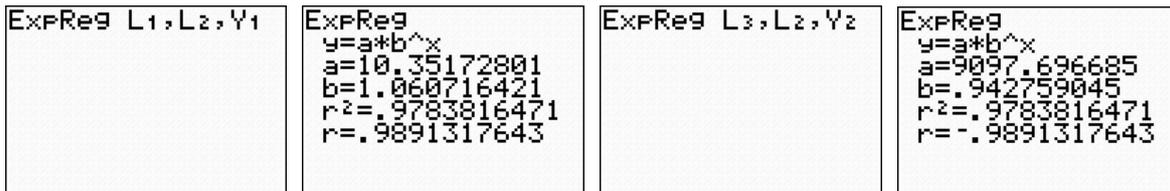
Very good correlation coefficient

Cox said, “All models are false, but some are useful.” The correlation coefficients above show that several models are suitable to model the Eiffel Tower, but we know that the model chosen by Gustave Eiffel is exponential.

### c. The exponential model

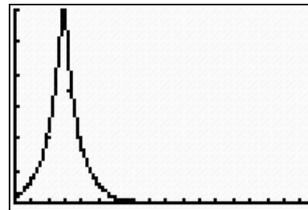
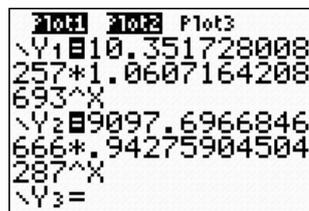
Why do we choose an exponential model?

Our eyes are trained to do this but pupils have to observe for themselves that if a function increases more and more (or decreases less and less) it describes probably an exponential an exponential process. In a situation of increasing less and less you should think about a logarithm function (chemistry).



The left-hand side

The right-hand side

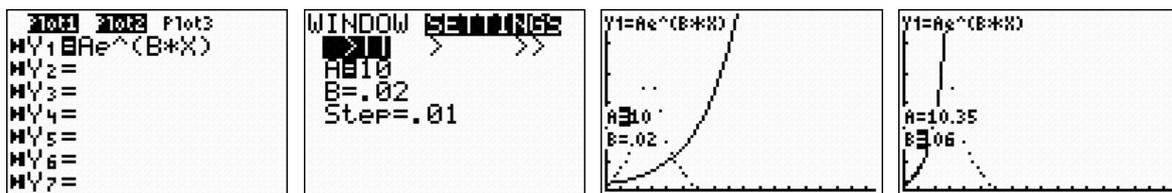


#### CAUTION

The accuracy has a very significant effect on its results. In some cases this influence is not important. But in case of a regression, fixing the decimal mode to one or two decimals can produce some strange results.

The function  $f(x) = 10.35 \times 1.06^x$  gives as a good exponential model for the left-hand side of the tower. This function can also be written in the following form  $f(x) = Ae^{Bx}$ .

We will use Transformation Graphing to determine the parameters  $A$  and  $B$ . We will start with the values  $A = 10$ ,  $B = 0.02$  and a step size of 0.01.



We can find the required values for  $A$  and  $B$  as follows algebraically.

- From the equation  $a \cdot b^x = a \cdot e^{x \ln b}$ ,
- We know that the graph has to pass through the points (25,48) and (54,280).

$$\text{This gives us the following system of equations: } \begin{cases} 48 = a \cdot e^{25b} \\ 280 = a \cdot e^{54b} \end{cases} \Leftrightarrow \begin{cases} a \approx 10.49 \\ b \approx 0.06 \end{cases}$$

#### d. Some decorations

Note that in reality, the bottom part (50 m) consists of straight-line segments. Therefore we will calculate, using regression, linear models for the following data.

x	5	10	15	20	25
y	10	22	29	37	48

$$y_1 = 1.82x + 1.9$$

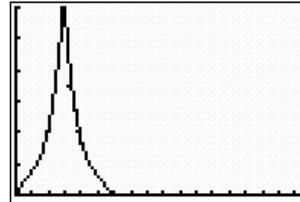
x	110	105	100	95	90
y	10	22	29	37	48

$$y_2 = -1.82x + 211$$

The left-hand (or right-hand) curve consists out of two parts: a straight segment followed by an exponential curve. The TI-84 Plus can plot these using piecewise functions.

```

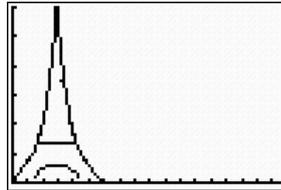
Plot1 Plot2 Plot3
\Y1(1.82X+1.9)*
(X<25)+(10.35*1.
06^X)*(25<=X)
\Y2(9097*.943^X
)*(X<95)+(-1.82X
+211)*(95<=X)
\Y3=
    
```



$Y_3$  defines the lower semicircle of the tower and  $Y_4$ <sup>4</sup> the floor of the first level. And add a little flag...

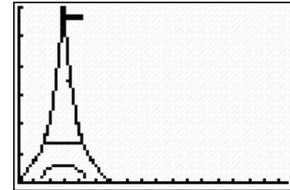
```

Plot1 Plot2 Plot3
)*(-1.82X
+211)*(95<=X)
\Y3(30^2-(X-57
.5)^2)
\Y4(30<=X)*70*(X
<=80)
\Y5=
    
```



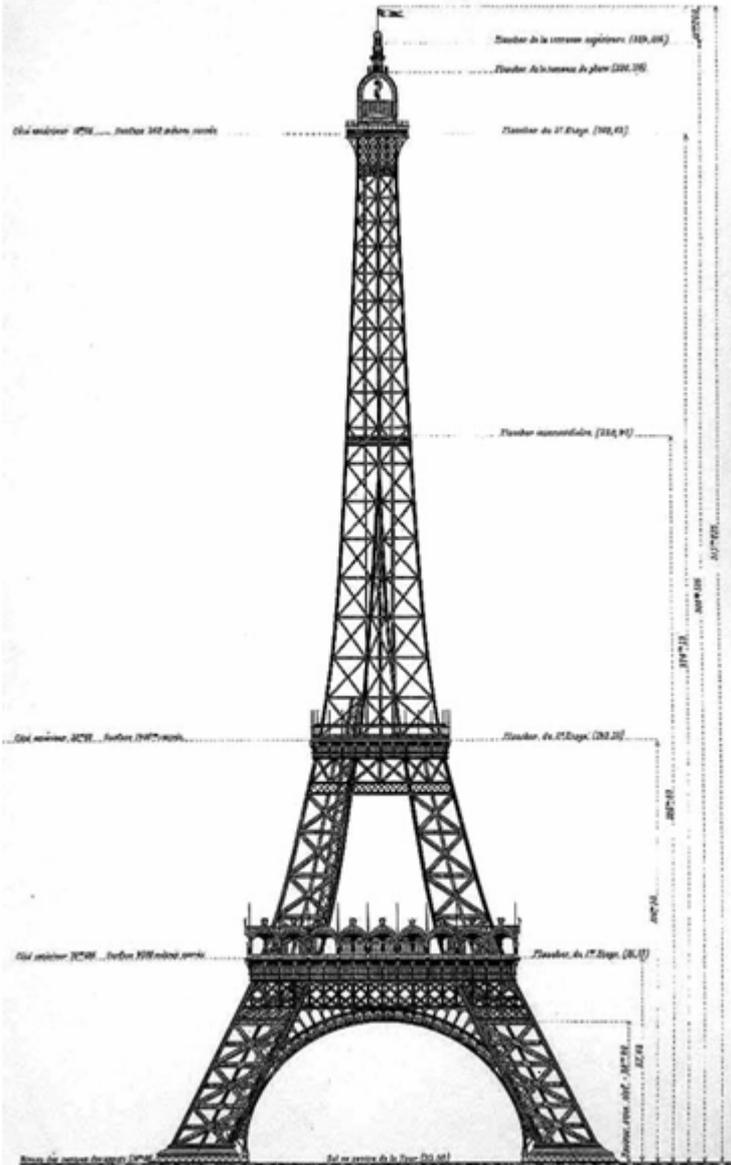
```

Plot1 Plot2 Plot3
\Y4(30<=X)*70*(X
<=80)
\Y5(58<=X)*280*(
X<=85)
\Y6(58<=X)*284*(
X<=85)
\Y7=
    
```



<sup>4</sup> Notice the choice of plot style for  $Y_4$ ,  $Y_5$  and  $Y_6$ .

AN ILLUSTRATION BY THE HAND OF GUSTAVE EIFFEL





### 3 Overview of applications

#### 3.1 Area Formulas

**CATEGORY**

Reference, Test, Practice

**DESCRIPTION**

Area Formulas reviews the definitions and the area formulas for the rectangle, square, parallelogram, triangle, trapezoid and circle.



**DIDACTICAL SUGGESTIONS**

Area Formulas explains the development of the formulas graphically by using animations and gives several examples of the calculation of the area for each shape. Additionally there is included a 15-question multiple-choice quiz providing practice at applying the formulas.

The application Area Formulas consists of two parts:

- Definitions and formulas – reference,
- Area quiz – test.

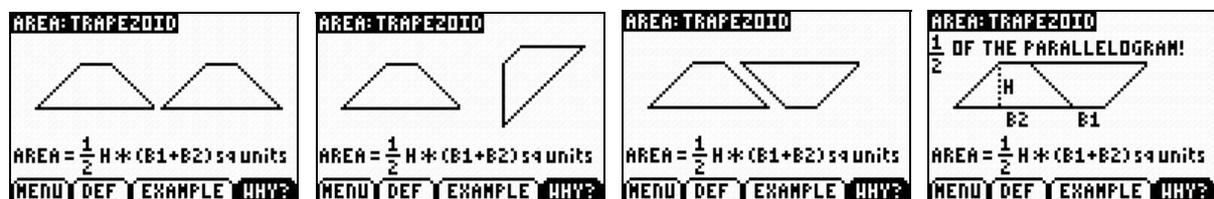
**a. Viewing definitions, formulas and examples**

To display information about shapes, select DEFINITIONS & FORMULAS from the SELECT A MODE menu. The SELECT A SHAPE menu will be displayed.

If you select 5: TRAPEZOID for example, you get information about the shape definition of a trapezoid. Use the function keys (F1, ..., F5) to return to the menu or to select AREA or EXAMPLE.

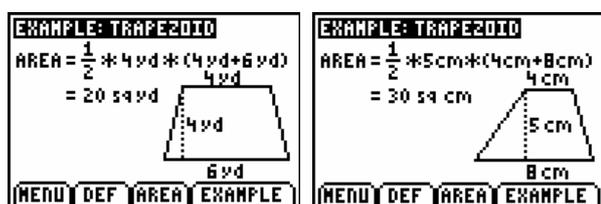


The WHY? option displays an explanation of the area formula.



Select EXAMPLE to display the calculation of the shape's area.

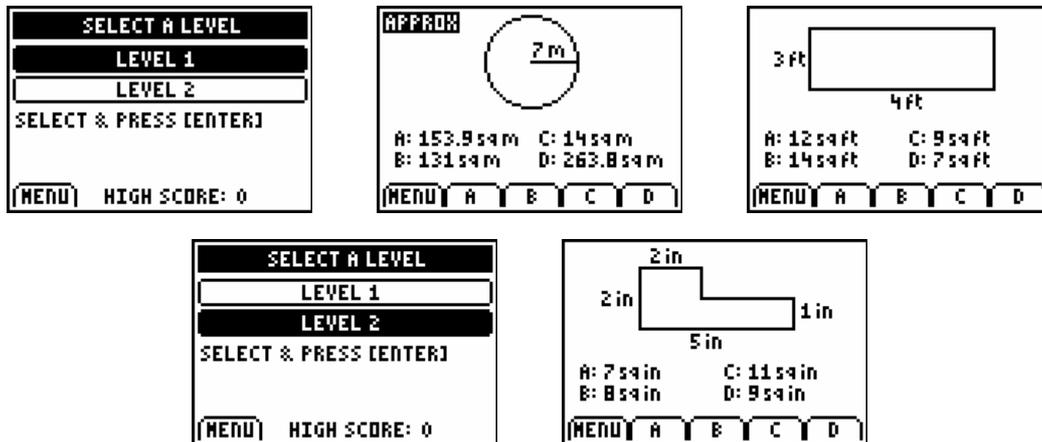
Select EXAMPLE a second time and you will get a different example for the same shape.



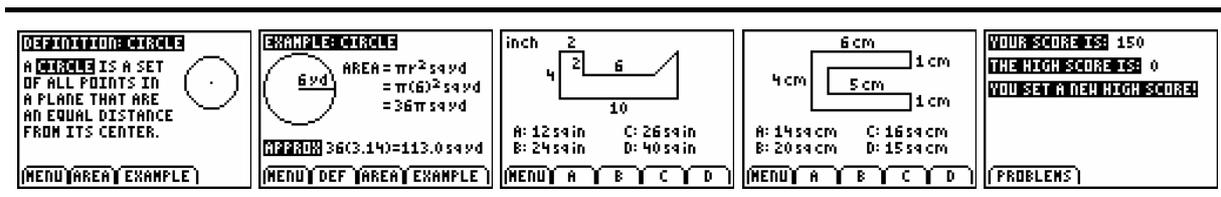
## b. Taking a quiz

The Area Formulas quiz contains two levels. Level one assesses the student's ability to compute the area of basic shapes. Level two is a bit more challenging, assessing the ability to compute the area of irregularly shaped objects.

Calculate the area and then select the letter that corresponds to your answer. Your answer will be checked and then the next problem will be displayed.



Each quiz consists out of 15 problems. Each correct answer in level one is worth 12 points and in level two 25 points. Incorrect answers are worth 0 points in both levels.



### POINT OF VIEW

Students like to deal with the 15 question multiple-choice quiz, which is included providing practice at applying the formulas. There are two levels of the quiz and high scores are saved. This is a very challenging feature for the students.

The application can be used for reviewing the definitions of special shapes, too, but only under certain circumstances. The development of the area formulas is shown by animation. Visualization and animation are very important didactical tools and students can see how to apply the area formula and why the formula is exactly this one and not another one. They can take part in the developing process in a very simple way by using this software. Experiences show that the students have to be made aware of this feature, because by themselves they only deal with the quiz.

At the end of the quiz, the student's score and the high score are displayed and that is very challenging for the students.

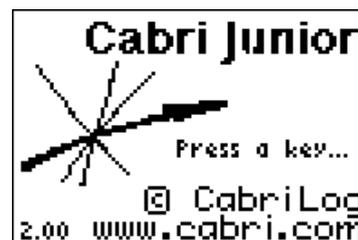
## 3.2 Cabri® Junior

### CATEGORY

Tool

### DESCRIPTION

This application performs analytic, transformational and Euclidean geometric functions and builds dynamical geometric constructions. It is possible to import and export figures to and from the TI-83/84 Plus and PC using Cabri Geometry™ II Plus.



### DIDACTICAL SUGGESTIONS

Cabri Junior enables students to explore geometric concepts that are difficult to express on paper. They can construct geometric objects and dynamically explore geometric properties, measurements and transformations.

With Cabri Junior, the Cabri version for the TI-83/84 Plus it is possible to:

- draw points, lines, segments, circles, triangles, and quadrilaterals,
- construct perpendiculars, parallel lines, perpendicular and angle bisectors and loci,
- carry out transformations like translations, reflections, rotations and dilations,
- compute lengths, perimeters, areas and angle values,
- display coordinates and equations (of lines and circles).

All the commands to do this are stored in menus which can be activated through the function keys F1 to F5. It's not necessary to press the ALPHA key first. A menu remains active until another will be selected or until you press CLEAR.

To navigate in a menu use the  $\blacktriangledown$   $\blacktriangle$  keys. Once a menu is activated use the  $\blacktriangleleft$   $\blacktriangleright$  keys to navigate through the menu. Use  $\blacktriangleright$  to open a submenu of a menu. To select a command first highlight it and then press ENTER. It is also possible to press the corresponding number although they are not shown on the screen.

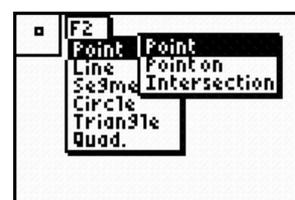
### F1: Tools

With this menu you can create, open and save Cabri Junior files. It also contains the Undo command and the Explore command to step through how a figure was made. Animate lets you set figures in motion and during an animation in Cabri Junior it is possible to change the independent objects.



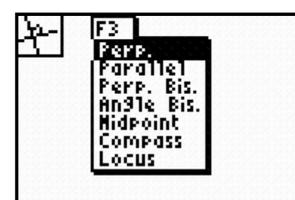
### F2: Figures

The drawing of lines, segments as well as circles is based on two points. For a circle this means a center point and a radius point. To start drawing put the cursor on the desired place and press ENTER to define the first point. Then replace the cursor to finalize the figure.



### F3: Constructions

Cabri Junior shows the objects that have to be constructed before you finally put them in the figure by pressing ENTER. The commands Midpoint and Perp. Bis. can be used for two points and for a segment. And with Compass you can create a circle with a radius equal to the distance between two points or the length of a segment.



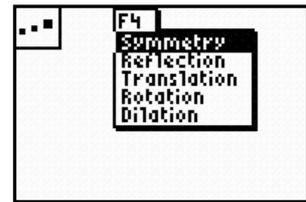
#### **F4: Transformations**

Symmetry rotates an object 180° with respect to a point and Reflection gives the mirror image across a line or segment.

Translation uses a segment to translate. The direction is defined by the beginning and end point of the segment.

And Rotation and DILATION use a point and a number (F5).

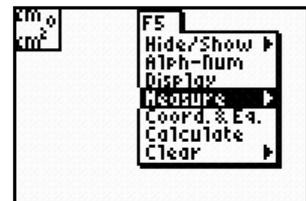
To do a transformation first select the object you want to transform.



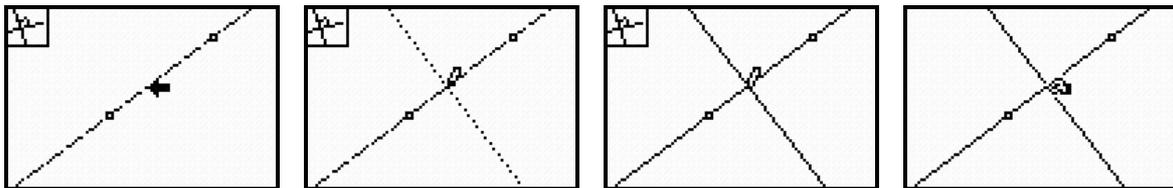
#### **F5: Measure & Display**

This menu allows the user to:

- hide and show objects,
- enter comments and numbers (Alph-Num),
- change the format of objects (Display),
- measure distance, length, area, angles and slopes,
- display coordinates and equations of lines and circles,
- do calculations using the numbers on the screen,
- delete objects (Clear).

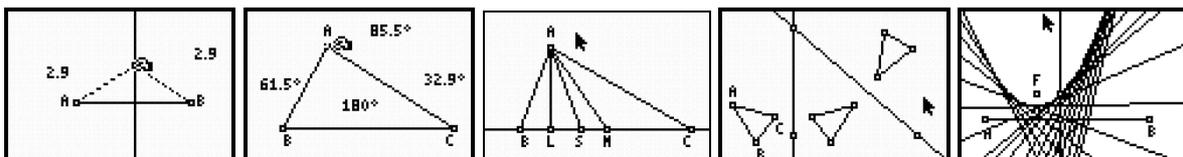


During the drawing of figures and making of constructions it is useful to look carefully at the shape of the cursor. The messages that appear in the computer version of Cabri are hidden in the lay out of the cursor. Each command has its own icon, shown in the upper left corner of the screen. Only when the pointer is active, no icon is shown.



Some extra functionality to end with:

- |           |  |
|-----------|--|
| CLEAR     | Quit a menu or activate the pointer.<br>3 x CLEAR = clear everything on the screen                     |
| DEL       | Clear objects. If DEL doesn't work, use F5 : CLEAR.  |
| 2nd       | Switch between choices in dialogue windows.<br>In version 2.0 it is also possible to use the ◀ ▶ keys. |
| ALPHA     | Drag and drop, the pointer has to be active.   |
| 2nd[QUIT] | Quit Cabri Junior and go to the home screen.   |



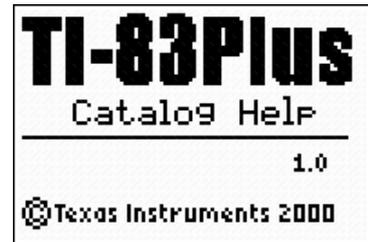
#### **POINT OF VIEW**

Although Cabri Junior has less functionality than the computer version it is a nice tool to let the students discover geometric properties in a daily class situation without going to a computer lab. The teacher can prepare basic figures on his computer (Cabri Geometry II Plus) to start a student's activity on the students TI-83/84 Plus.

### 3.3 Catalog Help

**CATEGORY**  
Help, Reference

**DESCRIPTION**  
Catalog Help is a reference for all commands and functions that are available through the catalog menu and the function menus.

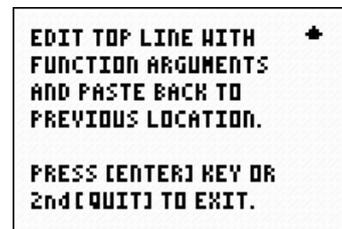
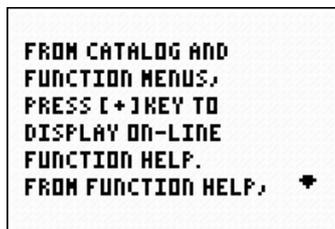


**DIDACTICAL SUGGESTIONS**

Although Catalog Help is primarily a reference tool, it may be used in didactical settings to have students explore the meaning of certain commands and functions. Another interesting didactical possibility is to have students explore what arguments are needed for certain functions and commands. Students can be asked to predict the arguments and to check their predictions with the application.

The Catalog Help application can be used as follows:

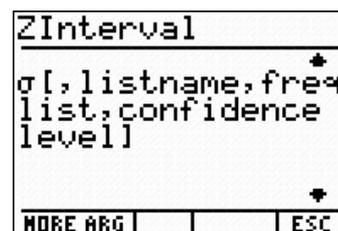
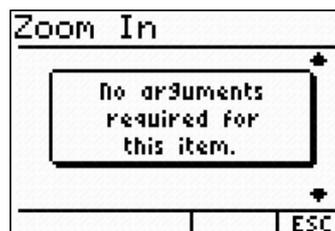
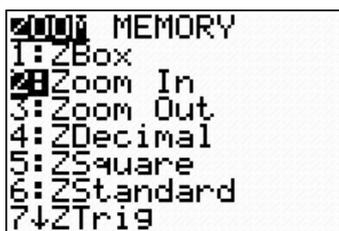
- From the catalog and function menus, you can use the [+] key to get help on the function or command. There is not always help available, so sometimes you may not get any help.
- When a function has arguments that are given in the help, you can edit these arguments and paste them back in the right location.



**Example 1**

Pressing [ZOOM] provides a menu for different zoom options on a graph. Choosing 2: Zoom In and then pressing [+], gives a text that says that no arguments are needed.

With the up and down arrow keys on the calculator, you can *walk* through the catalog (2nd[CATALOG]), and get the help for all the commands and functions. Press [Z] to go to the first function that starts with z. Then use the arrow down to go from ZBox to ZInterval to get help for the command Zinterval by pressing [+].



## Example 2

After 2nd MATRIX you get the menu for matrix settings and calculations. After choosing 3:dim( from the MATH menu and pressing [+], you get the help about the needed arguments for the dimension function.

- Choose MORE ARG to get the option for setting the dimensions sizes and the name of the matrix.
- Next you can edit the arguments of this function on the top line and after [ENTER], this command will appear on the home screen.



### POINT OF VIEW

This APP is very helpful to get a quick reminder of what the various commands and functions are about and more specifically what arguments in what syntax are needed. For students, Catalog Help is an easy reference card that can be used to encourage them to investigate various functions by exploring the syntax of the arguments.

### 3.4 CellSheet™

**CATEGORY**

TOOL



**DESCRIPTION**

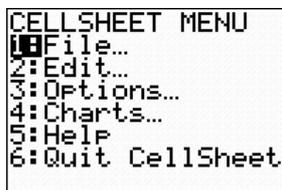
CellSheet combines spreadsheet functionality with the power of the TI-83/84 Plus. CellSheet can be useful in classes other than math, such as social studies, business, and science.

**DIDACTICAL SUGGESTIONS**

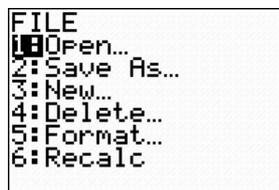
CellSheet can be used as a tool to perform spreadsheet calculations with data collected by a real data collection device, and also as a means to help develop conceptual understanding.

- Cells can contain:
- Integers
  - Real numbers
  - Formulas
  - Variables
  - Text and numeric strings
  - Functions

Each spreadsheet contains 999 rows and 26 columns. The amount of data you can enter is limited only by the available RAM on your TI-83/84 Plus. With the following screenshots we give a brief overview of the possibilities of CellSheet.



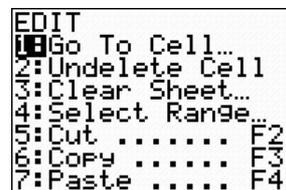
MAIN MENU



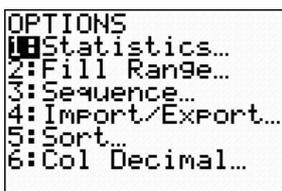
FILE MENU



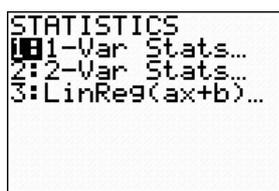
VARIOUS FORMATS



EDIT COMMANDS



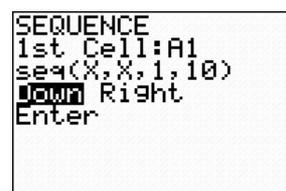
OPTIONS MENU



STATISTICS



RANGES



SEQUENCES

### Example 1

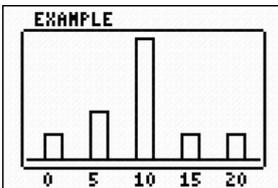
We use a simple chart with data on the number of points (column A) people scored on a quiz. Column B represents the frequency of each score. We show a bar graph and a pie graph.

GPD	A	B	C
1	0	2	
2	5	4	
3	10	10	
4	15	2	
5	20	2	
6			

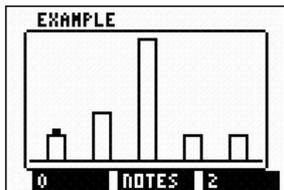
CHARTS  
 1: Scatter...  
 2: Scatter Window  
 3: Line...  
 4: Line Window...  
 5: Bar...  
 6: Bar Window...  
 7: Pie...

BAR CHART  
 Categories: A1:A5  
 Series1: B1:B5  
 Ser1Name: NOTES  
 Series2:  
 Ser2Name:  
 ↓

EXAMPLE

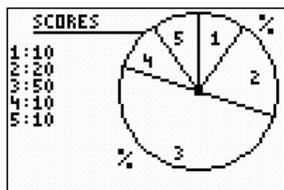


EXAMPLE

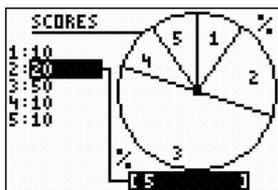


PIE CHART  
 Categories: A1:A5  
 Series: B1:B5  
 Number Percent  
 Title: SCORES  
 Draw

SCORES



SCORES



### Example 2

Suppose we can describe a falling object with the function  $x(t) = 5t^2$ . We will approximate the instantaneous velocity  $v(t) = \frac{dx(t)}{dt}$  by the average velocity  $v_{av} = \frac{x(t+h) - x(t)}{h}$  at  $t = 5$ .

First we define the function variable  $Y1 = 5X^2$ .

VEL	A	B	C
1	H		
2	.1		
3	.01		
4	.001		
5	1E-4		
6	1E-5		

A1: "H" [Menu]

VEL	A	B	C
1	H	XCT+H)	
2	.1	130.05	
3	.01		
4	.001		
5	1E-4		
6	1E-5		

B2: =Y1(C+A2) [Menu]

VEL	A	B	C
1	H	XCT+H)	
2	.1	130.05	
3	.01		
4	.001		
5	1E-4		
6	1E-5		

[Range] [Paste] [Menu]

VEL	A	B	C
1	H	XCT+H)	
2	.1	130.05	
3	.01		
4	.001		
5	1E-4		
6	1E-5		

B3:B6 [Paste] [Menu]

Select B2 and Copy (F3)      Select B3 and select Range B3:B6 (F1+▼)

VEL	A	B	C
1	H	XCT+H)	
2	.1	130.05	
3	.01	125.5	
4	.001	125.05	
5	1E-4	125.01	
6	1E-5	125	

B6: =Y1(C+A6) [Menu]

Paste (F4)

VEL	A	B	C
1	H	XCT+H)	
2	.1	130.05	50.5
3	.01	125.5	
4	.001	125.05	
5	1E-4	125.01	
6	1E-5	125	

C2: =(B2-Y1(C))/A2 [Menu]

Copy formula in C2 to range C3:C6

VEL	A	B	C
1	H	XCT+H)	
2	.1	130.05	50.5
3	.01	125.5	50.05
4	.001	125.05	50.005
5	1E-4	125.01	50.001
6	1E-5	125	50

C6: =(B6-Y1(C))/A6 [Menu]

### POINT OF VIEW

CellSheet is a simple spreadsheet application for the graphing calculator. Because of the small screen and the sometimes time consuming way to enter information and scroll over the spreadsheet it is not the most user friendly application. When a computer is at hand, a spreadsheet program like Excel is much easier.

However, when no computer is available (e.g. in a "normal" classroom or on a field trip with data collection devices) CellSheet can be very helpful. For relatively small spreadsheets and quick calculations, the application can be used as well. And to present the data you can still use Excel because a free CellSheet converter to Excel and vice versa is available.

### 3.5 Conic Graphing

**CATEGORY**

Reference



**DESCRIPTION**

Conic Graphing can be used to graph the four basic conic sections. The conic equations can be in function, parametric and polar forms. Conic Graphing does not address degenerate cases of conic sections.

**DIDACTICAL SUGGESTIONS**

This reference allows students to graph or trace circles, ellipses, hyperbolas and parabolas and solve for the conic's characteristics. They can learn about the dependence and causal relationships between equations, parameters and graphs and find out characteristics of conic sections.

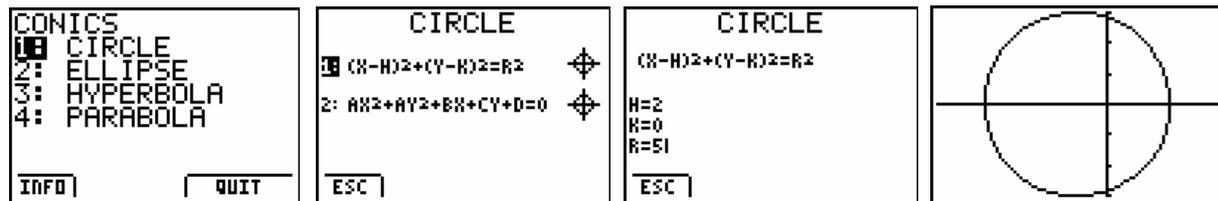
You can graph conic sections in function, parametric, or polar mode, based on your requirements. You can change the CONIC SETTINGS pressing the MODE-key while the application is running. There you can choose the TYPE of the Conic and the WINDOW SETTINGS.



Conic Graphing restores the original mode of the calculator (as before you started the application) when you exit the application.

Select from the four conic types, from the main menu, 1: CIRCLE.

When you graph a conic section, you select first the type of the equation and then you input values for H, K and R of the circle equation  $(X - H)^2 + (Y - K)^2 = R^2$ . Press [GRAPH] to plot the circle and [Y=] to go back to the definition screen.

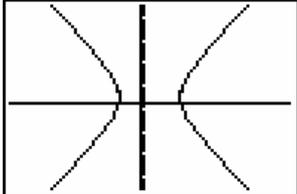


An *ellipse* is the set of points in a plane whose distances from two fixed points in the plane have a constant sum. The two fixed points are called the foci of the ellipse. Press ALPHA [SOLVE] when the definition screen is active to calculate the center and the foci.



A *hyperbola* is the set of points in a plane whose distances from two fixed points in the plane have a constant difference. The two fixed points are called the foci of the hyperbola and the line through the foci of the hyperbola the focal axis. The point on the axis halfway between the foci is the hyperbola's center. The points where the focal axis and hyperbola cross are the vertices.

Again ALPHA [ SOLVE ] you will get the center and the two foci as well as the two vertexes and the slope. By this option the student can find out characteristics of each conic.

<p>HYPERBOLA</p> $1: \frac{(X-H)^2}{A^2} - \frac{(Y-K)^2}{B^2} = 1$ $2: \frac{(Y-K)^2}{A^2} - \frac{(X-H)^2}{B^2} = 1$ <p>ESC</p>		<p>HYPERBOLA</p> <p>CENTER C=(3,2)</p> <p>VERTEX V1=(-6,2)</p> <p>VERTEX V2=(12,2)</p> <p>FOCUS F1=(-9.728,2)</p> <p>FOCUS F2=(15.728,2)</p> <p>SLOPE S= +-1</p> <p>ESC</p>
---	---	---

A set that consists of all the points in a plane equidistant from a given fixed point and a given fixed line in the plane is a *parabola*. The fixed point is the focus of the parabola and the fixed line the directrix. The point where the focal axis intersects the parabola is the vertex.

<p>PARABOLA</p> $1: (Y-K)^2 = 4P(X-H)$ $2: (X-H)^2 = 4P(Y-K)$ <p>ESC</p>		<p>PARABOLA</p> <p>VERTEX V=(0,0)</p> <p>FOCUS F=(2,0)</p> <p>DIRECTRIX X=-2</p> <p>ESC</p>
--	--	---

When the graph is displayed, press TRACE and move around on the curve to find the vertex graphically.

---

### POINT OF VIEW

This application is by itself only a reference and therefore not very challenging for the students, because they only can graph or trace conic functions and they cannot see at first sight the implemented didactical value for learning about conic functions. But teachers can transform this simple reference into a very useful learning material with the help of additional exercise sheets, where the students are told, what they should explore. The teacher has to make the student aware of the functionality the author of the software implemented in it.

## 3.6 EasyData™

### CATEGORY

TOOL

### DESCRIPTION

EasyData lets you collect, view, and analyze real-world data on the TI-83/84 Plus (SE) graphing calculators using Vernier USB sensors (only the TI-84 family) and through data collection devices, such as Texas Instruments CBR 2™ motion detector, CBL 2™ System, or the Vernier LabPro®.



### DIDACTICAL SUGGESTIONS

EasyData can be used as a tool to simplify data collection, and also as a means to help develop conceptual understanding.

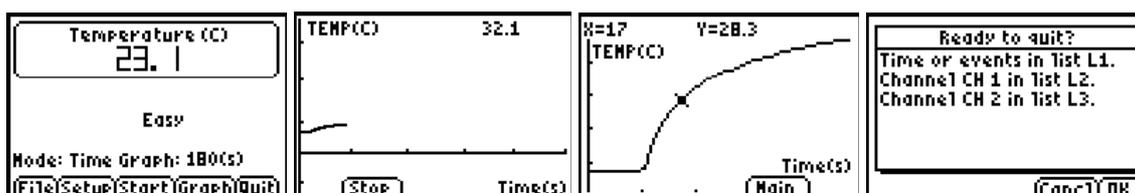
On a TI-84 Plus with operating system version 2.3 or later you can simply connect the Vernier USB sensors. A TI-83 Plus requires a separate data collection device, such as a CBL 2 system.

The major data collection tools to use with EasyData are:

- EasyTemp – Temperature probe
- EasyLink – USB interface to connect Vernier sensors to the TI-84 Plus (SE)
- CBR 2 – USB motion detector

When the calculator detects the data collection device, EasyData opens automatically and starts a default experiment appropriate for that device.

For example, we use the temperature sensor. Below left is the screen that is shown when the temperature sensor is plugged in.

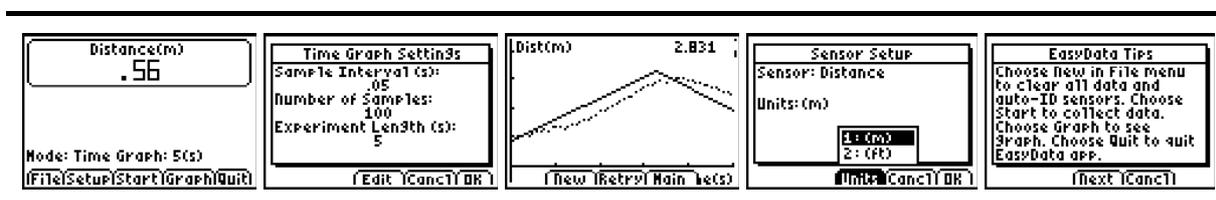


To collect data (with the EasyTemp) you can do the following steps:

- Select **Start** to start collecting data, and then wait five seconds.
- Hold the EasyTemp sensor for about 30 seconds.  
The graph shows the temperature as it changes.
- Select **Stop** to stop collecting data.  
EasyData displays a scaled graph of the sampled temperatures.
- Press **▶** repeatedly to scroll the cursor to the right, and note the temperature at each data point.
- When you finish exploring the graph, select **Main** to return to the EasyData main screen.
- Select **Quit**. A message indicates that the collected data has been stored in calculator lists.

When you quit EasyData, a message reminds you which calculator lists contain the collected data. You can then examine the data using your calculator or a computer.

- On your calculator, you can explore the data by viewing the data in the list editor. (On the TI-843/84 Plus, press [STAT] and then select Edit.)
- You can also perform statistical analysis (such as calculating mean, median, and standard deviation) on the data.
- By using TI Connect™ computer software and an appropriate TI Connectivity cable, you can copy the data to a computer and import the data into other software tools such as:
  - Spreadsheet software to analyze the data,
  - TI InterActive!™ for formal presentations.



**POINT OF VIEW**

**EasyData** is a wonderful application that makes collecting and analyzing real data much easier. Because of the simple way to connect the data collection devices to the calculator – especially on the TI-84 where you do not need a CBL 2 or other intermediate device any more – students can go any where (no extension cord needed!) to collect data and bring them back to school or home. Data can be analyzed with a graphing calculator, but with a link cable and TI Connect it is really easy to send the data to a computer for further analysis.

With EasyData the connection between science and mathematics is very natural. Also the didactical opportunities of a graphing calculator with EasyData contribute to thorough understanding of the underlying concepts.

## 3.7 Finance

### CATEGORY

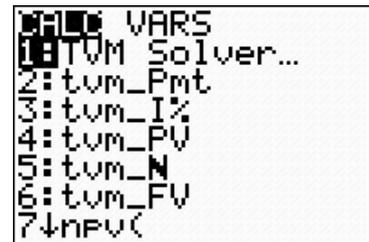
Tool and quick reference guide

### DESCRIPTION

The Finance application calculates the most common financial functions and values.

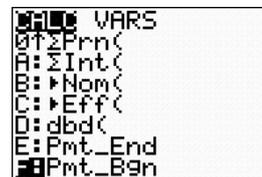
### DIDACTICAL SUGGESTIONS

This APP can contribute to the conceptual development of financial calculations and financial models, and can be used to better understand the concept underlying financial math.

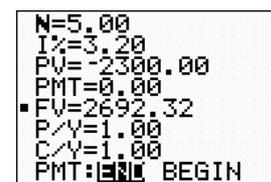


We recommend to set the calculator's display mode to two decimal places.

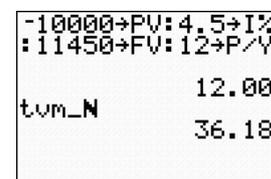
The FINANCE application consists of the following menus:



With the TVM Solver you can quickly solve 5 types of financial calculations in an easy and user-friendly way. For example, an amount of € 2300 is put in the bank at a compound interest rate of 3.2 % per year. What is the value after 5 years?

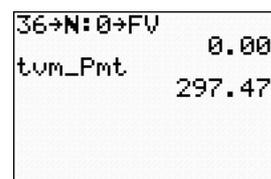


With the TVM Solver, the numbers can be entered as shown in the screen shot on the right with FV empty. Then put the cursor on the line with FV and press ALPHA [ENTER].



For the functions tvn\_N, tvn\_I%, tvn\_PV, tvn\_Pmt and tvn\_FV it is easier and more practical to use the TVM Solver.

The screen shots illustrate this for a loan of € 10,000 at a yearly interest rate of 4.5 %. How many monthly payments are needed to eventually pay back a total of about € 11,450?



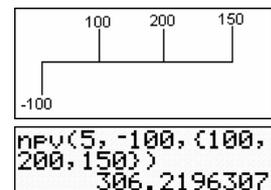
About 36 payments of € 297.47 are needed.

Please note that each function requires the values of other functions as parameters.

Two cash flow transaction:

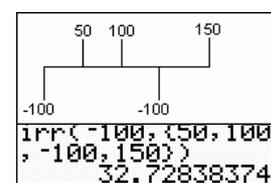
- npv (

Over a period of time with a constant rate of inflation of 5%, payments (or withdrawals) are made at different periods of the same length. The illustration on the right shows the cash flow. "What will be final present value?"



- irr (

An investment will be approved under the condition that the return will be bigger than 25%. The cash flow is -100, 50, 100, -100, 150. The answer 32.728... is bigger than 25%, and thus the investment can be approved.



In case of interest calculations using the proportional method,  $\blacktriangleright$ Nom ( and  $\blacktriangleright$ Eff ( can be used to convert an annual effective interest rate to a nominal rate and vice-versa.

$\blacktriangleright$ Nom( calculates the nominal interest rate.

For a revolving credit, the real rate is 18.86 % per year. Interest is accrued each day. What should be the declared nominal interest rate for this loan?

```

▶Nom(18.86,365)
      17.28
▶Eff(18.86,365)
      20.75
    
```

$\blacktriangleright$ Eff( calculates the effective interest rate

For a revolving credit, a “general effective rate” of 18.86 % per year is given Interest is charged each day. What is the real (effective) rate of interest for this loan?

The dbd ( function calculates the number of days between two dates (years between 1950 and 2049). Two formats are possible: MM.ddyy (US-format) or ddMM.yy (Europe). Two examples:

Number of days between 14/02/2004 and 31/12/2004 (leap year),  
 Number of days between 14/02/2006 and 31/12/2006 (normal year).

```

dbd(1402.04,3112
.04)
      321.00
dbd(1402.06,3112
.06)
      320.00
    
```

And how to calculate the payments of a loan?

Bal ( = outstanding (remaining) amount, after a certain number of payments.

$\Sigma$ Prn( = sum of the payments that have been paid between two periods  $p_i$  and  $p_j$ ,  $0 \leq i < j \leq n$ .

$\Sigma$ Int( = sum of the interest paid between two periods  $p_i$  and  $p_j$ ,  $0 \leq i < j \leq n$ .

Please note that also here you must first enter the parameters N, I%, PV, PMT and FV.

For a loan of € 4,000, with an annual interest rate of 3.9% and 24 monthly installments, what is the outstanding (remaining) amount to pay it back after 14 payments?

What have you paid so far and what is the interest that has been paid after 14 payments?

```

24▶N:3.9▶I%:4000
▶PV:-173.52▶PMT:
0▶FV
      0.00
    
```

```

bal(14)
      0.00
      1704.61
ΣPrn(1,13)
      -2127.95
ΣInt(1,13)
      -127.81
    
```

The functions that are direct available in Tvm Solver:		The variable TVM:	
Tvm_Pmt	Computes the amount of each payment.	N	Total number of payment periods
Tvm_I%	Computes the interest rate per year.	I%	Annual interest rate
Tvm_PV	Computes the present value.	PV	Present value
Tvm_N	Computes the number of payment periods.	PMT	Payment amount
Tvm_FV	Computes the future value.	FV	Future value
		P/Y	Number of payment periods per year
		C/Y	Number of compounding periods/year
Other financial functions:			
Npv(	Computes the net present value.	$\blacktriangleright$ Nom	Computes the nominal interest rate.
Irr(	Computes the internal rate of return.	$\blacktriangleright$ Eff(	Computes the effective interest rate.
Bal(	Computes the amortization sched. balance.	dbd(	Calculates the days between two dates.
$\Sigma$ Prn(	Computes the amort. sched. princ. sum.	Pmt_End	Selects ordinary annuity (end of period).
$\Sigma$ Int(	Computes the amort. sched. interest sum.	Pmt_Bgn	Selects annuity due (beginning of period).

### POINT OF VIEW

The Finance application provides the opportunity to quickly perform common financial calculations. In actual education, it is a helpful application that takes away the cumbersome calculations from the students. It is an application of how to use the geometrical progression and it naturally introduces spreadsheets.

## 3.8 Guess My Coefficients

### CATEGORY

Test, Practice

### DESCRIPTION

Guess My Coefficients provides a review of the graphs and equations of linear, quadratics and absolute value functions in a challenging game setting, where the user can set the number of problems in a game up to 99.



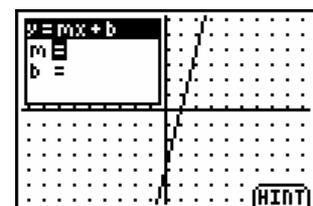
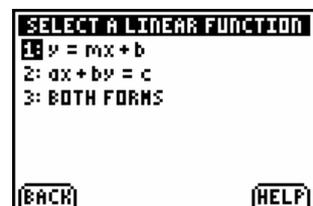
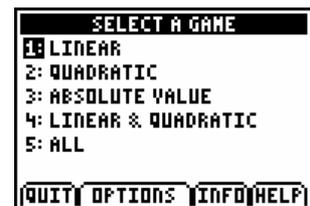
### DIDACTICAL SUGGESTIONS

Students can explore relationships between symbolic expressions and graphs and can get familiarized with various forms of linear, quadratic, and absolute value functions. By viewing the graph of the function they have to determine the coefficients and constants for this function eventually with the help of a HINT option, which shows points on the graph and lets them trace the graph.

First the function type has to be selected from the SELECT A GAME screen. And second the form of the equation of the function.

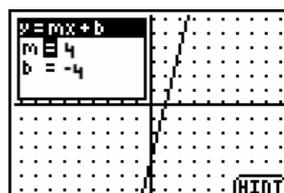
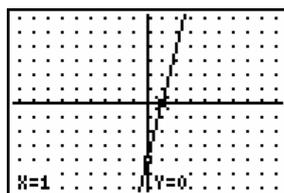
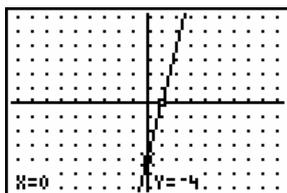
The following function forms are available:

- Linear
  - $y = mx + b$
  - $ax + by = c$
- Quadratic
  - $y = ax^2 + k$   $y = a \cdot x^2 + k$
  - $y = a(x - h)^2 + k$
  - $y = a(x - r)(x - s)$
- Absolute Value
  - $y = a|x| + k$
  - $y = a|x - h| + k$

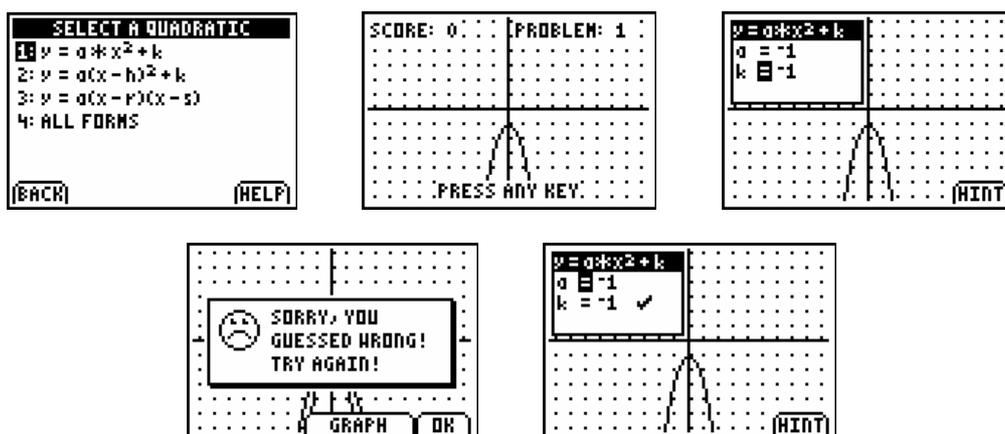


The game can also be played with multiple function types instead of just one type. In this case ALL has to be selected from the SELECT A GAME screen.

If you are not sure about the values of a graph, you can obtain a HINT, which lets you trace the graph.

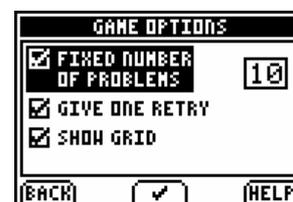


An example of a quadratic one.

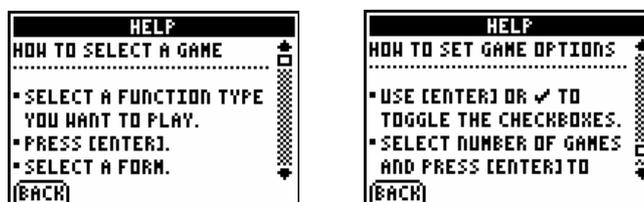


The score is computed based on whether the input is the correct answer on the first or second try and whether a hint is used to help to determine the correct answer. When you reach a high score, you can enter your initials in the high score table. The initials displayed are AAA until you change them.

You can set three different options for the games. The options that you select apply to every function type (game) on the SELECT A GAME screen. The game consists of the defined number of problems displayed on the right side of the screen, no matter how many incorrect answers you give. When this option is not selected, you play the game until you have given three incorrect answers.



Help is available for each game from the SELECT A GAME and OPTIONS screens, and from each SELECT A FUNCTION screen.



### POINT OF VIEW

At first sight and without additional learning materials this nice application can only be used for testing and practicing. Students like to work on it, because of the challenging game character of this application. Therefore it can be assumed to be very useful for the learning process of the students. If the teacher embeds the Guess My Coefficients application in a learning environment with additional exercise sheets, students can explore function problems with the help of this application. They can describe patterns and develop rules and even algebraic expressions and explain their conclusions.

### 3.9 Inequality Graphing

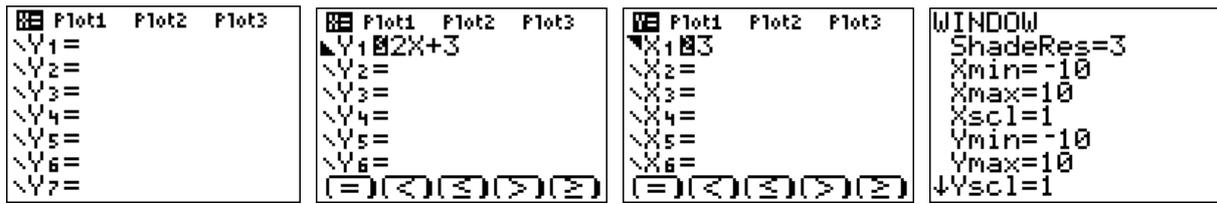
**CATEGORY**  
Graphing mode

**DESCRIPTION**  
Inequality Graphing enables the user to enter inequalities using symbols, even inequalities involving vertical lines in an X= editor. It is possible to plot the inequalities, including union and intersection shades, and to store the intersection points between the corresponding functions.

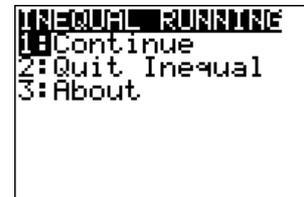


**DIDACTICAL SUGGESTIONS**  
With this application it is possible to add very easily a graphical approach to solving systems of linear equations (two variables) and linear programming.

Inequality Graphing is an application that once it is started it keeps running in the background. It changes the Y= window as follows and adds an X= editor to it. It also adds a shade resolution item (ShadeRes) to the WINDOW settings.



To quit Inequality Graphing you need to activate it again in the APPS menu and then select 2: Quit Inequal. Note that it is not possible to run Inequality Graphing and Transformation Graphing (3.14) at the same time.



The following two examples will show how Inequality Graphing works.

**Example 1**

We will determine the region of points  $(x, y)$  that satisfy:

$$\begin{cases} x + 2y \leq 4 \\ x + 4y \leq 6 \end{cases} \text{ and } \begin{cases} x \geq 0 \\ y \geq 0 \end{cases}$$

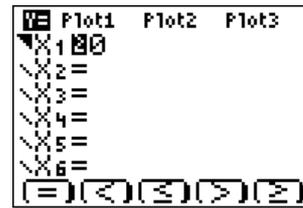
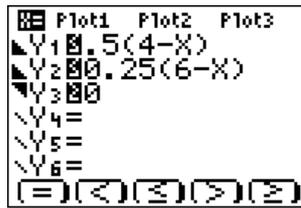
Therefore we define the linear functions  $Y1=0.5(4-X)$ ,  $Y2=0.25(6-X)$ ,  $Y3=0$ ,  $X1=0$  and plot them with the following WINDOW-settings (press TRACE CLEAR to remove the menu at the bottom of the screen).



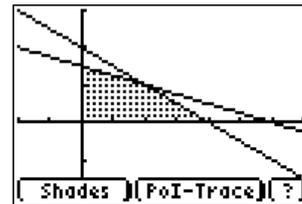
All the points in the enclosed area are solutions to our problem. It is possible to shade this area and to calculate its vertices.

To shade this area put the cursor on the equality signs to change them as follows into inequalities:

- [ALPHA] F1 → =
- [ALPHA] F2 → <
- [ALPHA] F3 → ≤
- [ALPHA] F4 → ≥
- [ALPHA] F5 → >

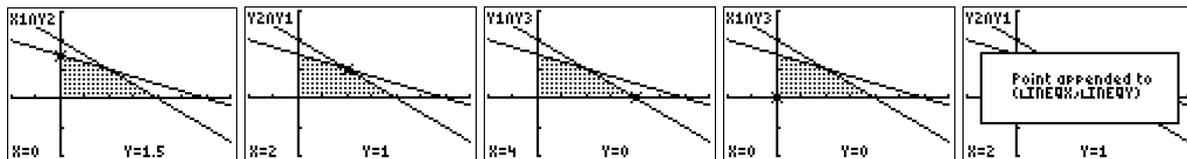


Press GRAPH, select Shades ([ALPHA] F1) and 1: Ineq Intersection.



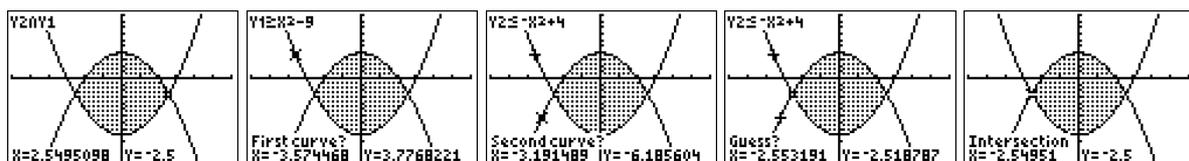
For linear inequalities it is possible to calculate the vertices the shaded area with PoI-Trace ([ALPHA] F3): ◀ ▶ = change the second function & ▲ ▼ = change the first function.

You can store a selected vertex by pressing STO ▶. The coordinates of the vertex will automatically be stored in the lists INEQX and INEQY.



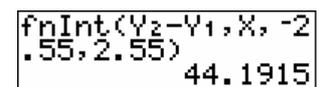
### Example 2

Let's try to find the area between the functions  $f(x) = x^2 - 9$  and  $g(x) = -x^2 + 4$ . For non linear functions it is not always possible to find the intersection points through Inequality Graphing. In such a case we need to use 5: intersect of the graphical CALC menu.



To approximate the area we can use the fnInt command. The calculations above are also numerical approximations of the intersection points  $x_1 = -\sqrt{\frac{13}{2}} \approx -2.55$  and  $x_2 = \sqrt{\frac{13}{2}} \approx 2.55$ .

$\int_{-2.55}^{2.55} (g(x) - f(x)) dx$  is a good approximation of this area.



### POINT OF VIEW

Inequality Graphing is a very good graphical extension. It helps students to visualize the solution of a system of equations and to find the area of the enclosed region by means of functions without doing a lot of calculations.

### 3.10 Polynomial Root Finder and Simultaneous Equation Solver

**CATEGORY**

Tool



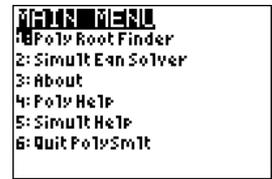
**DESCRIPTION**

This application enables the user to enter quickly coefficients for a polynomial or a system of linear equations and then to identify real and complex roots.

**DIDACTICAL SUGGESTIONS**

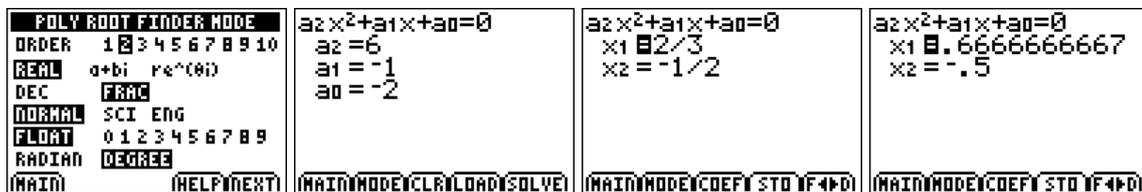
For polynomials it is possible to save the corresponding function in the Y= editor to make a visualisation of the solution, the roots of the saved function. And where possible this application also gives a symbolic representation of the infinite set of solutions of a system of linear equations.

As the name of this application tells, it consists of two parts: Polynomial Root Finder to solve polynomial equations and Simultaneous Equation Solver to solve systems of linear equations. The following example shows how to use this APP.

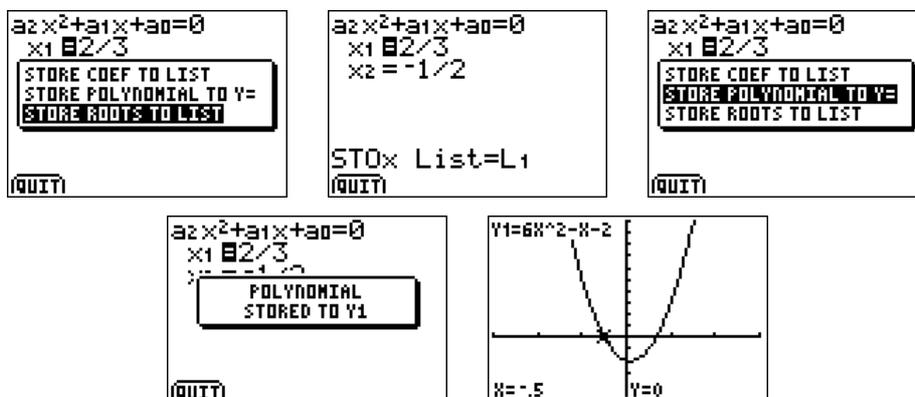


**1: Poly Root Finder**

To solve the equation  $6x^2 - x - 2 = 0$  first select the degree of the equation, press NEXT (= F5 or [GRAPH]), enter the coefficients of the polynomial and press SOLVE (F5 or [GRAPH]). F4/D gives a decimal approximation.



With the option STO the solution can be saved in an empty list and the polynomial in a Y variable to check the results graphically.



It is also possible to calculate complex roots. Therefore the MODE-settings need to be changed into a+bi (FLOAT 2) or re^θi.

<pre> POLY ROOT FINDER MODE ORDER 1 2 3 4 5 6 7 8 9 10 REAL 0+0i re^θ(i) DEC FRAC NORMAL SCI ENG FLOAT 0 1 2 3 4 5 6 7 8 9 RADIAN DEGREE (MAIN) (HELP/NEXT)         </pre>	<pre> a3x^3+...+a1x+a0=0 a3=1 a2=0 a1=0 a0=8 (MAIN/MODE/CLR/LOAD/SOLVE)         </pre>	<pre> a3x^3+...+a1x+a0=0 x1=-2.00 x2=1.00+1.73i x3=1.00-1.73i (MAIN/MODE/COEF/STO IF 4/D)         </pre>	<pre> a3x^3+...+a1x+a0=0 x1=-2.00 (MAIN/MODE/COEF/STO IF 4/D)         </pre>
	MODE: a+bi	MODE: REAL	

## 2: Simultaneous Equation Solver

Solving a system of linear equations is very similar to solving a polynomial equation. First you need to select the number of equations and variables (unknowns) and then enter the coefficients. Via F5 or [GRAPH] you get the solution. Some examples.

$$\bullet \begin{cases} x + y - z = 4 \\ 3x + y - z = 6 \\ x + y - 2z = 4 \end{cases}$$

<pre> SIMULT EQN SOLVER MODE EQUATIONS 2 3 4 5 6 7 8 9 10 UNKNOWN 2 3 4 5 6 7 8 9 10 DEC FRAC NORMAL SCI ENG FLOAT 0 1 2 3 4 5 6 7 8 9 RADIAN DEGREE (MAIN) (HELP/NEXT)         </pre>	<pre> SYSTEM MATRIX (3x4) [1 1 -1 4 ] [3 1 -1 6 ] [1 1 -2 4 ] (3,4)=4 (MAIN/MODE/CLR/LOAD/SOLVE)         </pre>	<pre> SOLUTION x1=1 x2=3 x3=0 (MAIN/MODE/SYSM/STO IF 4/D)         </pre>
--	---	--

With STO the system can be stored in an empty matrix and the solution in an empty list.

$$\bullet \begin{cases} x + 2y + 3z = 4 \\ 5x + 6y + 7z = 8 \end{cases}$$

<pre> SYSTEM MATRIX (2x4) [1 2 3 4 ] [5 6 7 8 ] (2,4)=8 (MAIN/MODE/CLR/LOAD/SOLVE)         </pre>	<pre> SOLUTION SET x1=-2+x3 x2=3-2x3 x3=x3 (MAIN/MODE/SYSM/STO/RREF)         </pre>	<pre> RREF (2x4) [1 0 -1 -2 ] [0 1 2 3 ] (MAIN/BACR/SYSM/STO/RREF)         </pre>
---	---	---

In this case the option F4/D is replaced by the option RREF that generates the Row Reduced Echelon Form.

$$\bullet \begin{cases} 2x - 6y + 14z = 11 \\ x - 3y + 7z = -3 \end{cases}$$

<pre> SYSTEM MATRIX (2x4) [2 -6 14 11 ] [1 -3 7 -3 ] (2,4)=-3 (MAIN/MODE/CLR/LOAD/SOLVE)         </pre>	<pre> SOLUTION NO SOLUTION FOUND (MAIN/MODE/SYSM/RREF)         </pre>	<pre> RREF (2x4) [1 -3 7 0 ] [0 0 0 1 ] (MAIN/BACR/SYSM/STO/RREF)         </pre>
---	---	--

---

### POINT OF VIEW

Polynomial Root Finder and Simultaneous Equation Solver is a very useful tool to solve equations and systems of equations especially in situations in which it is not necessary to do the calculations by hand.

### 3.11 Probability Simulation

CATEGORY: Tool

**DESCRIPTION**

Probability Simulation makes it possible to simulate very easy and fast some probability experiments on the TI-83/84 Plus.

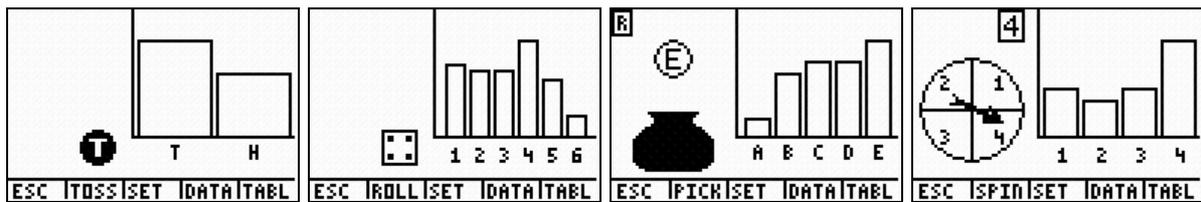
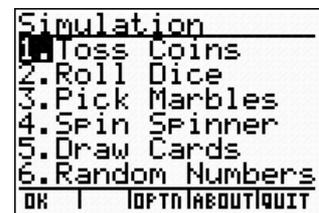


**DIDACTICAL SUGGESTIONS**

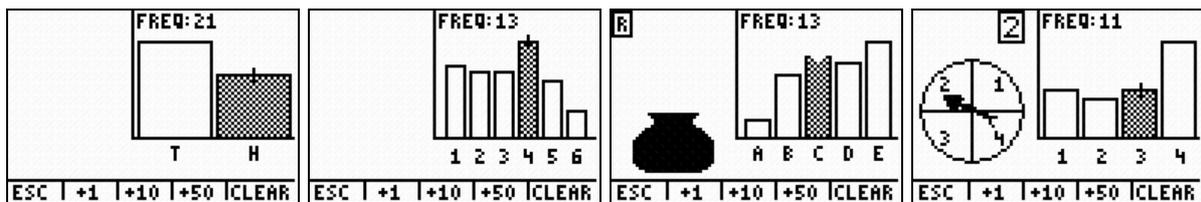
With Probability Simulation you can visualize several properties and laws of the Theory of Probability. The real data obtained out of the experiments can be used to introduce some discrete probability distributions and to do statistic.

Probability Simulation contains the following experiments:

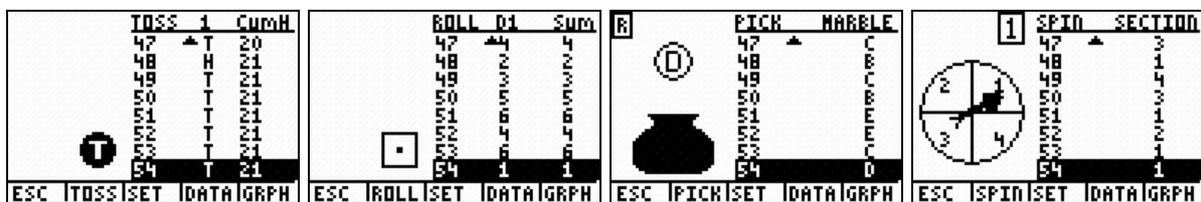
The menus of the experiments are more or less the same. With the following screen shots we will give a short overview of the experiments. While doing the first four experiments the following two menus are available.



And with the ◀ ▶ keys you display the frequency or probability (depending on the settings), a kind of trace mode.



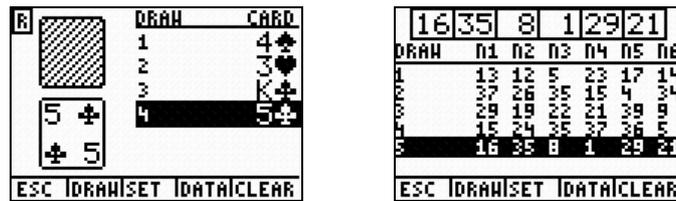
The histogram can be replaced by a table (TABL ↔ GRPH) to see the data of the simulation.



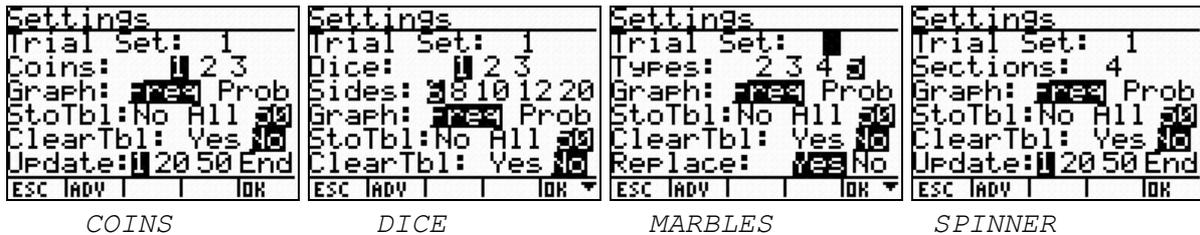
An action (Toss, Roll, Draw or Pick) can be stopped by pushing [ON].

And the data obtained out of the simulation can be stored in lists by the DATA option.

For the other two experiments only one menu is available and also only the table with the data.



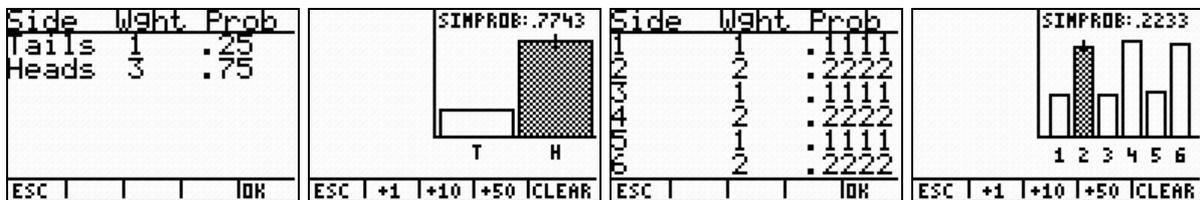
With the SET option you can manually change the settings of the experiments as follow:



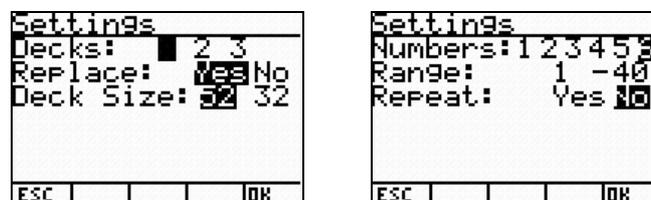
The experiments mentioned above have the following common options (sometimes you need to scroll down) and options to change the amount of coins, dice, marbles and sections:

- Trial** The amount of trials by pushing TOSS, ROLL, DRAW or SPIN
- Graph** The graph selection will display the frequency or the probability
- StoTbl** The viewable amount of trials in the table
- ClearTbl** Set this to yes to clear the data from the SET menu
- Update** The amount of trials before the graph/table will be updated.

With the ADV option on the Settings screen you can rig the experiments. Two examples:



For the experiments Draw Cards and Random Numbers you can only change the specific settings for the cards and the numbers.



### POINT OF VIEW

Probability Simulation can be used in several classroom situations: to introduce probability notion, to show sampling variations, to show the simulation of several experiments linked to chance, ...

## 3.12 Science Tools

### CATEGORY

Reference, Tool

### DESCRIPTION

Science Tools APP enhances the flexibility of the TI calculator, especially for the science classroom.



### DIDACTICAL SUGGESTIONS

The student can do unit conversions, as well as use the graphing and vector tool, handle constants and conversions and deal with significant figures easily. Science Tools APP as a tool as well as a reference can be used as a technological learning environment by dealing with science tasks in the classroom.

Science Tools consists of four tools:

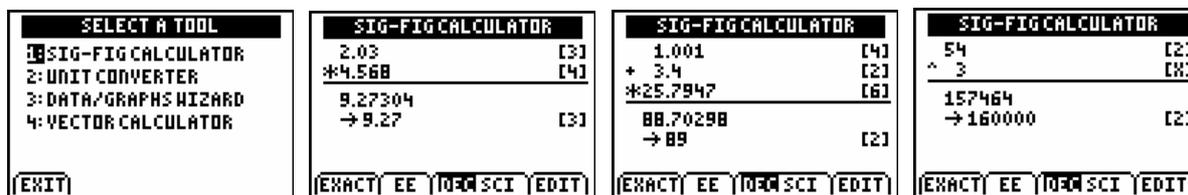
- the Sig-Fig Calculator,
- the unit Converter,
- the Data/Graphs Wizard,
- the Vector Calculator.

#### a. The SIG-FIG CALCULATOR

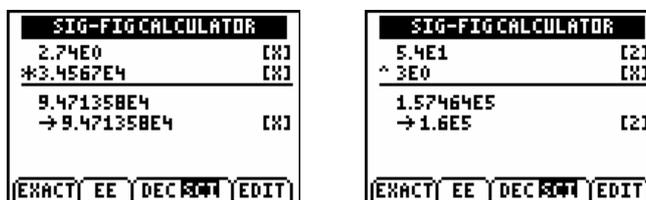
The results of this calculator tool are rounded to the correct number of significant figures.

Any combination of addition, subtraction, multiplication, division and raising a value to a power can be used. Multiplication and division are performed first if no parentheses are used.

If a value is set as EXACT, it will not be subject to rounding.



If you have to calculate  $2.74 \times 34567$  in scientific notation, you have to select SCI.



A conventional calculator doesn't consider precision. It shows results with the maximum number of digits that it can display. The calculated results should be rounded to the correct number of significant figures. The Sig-Fig Calculator tool automatically applies the common rounding rules.

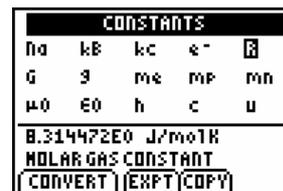
## b. The UNIT CONVERTER

The conversions are displayed at the bottom the screen. If EXPT is selected, the result will be pasted (exported) to the home screen.



The CONSTANT screen displays the following options:

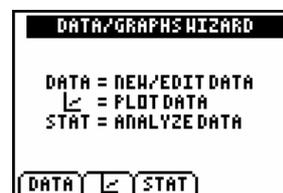
CONVERT returns to the UNIT CONVERTER menu  
 EXPT pastes the constant to the home screen  
 EDIT copies the constant to a conversion screen



## c. The DATA/GRAPH WIZARD

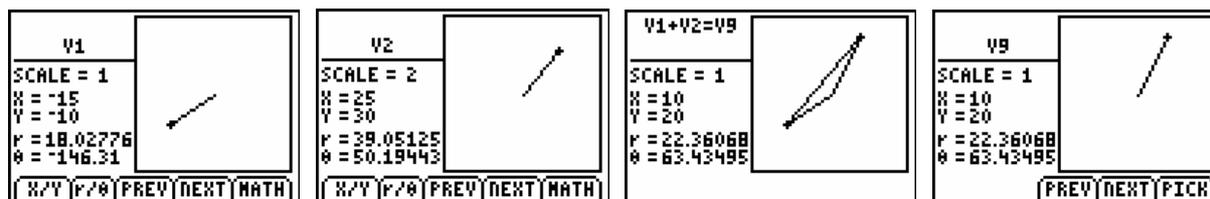
This wizard provides an easy way to perform basic, common tasks associated with:

- Entering, viewing or editing data,
- Viewing and analyzing data graphically,
- Finding a best fit function for the data,
- Performing basic statistical analysis of the data.



## d. The VECTOR CALCULATOR

The vector calculator allows the user to construct vectors and perform basic vector operations. Vectors are graphically displayed on the screen and stored to V1 through V9. After creating vectors the following vector operations can be performed: addition, subtraction, scalar multiplication (dot product) or vector multiplication (cross product).



## POINT OF VIEW

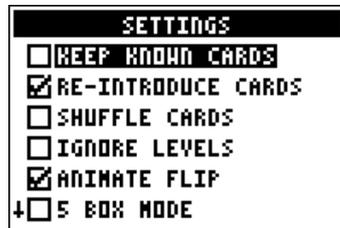
If students want to get used to this diverse application they have to deal intensively with it. It provides a lot of possibilities and calculations, and students have to invest a big amount of time to learn about the right use of the software.

Only with the use of the Sig-Fig Calculator tool the students can display the significant figures of entered values, perform mathematical operations using either decimal or scientific notation and display the results with the correct number of significant figures or convert entries in decimal notation to scientific notation and vice versa. And the Sig-Fig Calculator tool is only one out of the four different tools of the Science Tools APP. Unfortunately this useful and powerful software isn't very widely used, because of the time necessary to get used to the full scale of the tool.



With the arrow keys, you can move on to the next question or move back or switch between different parts of the card like different alternative answers and with the left-right arrows you can move to the previous or next card in the stack.

Through the MAIN MENU (a choice in the MENU window), you can change the settings for the way the cards and the stacks are presented (both the view and the order).



### POINT OF VIEW

For doing quizzes and answering other multiple choice questions, a computer with more pixels, faster, easier to read, more animation, ... may be more user friendly. However, StudyCards is an application that can be used in situations when no other technology is at hand.

### 3.14 Transformation Graphing

**CATEGORY**  
Graphing mode

**DESCRIPTION**  
Transformation allows visualizing dynamically how changes in a function's parameters affect its graph.



**DIDACTICAL SUGGESTIONS**  
This application enables students to discover several properties in terms of a function's parameters: roots, increasing and decreasing, symmetry, period, ... It can also be used for modeling by manipulating coefficients to fit equations to data points.

Transformation Graphing is an application that once it's started it keeps running in the background. It changes the Y= window as follows and adds the **SETTINGS** menu to the **WINDOW** screen.

```
Plot1 Plot2 Plot3
MY1=
MY2=
MY3=
MY4=
MY5=
MY6=
MY7=
```

```
WINDOW SETTINGS
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=3
```

```
WINDOW SETTINGS
>|| > >>
A=.1
B=.2
C=.5
D=1
Step=1
```

To quit Transformation Graphing you need to activate it again in the **APPS** menu and then select 1: Uninstall. Note that it is not possible to run Transformation Graphing and Inequality Graphing (3.9) at the same time.

```
TRANSFORM APPS
1:Uninstall
2:Continue
```

With Transformation Graphing is possible to observe the effects of changing parameter values on the graph without leaving the graph screen. It is only available in the function mode and when it's active it's only possible to plot one function.

Transformation Graphing allows the use of four parameters: A, B, C, and D. All the others act like constants, using the value in the RAM memory.

Transformation Graphing has three play types.

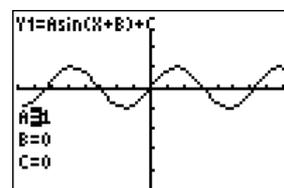
- PLAY-PAUSE (>||) lets you change the parameter and plot the graph.
- PLAY (>) stores a series of changes and shows the corresponding graphs in a continuous slide show.
- PLAY-FAST (>>) stores a series of changes and shows the corresponding graphs in a fast continuous slide show.

We will use the function  $f(x) = A\sin(Bx) + C$  to illustrate how Transformation Graphing works. We will start with the following **WINDOW** settings.

```
Plot1 Plot2 Plot3
MY1: A sin(X+B)+C
MY2=
MY3=
MY4=
MY5=
MY6=
MY7=
```

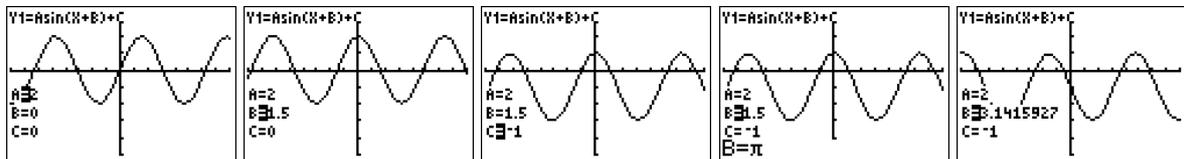
```
WINDOW SETTINGS
Xmin=-8
Xmax=8
Xscl=1
Ymin=-5
Ymax=3.5
Yscl=1
Xres=3
```

```
WINDOW SETTINGS
>|| > >>
A=.1
B=0
C=0
Step=.5
```



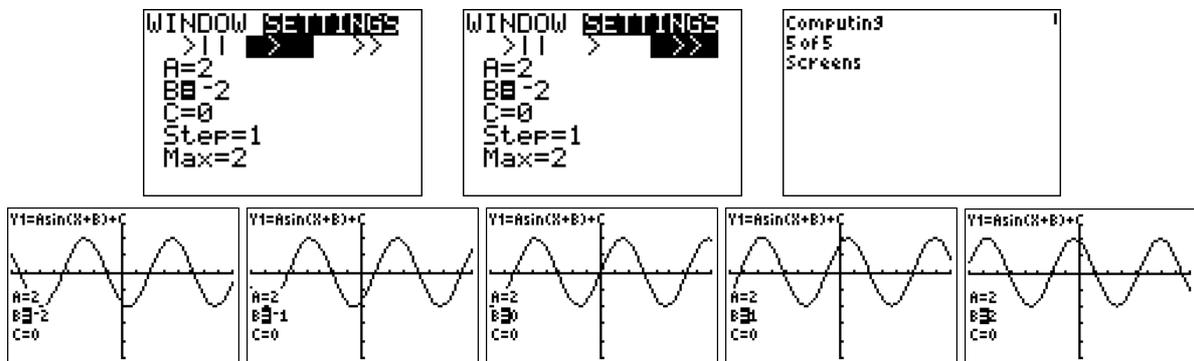
### PLAY-PAUSE (>||)

Press  $\leftarrow$   $\rightarrow$  to change the selected parameter and  $\blacktriangle$   $\blacktriangledown$  to select a different parameter. The graph will change automatically. It is also possible to enter a value manually. Select the parameter, enter the value and press ENTER.



### PLAY (>) and PLAY-FAST (>>)

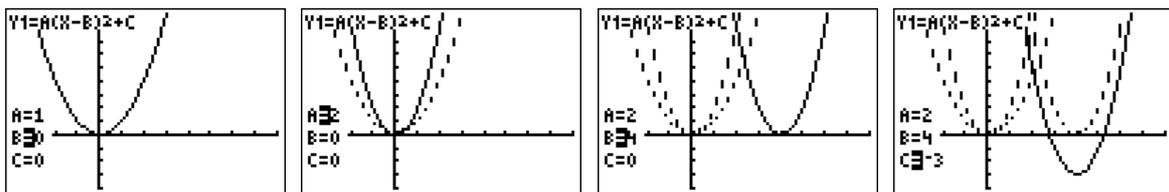
With these options you can define a slide show per parameter. By putting the cursor on the equality sign and pressing enter you can select another parameter. Press [GRAPH] to start generating the screens for the slide show. The definitions below will generate 5 screens for the parameter B: from -2 to 2 in steps of size 1.



Press ENTER to pause the show and again to resume it and press and hold ON to stop.

Transformation Graphing also adds an extra setting to the graph format screen, 2nd[FORMAT]: TrailOff or TrailOn.

With TrailOn you will see better the effect of changing a parameter because the previous graphs stay on the screen in a dotted format.




---

### POINT OF VIEW

Transformation graphing is a dynamical tool with which students can independently discover properties of real functions by means of a graphical approach. It will give the students a better and deeper insight.

## 4 Additional information

### 4.1 Companion software

#### TI Connect™

**Free**



TI Connect is the link software, which makes the connectivity between the TI-83/84 Plus (and other TI graphing calculators) and computer quick and easy: downloading and transferring data, OS updates, installation of applications (APPS).

The most important tools included in TI Connect are:



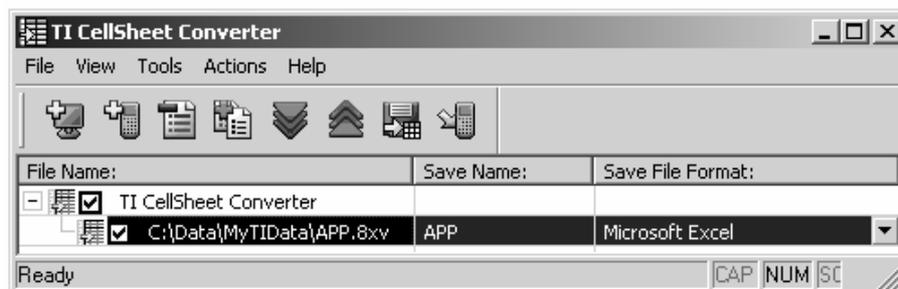
#### TI CellSheet™ Converter

**Free**



The TI CellSheet Converter provides the ability to convert spreadsheet files from one format to another. Supported file formats are the CellSheet APP, Microsoft® Excel, AppleWorks® and tab-delimited text.

You can also use the TI CellSheet Converter to convert spreadsheets from one TI device format into another device format (e.g. TI-84 Plus into Voyage™ 200).



TI Connect must be installed prior to installing the TI CellSheet Converter software for the TI CellSheet Converter software to operate properly.

#### TI StudyCards™ Creator

**Free**



TI StudyCards is the software teachers and students can use to create stacks of electronic flash cards that can be viewed on a TI graphing calculators using the StudyCards APP. Cards can contain text and images. You can create stacks for use in any subject and for each class!

Afterwards you can transfer the stacks of cards from computer to the calculator using the TI StudyCards Creator or TI Connect.

An example of a stack of 14 cards:

Card: 7 of 14

Name of Card: Doubletrouble

Front of Card Screen 1 of 2

\* Down arrow to choices \*  
What is the domain of the function

$$f(x) = \frac{4}{\sqrt{x-2}}$$

Back of Card Screen 1 of 1

This function has two problems - possible zero denominator AND workable square root values. The values  $x \geq 2$  will satisfy the square root, but  $x=2$  will cause a zero denominator.

Number of choices: 4

The correct answer is: 3

## TI-SmartView™ Purchase

## Purchase



TI-SmartView™ software gives you a TI-84 Plus calculator, full functionality including applications, on your computer. This emulator of the TI-84 Plus (Silver Edition) is an easy-to-use demonstration tool that offers many unique instructional capabilities. It is possible to include, only by drag and drop, screens from TI-SmartView in teaching material.

### SOME FEATURES

#### Key Press History

As keys are selected key images can be projected to the class. These keys can also be copied to teaching materials.



#### Scripts

Teachers can prerecord their own key press history into scripts for playback in class. It is easy to create, edit, play, pause and modify speed.

#### CBL 2™ / CBR 2™

You can use the TI-SmartView, connected to a CBL 2 or CBR 2, to collect real world data.

#### View3™ Feature

Teachers can simultaneously project up to three representations of graph, table, equation, list and stat plot window, together with a large version of the current calculator screen.

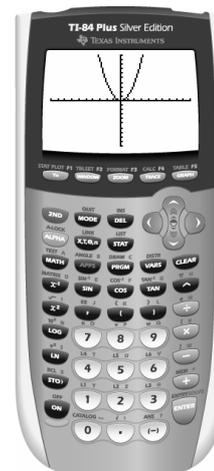
#### Screen Capture

It's easy to create and save multiple screen captures.

#### SmartPad Application

The combination of TI-SmartView 2.0 and the SmartPad APP allows you to use your TI-84 Plus (Silver Edition) as a remote input device.

**TI-SmartView is only available for educators.**



P1ot2 P1ot3

Y1 X^2

Y2 =

Y3 =

Y4 =

Y5 =

Y6 =

Y7 =

Equation

X	Y1
0	0
1	1
4	16
9	81
16	256

X=0

Table

WINDOW

Xmin=-10

Xmax=10

Xscl=1

Ymin=-10

Ymax=10

Yscl=1

Xres=1

## 4.2 How to install and start up applications

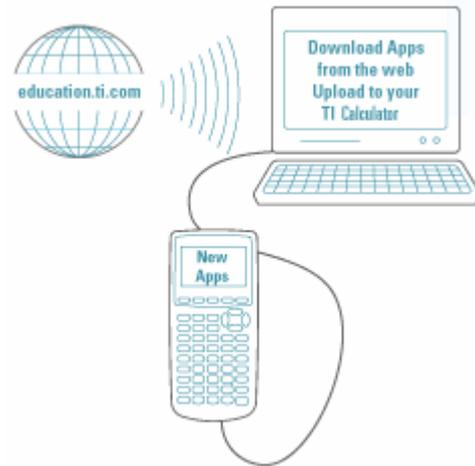
Before you can install applications on your TI-84 Plus Silver Edition be sure the software TI Connect is installed on your computer. If not, you can download it for free from the TI website: [www.education.ti.com](http://www.education.ti.com) → Downloads.

It's also a good idea to check first if you have the latest Operating System installed on your calculator. To check which OS is on your calculator press  $2^{nd}$ [MEM] and select 1:About. You will find the latest OS via the website mentioned above and the installation of a new OS works the same as the installation of an application, see further.

Be sure your calculator is physically connected to your computer with a TI-GRAPH LINK™ cable or a USB Cable.

Now you are ready to install an application.

- A. At [www.education.ti.com](http://www.education.ti.com), go to the page to download APPS and download for free the desired APP to your computer.
- B. Browse for the location of the downloaded APP file.
  - For Windows users drag and drop the APP file onto the TI Connect icon. It is also possible to open the TI DeviceExplorer of TI connect and drag and drop the APP file into this window.

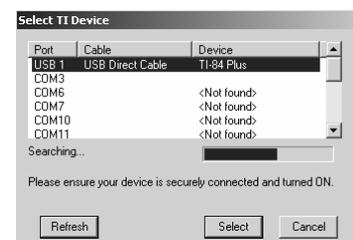


A third possibility is to right click on the APP file and choose Send to TI Device ... From there just do what the wizards will ask you to do.

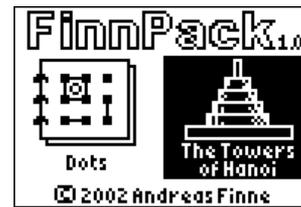


- For Mac OS X users launch the TI Device Explorer of TI Connect and drag and drop the APP file into the device window.

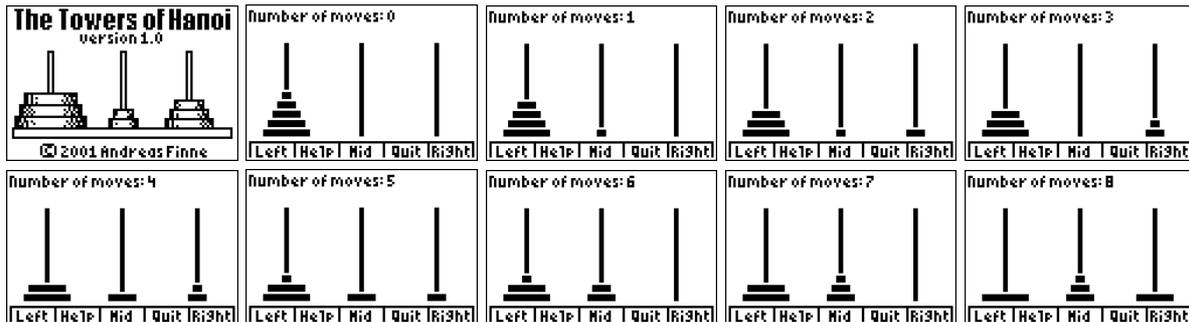
The APP will be installed automatically into the Archive memory. Sometimes you need to select (e.g. when you switch calculators) the calculator before an APPS transfer can start.



Once an APP is installed on the calculator you can use it by pressing the APPS button, selecting the APP and pressing ENTER.



Some screen captures of The Towers of Hanoi.



How many applications you can install on your calculator depends on which calculator you have and which APPS you want to install.

The Archive memory, in which the APPS are installed, of the TI-83 Plus, TI-84 Plus or TI-84 Plus Silver Edition calculator is divided into slots into which you can load APPS. Some APPS take up only one slot and other APPS can take up to four. A summary:

Graphing Calculator	Flash ROM	Applications (±)
TI-83 Plus	160 KB	10
TI-84 Plus	480 KB	30
TI-84 Plus Silver Edition	1.5 MB	94

## 5 About the authors

### ***SERGE ETIENNE***

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has been a math teacher since 1978. He publishes many articles in several fields: mathematics, electronics, computer science and pocket calculators. Since 1990 he has taught classes related to computer science. He gives adult education classes in computer science for the Conservatoire National des Arts et Métiers (CRA-CNAM) (National French Engineering School). At the end of 1990s he got involved in the design and realization of the JADE software for evaluation of the French Education Department (France). After having taken care of teachers' day release courses for a few years, he now teaches mathematics at FESCH middle school in Ajaccio. Personal website: [perso.wanadoo.fr/serge-etienne](http://perso.wanadoo.fr/serge-etienne).

### ***KOEN STULENS***

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is an educational consultant for Texas Instruments, a T<sup>3</sup> instructor in Flanders, Belgium and connected to the Math department of the Hasselt University (Belgium). He is co-author of the books *Statistics with a Graphical Calculator* and *Discover Mathematics with Derive*. He has also developed quite a lot of material (Dutch) for doing mathematics with technology for secondary education available via the websites [www.scholennetwerk.be](http://www.scholennetwerk.be) and [www.t3vlaanderen.be](http://www.t3vlaanderen.be).

### ***HILDEGARD URBAN-WOLDRON***

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has a Ph.D. in Physics/Mathematics from Vienna University, a Diploma from Donau University Krems and an ECHA Diploma from the University of Nijmegen. She teaches Physics and Mathematics in High School classes teaches Physics to future middle school teachers. Additionally, she also lectures in teacher education programs and participates in a Ministry of Education project group for basic education output, education standards and learning and teaching with new media and technology. Her special interests are teaching and learning with new media, didactics of media and physics and special classes for gifted students.

### ***MARTIN VAN REEUWIJK***

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has been an educational developer and researcher in the field of mathematics at the Freudenthal Institute in The Netherlands for over fifteen years. In the 1990s, he coordinated (together with Els Feijs) the development of the Mathematics in Context middle school mathematics curriculum for American students. Since then he has been involved in projects and contacts in the United States and other countries throughout the world. Martin's major interests are (early) school algebra, assessment, and the use of technology in the learning and teaching of mathematics. He directed various projects in which the possibilities of the Internet (like e.g. java applets) were explored, investigated and developed. Since the early nineties Martin has contributed as an editor to the Dutch TI newsletter, and, together with Monica Wijers, he reviews all the Dutch translations of manuals with educational products from TI.

